Shape and topology optimal design problems in electromagnetic casting*

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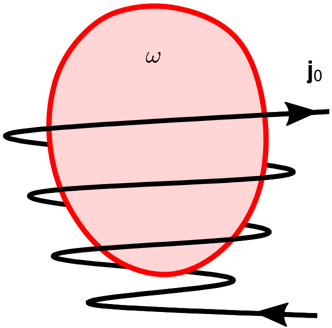
Electromagnetic Casting Problem

$$\Omega = \mathbb{R}^3 \setminus \omega$$



We have certain mass of liquid metal in the domain ω that levitates above an electromagnetic field.

We assume that the electric current frequency is very high. Then the magnetic field penetrates a negligible distance into the liquid metal (skin effect).

Magnetic Field Equations in the 3D problem

The magnetic field satisfies the following equations:

$$\begin{cases} \nabla \times \mathbf{B} & = \mu_0 \mathbf{j}_0 & \text{in } \Omega \\ \nabla \cdot \mathbf{B} & = 0 & \text{in } \Omega \\ \mathbf{B} \cdot n & = 0 & \text{on } \Gamma \\ \|\mathbf{B}\| & = O(\|x\|^{-1}) & \text{as } \|x\| \to \infty \end{cases} \tag{1}$$

ω: domain occupied by the liquid metal.
Γ: boundary of ω.
$\Omega = \mathbb{R}^3 \setminus \omega$ is the exterior of the liquid metal.
$\mathbf{j}_0$ is the electric current density.
$\mathbf{B}$ is the magnetic flux density field.
μ_0: magnetic permeability of the vacuum.
n: outward-pointing unit normal vector of Γ.

Equilibrium and volume constraints

In addition to the field equations, the liquid metal shape ω in equilibrium satisfy:

$$\frac{1}{2\mu_0}\|\mathbf{B}\|^2 + \gamma\mathcal{H} + \rho g x_3 = p_0 \quad \text{constant on } \Gamma \tag{1}$$

and the volume constraint:

$$\int_\omega \mathrm{d}v = V_0 \tag{2}$$

γ: surface tension of the liquid metal.
$\mathcal{H}$: mean curvature of Γ.
ρg: specific weight of the liquid.
p_0: pressure in the liquid metal at height $x_3 = 0$.

- In the forward problem we have to find the domain ω such that the solution **B** to the exterior problem satisfies also the equilibrium equation.

2D Electromagnetic Casting Problem

In the two-dimensional problem we assume $\mathbf{j}_0 = (0, 0, j_0)$ and $\mathbf{B} = (B_1, B_2, 0)$. In addition, j_0 is compactly supported and the total current zero: $\int_\Omega j_0 \, \mathrm{d}a = 0$.

Then, there exists the magnetic flux funtion $\varphi : \Omega \to \mathbb{R}$ such that $\mathbf{B} = (\frac{\partial \varphi}{\partial x_2}, -\frac{\partial \varphi}{\partial x_1}, 0)$ where φ is solution to the state equations:

$$\begin{cases} -\Delta\varphi = \mu_0 j_0 & \text{in } \Omega \\ \varphi = 0 & \text{on } \Gamma \\ \varphi(x) = O(1) & \text{as } \|x\| \to \infty \end{cases} \tag{1}$$

The equilibrium in terms of the magnetic flux φ becomes:

$$\frac{1}{2\mu_0} \left| \frac{\partial \varphi}{\partial n} \right|^2 + \gamma \mathcal{C} = p_0 \quad \text{constant on } \Gamma \tag{2}$$

$\mathcal{C}$: Curvature of Γ.

The area constraint is

$$\int_\omega \mathrm{d}a = A_0 \tag{3}$$

More information about the forward problem

For more information about the forward problem see:

- Mestel, A.J. (1982), *Magnetic levitation of liquid metals*, Journal of Fluid Mechanics, Vol. 117 pp. 27–43.
- Sneyd, A.D. and Moffatt, H.K. (1982), *Fluid dynamical aspects of the levitation-melting process*, Journal of Fluid Mechanics, Vol. 117 pp. 45–70.
- Etay, J. and Garnier M. (1982), *Sur le contrôle électro-magnétique des surfaces métalliques liquides et ses applications.* J.M.T.A. Vol 1 No. 6 pp. 911–925.
- Brancher, J.-P., Etay, J. and Séro-Guillaume, O.E. (1983), *Formage d'une lame métallique liquide.*, Calculs et expériences. Journal de Mecanique Theorique et Appliquee, Vol. 2 No. 6, pp. 977-989.

Solution of the forward problem (2D case):

Analitical solutions (2D problem using complex variables):

- Shercliff, J.A. (1981), *Magnetic shaping of molten metal columns*, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, Vol. 375 No. 1763, pp. 455-473.

