Inductor design in electromagnetic casting

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Outline

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Introduction

- The Electromagnetic Casting (EMC) is an important technology in the metallurgical industry.
- It is based on the repulsive forces that an alternating electromagnetic field produces on the surface of diamagnetic liquid metals.
- It makes use of the electromagnetic field for contactless heating, shaping and control of solidification of hot melts.

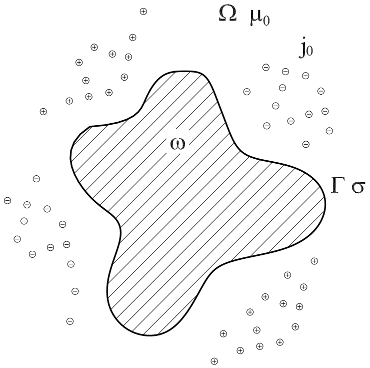
Aim:

- Define a numerical method based on nonlinear optimization to design suitable inductors.

Outline

Electromagnetic Casting Problem

- The EMC problem studied here concerns the case of a vertical column of liquid metal falling down into an electromagnetic field created by vertical inductors.
- We consider an alternating electric current of high frequency, so that the magnetic field penetrate a negligible distance into the liquid metal.



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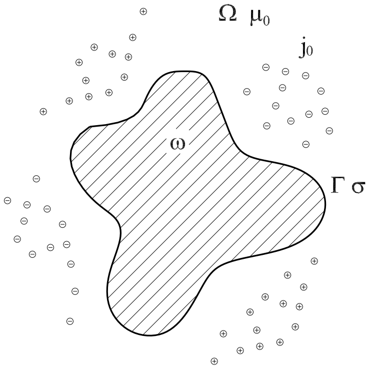


Photograph of a liquid metal drop

Magnetic field Equations

Michel Pierre, Jean R. Roche (1991)

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}_0 \quad \text{in } \Omega \quad (1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad \text{in } \Omega \quad (2)$$

$$\mathbf{B} \cdot \nu = 0 \quad \text{on } \Gamma \quad (3)$$

$$\|\mathbf{B}\| = O(\|x\|^{-1}) \quad \text{as } \|x\| \to \infty \text{ in } \Omega \quad (4)$$

ω: closed domain occupied by the liquid metal.
Γ: boundary of ω.
$\Omega = \mathbb{R} \setminus \omega$ exterior of the liquid metal.
$\mathbf{j}_0 = (0, 0, j_0)$ electric current density vector.
$\mathbf{B} = (B_1, B_2, 0)$ magnetic field.
μ_0: magnetic permeability of the vacuum.
ν: outward unit normal vector.

Equilibrium and constraints

We also have the equilibrium equation on the boundary:

$$\frac{1}{2\mu_0}\|\mathbf{B}\|^2 + \sigma\mathcal{C} = \bar{p} \quad \text{constant in } \Gamma \tag{5}$$

The volume constraint:

$$\int_{\omega} \mathrm{d}\Omega = S_0 \tag{6}$$

and we assume that the current j_0 has a compact support and satisfies:

$$\int_{\Omega} j_0 \,\mathrm{d}\Omega = 0 \tag{7}$$

Magnetic Flux function

Given (1)-(7), we can prove that exists the Magnetic Flux function $\varphi : \Omega \to \mathbb{R}$ such that:

$$\mathbf{B} = \left(\frac{\partial \varphi}{\partial x_2}, -\frac{\partial \varphi}{\partial x_1}, 0 \right) \tag{8}$$

Thus, φ is the solution of the state equations:

$$-\Delta\varphi = \mu_0 j_0 \quad \text{in } \Omega \tag{9}$$

$$\varphi = 0 \quad \text{in } \Gamma \tag{10}$$

$$\varphi(x) = O(1) \quad \text{as } \|x\| \to \infty \tag{11}$$

The equilibrium equation on the boundary takes the form:

$$\frac{1}{2\mu_0} \|\nabla\varphi\|^2 + \sigma\mathcal{C} = \bar{p} \quad \text{constant on } \Gamma \tag{12}$$