Numerical solutions for the 2D problem based on series expansions:

- Brancher, J.-P., Etay, J. and Séro-Guillaume, O.E. (1983), *Formage d'une lame métallique liquide. Calculs et expériences*. Journal de Mecanique Theorique et Appliquee, Vol. 2 No. 6, pp. 977-989.

Numerical solutions based on optimization for the 2D problem:

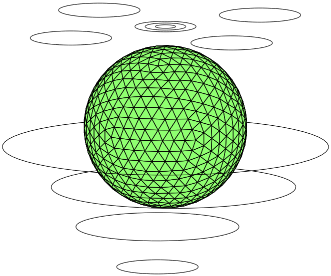
- Pierre, M. and Roche, J.-R. (1991), *Computation of free surfaces in the electromagnetic shaping of liquid metals by optimization algorithms*, European Journal of Mechanics - B/Fluids, Vol. 10 No. 5, pp. 489–500.

Solution of the forward problem:

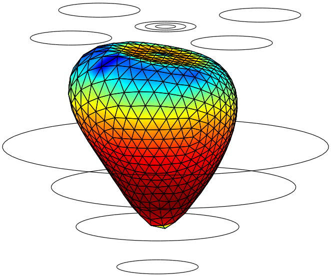
Numerical solutions based on optimization for the 3D problem:

- Séro-Guillaume, O.E., Zouaoui, D., Bernardin, D. and Brancher, J.P. (1992), *The shape of a magnetic liquid drop*, Journal of Fluid Mechanics, Vol. 241, pp. 215–232.
- Pierre, M. and Roche, J.R. (1993), *Numerical simulation of tridimensional electromagnetic shaping of liquid metals*, Numerische Mathematik, Vol. 65 No. 2, pp. 203–217.
- Novruzi, A. and Roche, J.R. (1995), *Second Order Derivatives, Newton Method, Application to Shape Optimization*, Research Report RR-2555, INRIA, Nancy.
- Eppler, K. and Harbrecht, H. (2005), *Fast wavelet BEM for 3d electromagnetic shaping*, Applied Numerical Mathematics, Vol. 54 Nos 3-4, pp. 537–554.
- Canelas, A., Pereira, A., Roche, J.R. and Brancher, J.P. (2019), *Solution of the equilibrium problem in electromagnetic casting considering a solid inclusion in the melt*, Mathematics and Computers in Simulation, Vol. 160, pp. 126–137.

Solution of the forward problem



Initial guess



final equilibrium shape

The colors of the right hand side figure indicate the intensity of the magnetic pressure.

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Inverse Problem

- In the inverse problem certain target shape ω^* of boundary Γ^* is given, and we have to find the configuration electric currents $\mathbf{j}_0$ such that ω^* is in equilibrium.
- For instance, in the 2D problem we have to find j_0 such that the solution φ of the state equations

$$\begin{cases} -\Delta\varphi &= \mu_0 j_0 \quad \text{in } \Omega^* \\ \varphi &= 0 \quad \text{on } \Gamma^* \\ \varphi(x) &= O(1) \quad \text{as } \|x\| \to \infty \end{cases} \tag{1}$$

satisfies also the equilibrium equation:

$$\frac{1}{2\mu_0}\left|\frac{\partial\varphi}{\partial n}\right|^2 + \gamma\mathcal{C} = p_0 \quad \text{constant on } \Gamma^* \tag{2}$$

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Inverse Problem

Papers about existence solutions and characterization of shapable shapes:

- Henrot, A. and Pierre, M. (1989), *Un problème inverse en formage des métaux liquides*, RAIRO Modélisation mathématique et analyse numérique, Vol. 23 No. 1, pp. 155–177.
- Henrot, A. and Pierre, M. (1991), *About existence of equilibria in electromagnetic casting*, Quarterly of Applied Mathematics, Vol. 49 No. 3, pp. 563–575.
- Felici, T.P. (1991), *The inverse problem in the theory of electromagnetic shaping*, PhD Thesis, University of Cambridge.
- Felici, T.P. and Brancher, J.-P. (1991), *The inverse shaping problem*, European Journal of Mechanics - B/Fluids, Vol. 10 No. 5, pp. 501–512.
- Pierre, M. and Rouy, E. (1996), *A tridimensional inverse shaping problem*, Communications in Partial Differential Equations, Vol. 21 Nos 7-8, pp. 1279–1305.