Variational formulation of the EMC Problem

The variational formulation of the Direct EMC Problem consists in finding ω as a stationary point of the Total Energy:

$$E(\omega) = -\frac{1}{2\mu_0}\int_{\Omega} \|\nabla\varphi_\omega\|^2 \, d\Omega + \sigma \int_{\Gamma} d\Gamma \,, \tag{13}$$

subject to the area constraint:

$$S(\omega) = \int_{\omega} d\Omega = S_0 \,. \tag{14}$$

where φ_ω satisfies:

$$-\Delta\varphi_\omega = \mu_0 j_0 \quad \text{in } \Omega \tag{15}$$

$$\varphi_\omega = 0 \quad \text{on } \Gamma \tag{16}$$

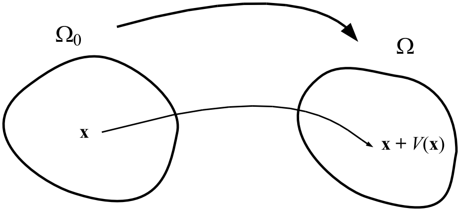
$$\varphi_\omega(x) = O(1) \quad \text{as } \|x\| \to \infty \tag{17}$$

Differentiation w.r.t. the shape

To characterize the stationary points, i.e., the equilibrium configurations, we use the concept of shape derivatives:

Given a reference domain Ω_0, we consider the transformations:

$$T = Id + V\,, \quad \text{with} \quad V \in W^{1,\infty}(\mathbb{R}^2, \mathbb{R}^2)\,, \quad \|V\|_{W^{1,\infty}(\mathbb{R}^2,\mathbb{R}^2)} < 1\,, \tag{18}$$



Domain transformed by the vector field V.

Equilibrium Condition

The lagrangian function is:

$$L(\omega,\bar{p}) = E(\omega) - \bar{p}(S(\omega) - S_0)\,, \tag{19}$$

where $\bar{p}$ is the Lagrangian multiplier associated to the area constraint.

The stationary points satisfy:

$$L'(\omega,\bar{p})(V) = 0\,, \quad \forall\, V \in W^{1,\infty}(\mathbb{R}^2,\mathbb{R}^2)\,. \tag{20}$$

Theorem:
The equilibrium condition of the Variational Problem is:

$$\int_\Gamma \left(\frac{1}{2\mu_0} \|\nabla\varphi\|^2 + \sigma\mathcal{C} - \bar{p} \right) (V\cdot\nu)\, d\Gamma = 0 \quad \forall\, V \text{ in } W^{1,\infty}(\mathbb{R}^2,\mathbb{R}^2)\,. \tag{21}$$

Solution of the state equation

For the state equation we use the particular solution φ_1:

$$\varphi_1(x) = -\frac{\mu_0}{2\pi} \int_{\mathbb{R}^2} \ln \|x - y\| j_0(y)\, d\Omega \tag{22}$$

Then, the magnetic flux φ can be computed as:

$$\varphi(x) = v(x) + \varphi_1(x) \tag{23}$$

where the function v is the solution of:

$$-\Delta v(x) = 0 \quad \text{in } \Omega \tag{24}$$

$$v(x) = -\varphi_1(x) \quad \text{on } \Gamma \tag{25}$$

$$v(x) = O(1) \quad \text{as } \|x\| \to \infty \tag{26}$$

Solution of the homogeneous equation

An integral representation of v is given by:

$$v(x) = -\frac{1}{2\pi}\int_\Gamma q(y)\ln\|x-y\|\,d\Gamma + c \tag{27}$$

where c is the value at the infinity, and $q \in H^{-1/2}(\Gamma)$ must satisfy:

$$\int_\Gamma q(x)\,d\Gamma = 0 \tag{28}$$

the boundary conditions on Γ are imposed weakly:

$$-\frac{1}{2\pi}\int_\Gamma g(x)\int_\Gamma q(y)\ln\|x-y\|\,d\Gamma\,d\Gamma + c\int_\Gamma g(x)\,d\Gamma$$
$$= -\int_\Gamma \varphi_1(x)g(x)\,d\Gamma \quad \forall g \in H^{-1/2}(\Gamma) \tag{29}$$

Summary of the equations of the Direct Problem

1) State equations:

$$-\frac{1}{2\pi}\int_\Gamma g(x)\int_\Gamma q(y)\ln\|x-y\|\,\mathrm{d}\Gamma\,\mathrm{d}\Gamma + + c\int_\Gamma g(x)\,\mathrm{d}\Gamma = -\int_\Gamma \varphi_1(x)g(x)\,\mathrm{d}\Gamma \quad \forall g\in H^{-1/2}(\Gamma) \tag{30}$$

$$\int_\Gamma q(x)\,\mathrm{d}\Gamma = 0 \tag{31}$$

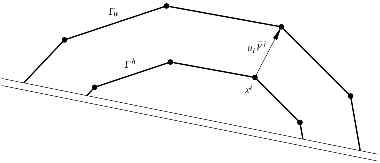
2) Equality constraint regarding the area of ω:

$$\int_\omega \mathrm{d}\Omega = S_0 \tag{32}$$

3) Equilibrium equation on the boundary:

$$\int_\Gamma \left(\frac{1}{2\mu_0}\|\nabla\varphi\|^2+\sigma\mathcal{C}-\bar{p}\right)(V\cdot\nu)\,\mathrm{d}\Gamma = 0 \quad \forall V \text{ in } W^{1,\infty}(\mathbb{R}^n,\mathbb{R}^n) \tag{33}$$

Discretization of the boundary



The parametric transformation $T_{\mathbf{u}}$ is defined as:

$$T_{\mathbf{u}}(x) = x + V_{\mathbf{u}}(x) \tag{34}$$

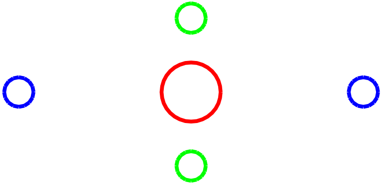
$$V_{\mathbf{u}}(x) = \sum_{i=1}^{n} u_i V^i(x) \tag{35}$$

where $\mathbf{u}^T = (u_1, \ldots, u_n) \in \mathbb{R}^n$ is the vector of shape parameters. Then, the updated boundary $\Gamma_{\mathbf{u}}$ is given by:

$$\Gamma_{\mathbf{u}} = \left\{ X \mid X = x + V_{\mathbf{u}}(x);\ x \in \Gamma^h \right\} \tag{36}$$

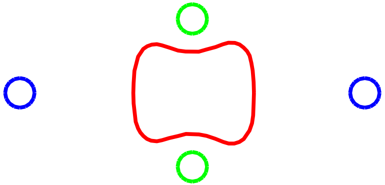
Example

Iter 0



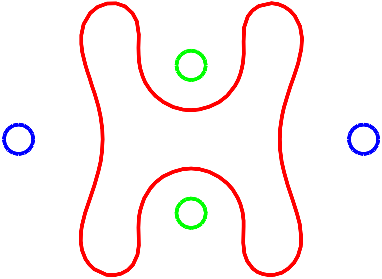
Example

Iter 4



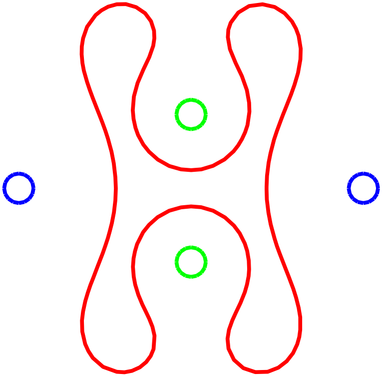
Example

Iter 10

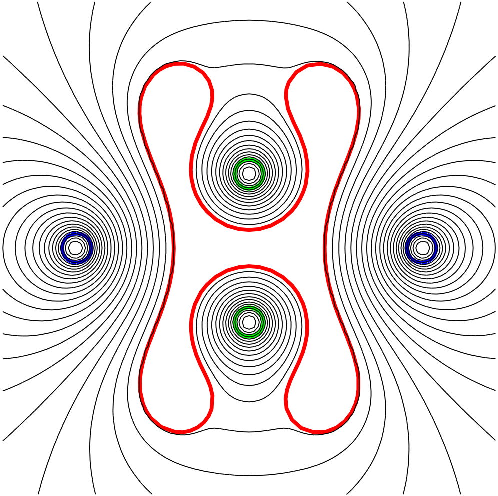


Example

Iter 29



Example



Outline

Inverse Problem

- In the Inverse Problem we have to find the configuration of inductors to have ω approximately equal to a target shape ω^*.
- We propose to formulate the Inverse Problem as a nonlinear optimization problem:
  - Minimize a "distance" between the equilibrium shape and the target one.
- For this purpose we consider the shape optimization of the inductors.