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Location of point currents in the 2D Case

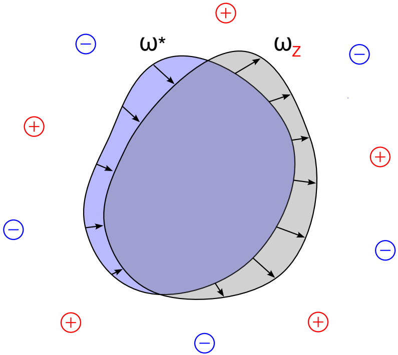
Works made in collaboration with José Herskovits:

- A. CANELAS; J. R. ROCHE; J. HERSKOVITS; The inverse electromagnetic shaping problem. Structural and Multidisciplinary Optimization, v. 38 n. 4, p. 389-403, 2009.
- A. CANELAS; J. R. ROCHE; J. HERSKOVITS; Inductor shape optimization for electromagnetic casting. Structural and Multidisciplinary Optimization, v. 39 n. 6, p. 589-606, 2009.

Formulations of the inverse problem

First formulation:

Let $\mathbf{Z}$ be a vector field on Γ^*, and $\omega_{\mathbf{Z}} = (\mathbf{I}_d + \mathbf{Z})(\omega^*)$. Then:



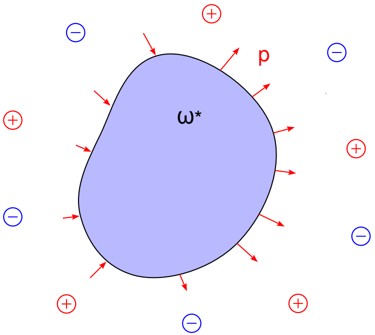
$$\left\{ \begin{array}{l} \min\limits_{j_0,\mathbf{Z}} \|\mathbf{Z}\|^2_{L^2(\Gamma^*)}, \\ \text{subject to:} \\ \omega_{\mathbf{Z}} \text{ is in equilibrium for } j_0. \end{array} \right. \quad (1)$$

Formulations of the inverse problem

Second formulation:

In a second approach we introduce an artificial surface force p on Γ^*:

$$\frac{1}{2\mu_0}\left|\frac{\partial\varphi}{\partial n}\right|^2 + \gamma\mathcal{C} + p = p_0 \quad \text{constant on } \Gamma^*$$



$$\left\{\begin{array}{l} \min\limits_{j_0,p} \|p\|^2_{L^2(\Gamma^*)}, \\ \text{subject to:} \\ \omega^* \text{ is in equilibrium for } j_0 \text{ and } p. \end{array}\right. \tag{2}$$

Optimization algorithm

- In the discrete version of the optimization problem the optimization variables are the points x_i such that:

$$j_0 = \sum_{i=1}^{m} \alpha_i \delta_{x_i} \,, \quad \alpha_i = \pm 1 \,. \tag{1}$$

  and the nodal values of the displacement **Z** or the artificial pressure p.
- We use the optimization algorithm FAIPA of José Herskovits:

  Mathematical programming models and algorithms for engineering design optimization, J. Herskovits, P. Mappa, E. Goulart, C.M. Mota Soares. Comput. Methods Appl. Mech. Engrg. 194 3244–3268, 2005.

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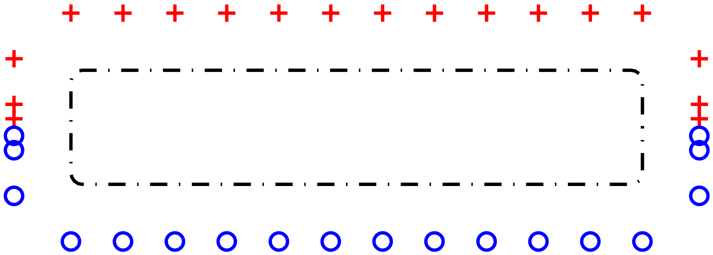
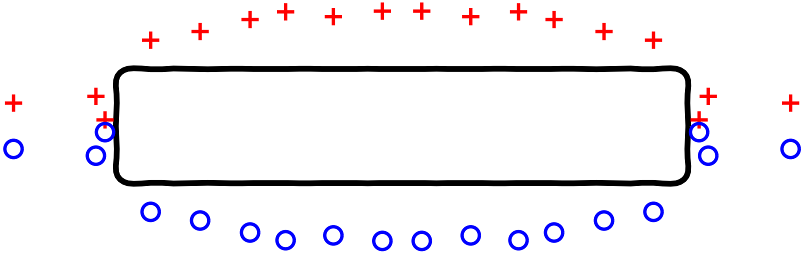
Example

Target shape and initial configuration of electric currents:



Example

Solution obtained:



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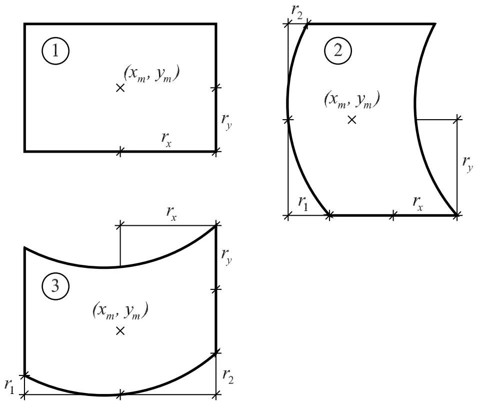
Shape optimization of the inductors

- In this case, instead of optimizing the location of some point currents, we optimize a few parameters that define the shape of the inductors:



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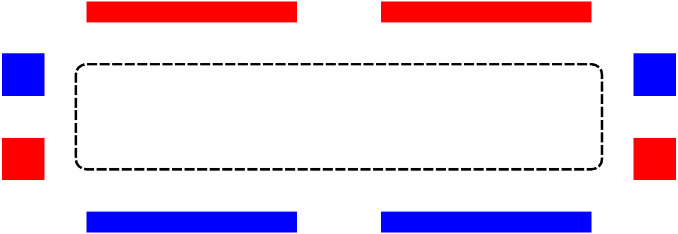
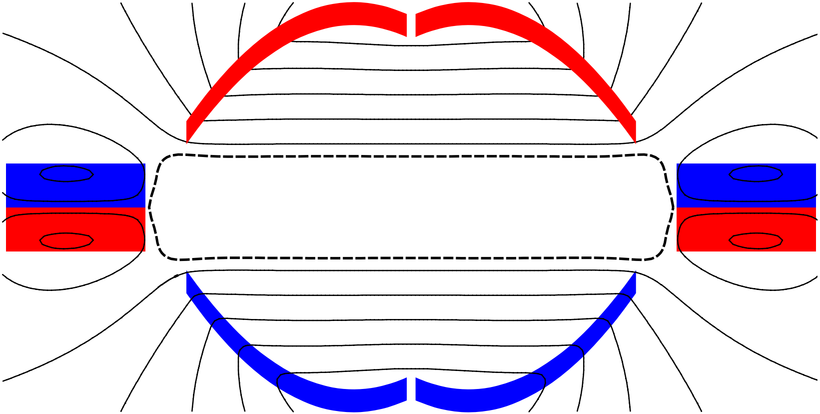
Shape optimization of the inductors

Target shape and initial configuration of inductors:



Example

Solution obtained:



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Topology optimization in the 2D Case

Works made in collaboration with Antonio A. Novotny:

- A. CANELAS; A. A. NOVOTNY; J. R. ROCHE; A new method for inverse electromagnetic casting problems based on the topological derivative. Journal of Computational Physics, v. 230 n. 9, p. 3570-3588, 2011.
- A. CANELAS; A. A. NOVOTNY; J. R. ROCHE; Topology design of inductors in electromagnetic casting using level-sets and second order topological derivatives. Structural and Multidisciplinary Optimization, v. 50 n. 6, p. 419-435, 2014.

In this case we will look for:

a domain Ω^+ with electric current density $+I$ and
a domain Ω^- with electric current density $-I$

by means of a topology optimization formulation.

Topology optimization in the 2D Case

Consider the equilibrium equation:

$$\frac{1}{2\mu_0}\left|\frac{\partial\varphi}{\partial n}\right|^2 + \gamma\mathcal{C} = p_0 \quad \text{constant on } \Gamma^* \tag{1}$$

Then:

$$\frac{\partial\varphi}{\partial n} = \varkappa\sqrt{2\mu_0(p_0 - \gamma\mathcal{C})} \quad \text{with} \quad \varkappa = \pm 1\,. \tag{2}$$

Therefore $p_0 \geq \max_{\Gamma^*}\gamma\mathcal{C}$. It can be shown that $\frac{\partial\varphi}{\partial n}$ is zero at some points, therefore:

$$p_0 = \max_{\Gamma^*}\gamma\mathcal{C}\,. \tag{3}$$

Calling $\bar{p} = \sqrt{2\mu_0(p_0 - \gamma\mathcal{C})}$ we have:

$$\frac{\partial\varphi}{\partial n} = \varkappa\bar{p} \quad \text{on } \Gamma^*\,, \tag{4}$$

with the sign changes of $\varkappa$ at the zeros of $(p_0 - \gamma\mathcal{C})$, that is at the points of maximum curvature.

Topology optimization in the 2D Case

We can formulate the problem as: find j_0 such that the system

$$\begin{cases} -\Delta\varphi &= \mu_0 j_0 & \text{in } \Omega^* \,, \\ \varphi &= 0 & \text{on } \Gamma^* \,, \\ \dfrac{\partial\varphi}{\partial n} &= \varkappa\,\bar{p} & \text{on } \Gamma^* \,, \\ \varphi(x) &= O(1) & \text{as } \|x\| \to \infty \,, \end{cases} \tag{1}$$

has a solution $\varphi \in W_0^1(\Omega^*)$.

We known that for a simply connected ω^*, with Γ^* an analitic Jordan curve, and $p_0 = \max_{\Gamma^*} \gamma\mathcal{C}$ (the maximum must be attained at an even number of points) then (Henrot and Pierre 1989):

(i) there exists a solution for the inverse problem,
(ii) the solution is not unique for j_0.

Topology optimization in the 2D Case

We proposed to minimize a Kohn-Vogelius-like functional:

$$J(\phi) = \frac{1}{2}\|\phi\|^2_{L^2(\Gamma^*)} = \frac{1}{2}\int_{\Gamma^*} |\phi|^2 \,\mathrm{d}s\,, \tag{1}$$

where the auxiliary function ϕ depends implicitly on j_0 through the solution of the problem:

$$\begin{cases} -\Delta\phi &= \mu_0 j_0 & \text{in } \Omega^*\,, \\ \dfrac{\partial\phi}{\partial n} &= \varkappa\,\bar{p} - d(j_0) & \text{on } \Gamma^*\,, \\ \phi(x) &= O(1) & \text{as } \|x\| \to \infty\,, \\ \int_{\Gamma^*}\phi\,\mathrm{d}s &= 0\,. \end{cases} \tag{2}$$

Where there is a compatibility condition:

$$\int_{\Gamma^*} \varkappa\,\bar{p}\,\mathrm{d}s = 0\,, \quad \text{and } d(j_0) = |\Gamma^*|^{-1}\int_{\Gamma^*} \mu_0 j_0\,\mathrm{d}s \tag{3}$$

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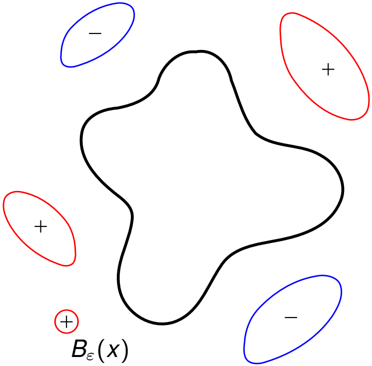
Computing the topological expansion

We have the target shape and certain currents, and introduce a circular perturbation:



Computing the topological expansion

The perturbed electric current density is:

$$j_\varepsilon = j_0 + \alpha I \chi_{B_\varepsilon(\hat{x})} \,. \tag{1}$$

Then, for the functional:

$$\psi(\varepsilon) = J(\phi_\varepsilon) = \frac{1}{2} \int_{\Gamma^*} |\phi_\varepsilon|^2 \, \mathrm{d}s \,, \tag{2}$$

Theorem: the topological expansion of ψ is:

$$\psi(\varepsilon) = \psi(0) + (\pi\varepsilon^2) D_T^1 \psi + (\pi^2 \varepsilon^4) D_T^2 \psi \,, \tag{3}$$

Despite it has only three terms, the expansion is exact.

Computing the topological expansion

The topological derivatives are:

$$D_T^1\psi(\hat{x}) = \alpha I \int_{\Gamma^*} \phi f \,\mathrm{d}s\,, \tag{1}$$

$$D_T^2\psi(\hat{x}) = \frac{1}{2} I^2 \int_{\Gamma^*} f^2 \,\mathrm{d}s\,. \tag{2}$$

Where the function f is the solution to:

$$\begin{cases} -\Delta f & = \delta_{\hat{x}} & \text{in } \Omega^*\,, \\ \dfrac{\partial f}{\partial n} & = -\|\Gamma^*\|^{-1} & \text{on } \Gamma^*\,, \\ f(x) & = O(1) & \text{as } \|x\| \to \infty\,, \\ \displaystyle\int_{\Gamma^*} f\,\mathrm{d}s & = 0\,. \end{cases} \tag{3}$$

$D_T^1\psi(\hat{x})$ can be computed very efficiently by the adjoint method. $D_T^2\psi(\hat{x})$ cannot be computed by the adjoint method, but it does not depend on the current configuration of electric currents.

Contents

Heuristic optimization algorithm

The optimization algorithm is as follows:

- **Step 0:** Divide the domain of interest in a mesh of cells, and initialize the integer variable N. Compute the initial value of the functional J.
- **Step 1:** Compute the topological derivative.
- **Step 2:** For the cells with current density $\alpha_p \leq 0$ set $\alpha_p := \alpha_p + 1$ to the N cells with the minimum topological derivative. For the cells with current density $\alpha \geq 0$ set $\alpha_p := \alpha_p - 1$ to the N cells with the maximum topological derivative.
- **Step 3:** Compute the updated value of the functional J, if it does not decrease then set $N := N/2$ and return to Step 2.
- **Step 4:** Stop or return to Step 1 according to an optimality criterion.

Contents

Level-set procedure

The region Ω^+ of positive electric current density:

$$\Omega^+ = \{x \in \Omega,\ \psi^+(x) < 0\}\,, \tag{1}$$

Analogously, the region Ω^- of negative electric current density:

$$\Omega^- = \{x \in \Omega,\ \psi^-(x) < 0\}\,. \tag{2}$$

The region of electric current zero is:

$$\Omega^0 = \Omega - (\Omega^+ \cup \Omega^-)\,. \tag{3}$$

In addition, the 'expected variation' EV of the objective functional produced by the introduction of a small ball of electric current of sign α:

$$EV(\hat{x}, \varepsilon, \alpha) = (\pi\varepsilon^2) D_T^1 \psi(\hat{x}) + (\pi^2 \varepsilon^4) D_T^2 \psi(\hat{x})\,. \tag{4}$$

Sufficient optimality conditions

Sufficient optimality conditions (for the perturbations considered here):

$$EV(\hat{x},\varepsilon,\alpha) > 0\,, \quad \forall \hat{x} \in \Omega^+\,, \text{ and } \alpha = -1\,, \tag{1}$$

$$EV(\hat{x},\varepsilon,\alpha) > 0\,, \quad \forall \hat{x} \in \Omega^-\,, \text{ and } \alpha = +1\,, \tag{2}$$

$$EV(\hat{x},\varepsilon,\alpha) > 0\,, \quad \forall \hat{x} \in \Omega^0\,, \text{ and } \alpha = \pm 1\,. \tag{3}$$

Hence, we define:

$$g^+(x) = \begin{cases} -EV(\hat{x},\varepsilon,-1) & \text{if } \hat{x} \in \Omega^+\,, \\ EV(\hat{x},\varepsilon,+1) & \text{if } \hat{x} \in \Omega^0 \cup \Omega^-\,, \end{cases} \tag{4}$$

$$g^-(x) = \begin{cases} -EV(\hat{x},\varepsilon,+1) & \text{if } \hat{x} \in \Omega^-\,, \\ EV(\hat{x},\varepsilon,-1) & \text{if } \hat{x} \in \Omega^0 \cup \Omega^+\,. \end{cases} \tag{5}$$

The algorithm updates ψ^+ and ψ^- iteratively to obtain:

$$\exists\, \tau^+ > 0 \quad \text{s.t.} \quad g^+ = \tau^+ \psi^+\,, \tag{6}$$

$$\exists\, \tau^- > 0 \quad \text{s.t.} \quad g^- = \tau^- \psi^-\,, \tag{7}$$

Topology optimization procedure

The algorithm is based on the proposed by Amstutz and Andrä (2006).

We can start from $\psi_0^+ = \psi_0^- = 1/\|1\|_{L_2(\Omega)}$ to obtain $\Omega_0 = \Omega$.

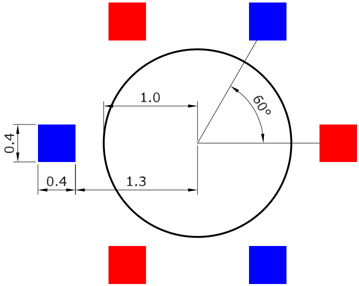
- **Step 1:** Compute the topological derivatives and the expected variations $EV(\hat{x}, \varepsilon, +1)$ and $EV(\hat{x}, \varepsilon, -1)$.
- **Step 2:** Compute g_n^+ and g_n^-.
- **Step 3:** find the real value $t_n \in [0, 1]$ such that the following sets minimize the objective functional:

$$\left.\begin{aligned} \psi_{n+1}^+ &= (1 - t_n)\psi_n^+ + t_n g_n^+ \\ \psi_{n+1}^- &= (1 - t_n)\psi_n^- + t_n g_n^- \end{aligned}\right\} , \tag{1}$$

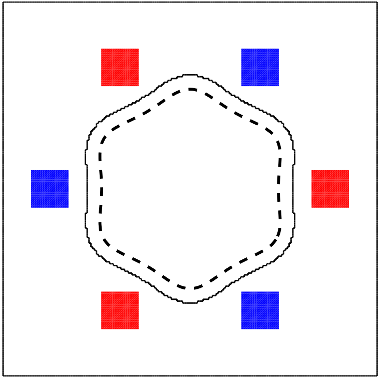
- **Step 4:** Stop or increase n and return to Step 1 according to an optimality criterion.

Contents

Example 1



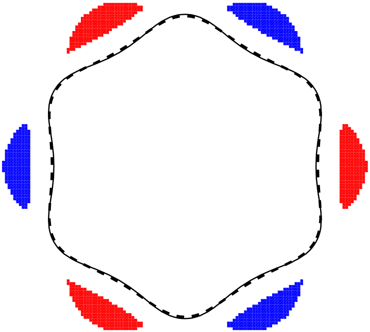
(a)



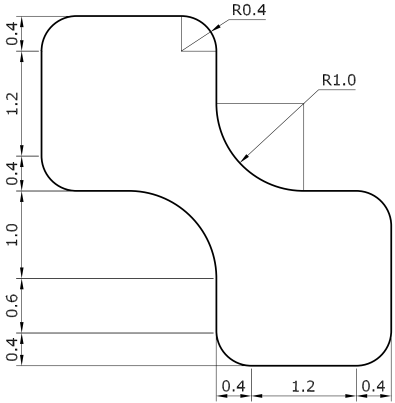
(b)

Example 1

Solution obtained:

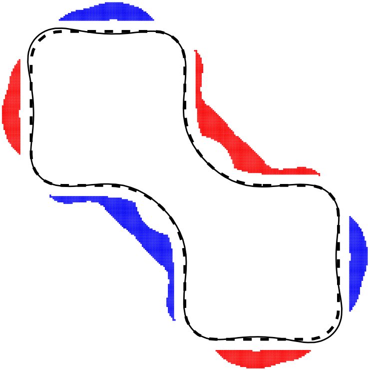


Example 2

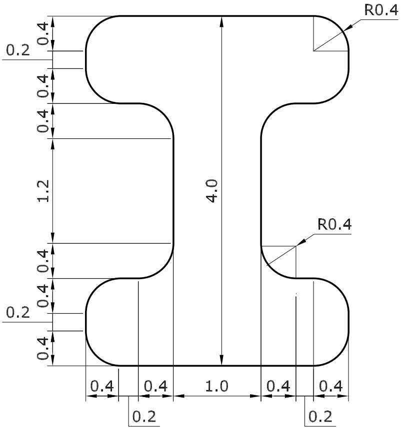


Example 2

Solution obtained:

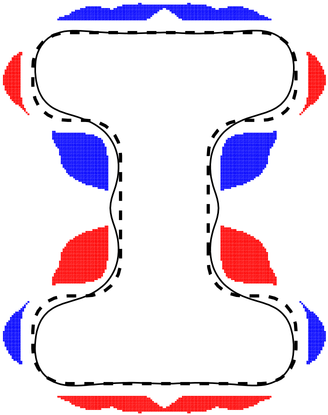


Example 3



Example 3

Solution obtained:



Contents

Topology optimization via quadratic programming

- A. CANELAS; J. R. ROCHE; Topology optimization in electromagnetic casting via quadratic programming. Inverse Problems in Science and Engineering, v. 22 n. 3, p. 419-435, 2014.
- A. CANELAS; J. R. ROCHE; Solution to a three-dimensional axisymmetric inverse electromagnetic casting problem. Inverse Problems in Science and Engineering, v. 27 n. 10, p. 1451-1467, 2019.

In this case we minimize the same functional $J(\phi)$, which is quadratic in the variable ϕ, subject to linear equality and inequality constraints.

The approach is more expensive from the computational point of view, but there are very efficient quadratic programming algorithms that ensure convergence to a global minimum.

Topology optimization via quadratic programming

The optimization formulation is:

$$
\begin{aligned}
&\min_{j_0,\phi} \quad J(\phi) + \lambda\psi(j_0)\,,\\
&\text{s.t.} \quad \left\{\begin{array}{rcll} -\Delta\phi &=& \mu_0 j_0 & \text{in } \Omega^*\,,\\ \dfrac{\partial\phi}{\partial n} &=& \kappa\,\bar{p} & \text{on } \Gamma^*\,,\\ \phi(x) &=& O(1) & \text{as } \|x\|\to\infty\,, \end{array}\right.\\
&\qquad\int_{\Omega^*} j_0\,dx = 0\,,\\
&\qquad |j_0| \le I\,,
\end{aligned}
\tag{1}
$$

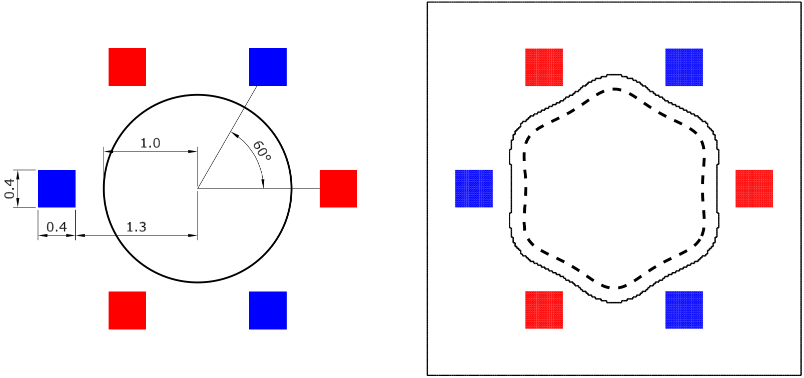
where λ is a penalty parameter and $\psi(j_0)$ is the total current defined as:

$$
\psi(j_0) = \int_{\Omega^*} |j_0|\,dx\,. \tag{2}
$$

To obtain a quadratic programming formulation we use a change of variables: $j_0 = j_0^+ - j_0^-$, $|j_0| = j_0^+ + j_0^-$, where $j_0^+ \ge 0$ and $j_0^- \ge 0$.

Contents

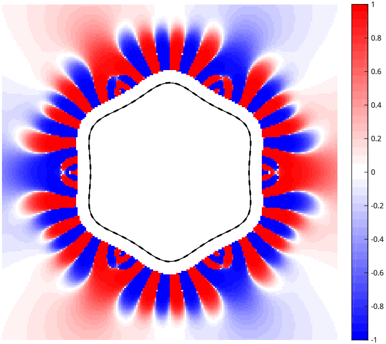
Example 1



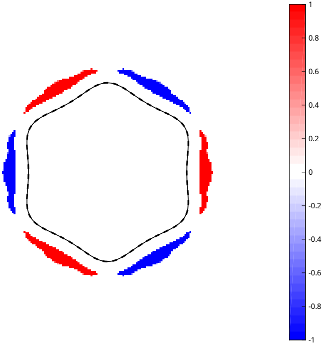
(a) (b)

Example 1

Solution obtained:

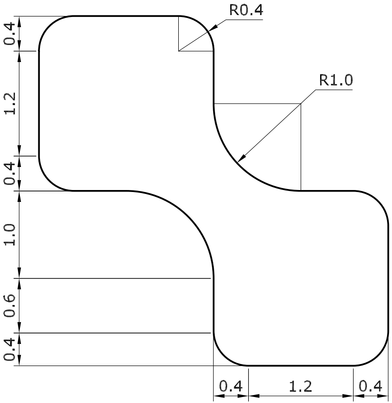


$\lambda = 0$



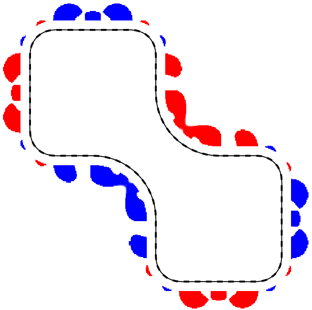
$\lambda = 10^{-7}$

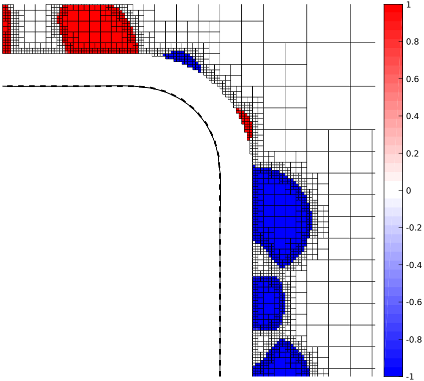
Example 2



Example 2

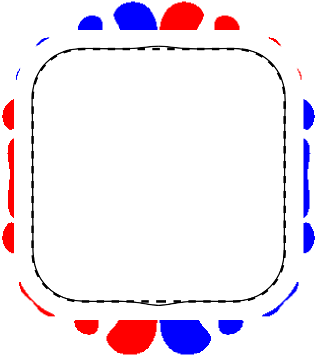
Solution obtained:

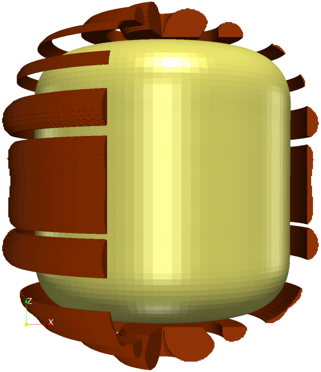




Example 3: 3D axisymmetric target shape

Solution obtained:





Contents

Conclusions

- We have addressed the inverse electromagnetic casting problem, and proposed different formulations to obtain numerical approximate solutions.
- The nonlinear formulations with point currents require “good” initial guesses.
- The solutions obtained through shape optimization have the same disadvantage.
- The topology optimization techniques start from scratch, and find better solutions.
- The topology optimization algorithms that use the topological derivative are heuristic but efficient.
- The quadratic programming formulations are less efficient but ensure convergence to a global optimum.

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