Inverse Problem

The proposed formulation considers a deformation of the target shape ω^* defined by the following mapping:

$$T_Z(x) = (Id + Z)(x), \quad \forall x \in \mathbb{R}^2 \tag{37}$$

where Z is smooth and has a compact support in $\mathbb{R}^2$. Defining:

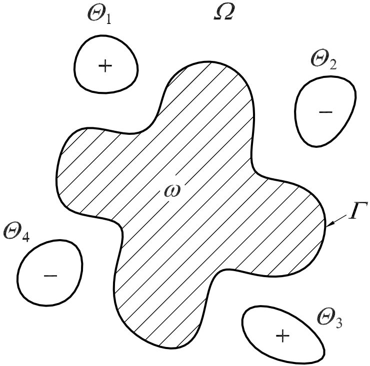
$$\omega_Z = T_Z(\omega^*) \tag{38}$$

$$\Gamma_Z = T_Z(\Gamma^*) \tag{39}$$

The Inverse Problem is formulated as:

$$\begin{aligned} &\min_{j_0, Z} \|Z\|^2_{L^2(\Gamma^*)} \\ &\text{subject to:} \\ &\omega_Z \text{ is equilibrated under } j_0 \end{aligned} \tag{40}$$

Shape optimization of the inductors



We assume that the current density is uniform on some domains Θ_p. This hypothesis is valid for inductors composed of multiple insulated strands, twisted or woven together (Litz-Wire).

Shape optimization of the inductors

The electric current density j_0 is:

$$j_0 = I \sum_{i=1}^{n_c} \alpha_i \chi_{\Theta_i} \, , \tag{41}$$

In this case the particular solution φ_1 is:

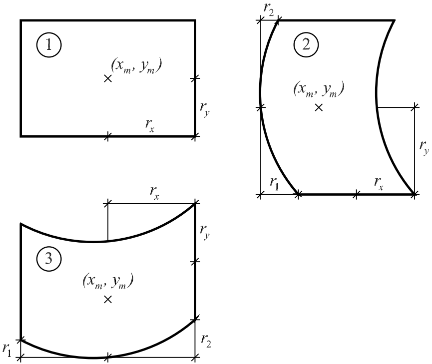
$$\varphi_1(x) = -\frac{\mu_0 I}{2\pi} \sum_{i=1}^{n_c} \alpha_i \int_{\Theta_i} \ln \|x - y\| \, \mathrm{d}\Omega_y \, . \tag{42}$$

Let $w : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$ be $w(x, y) = (1/4)(1 - 2 \ln \|x - y\|)(x - y)$.
Then, φ_1 can be computed as:

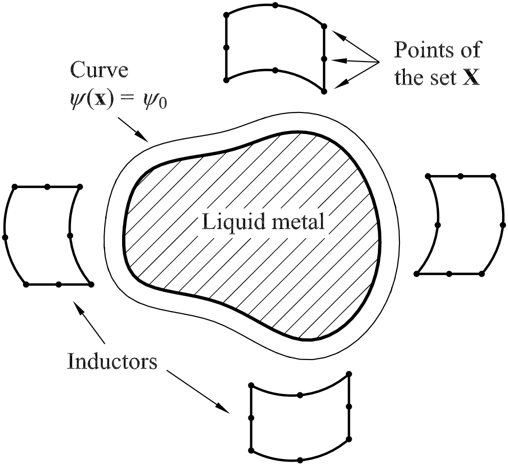
$$\varphi_1(x) = -\frac{\mu_0 I}{2\pi} \sum_{i=1}^{n_c} \alpha_i \int_{\Gamma_i} w(x, y) \cdot \nu \, \mathrm{d}\Gamma_y \, . \tag{43}$$

Inductors

We consider the parametric shapes that are shown by the figure:



Geometric Constraints



Geometric Constraints

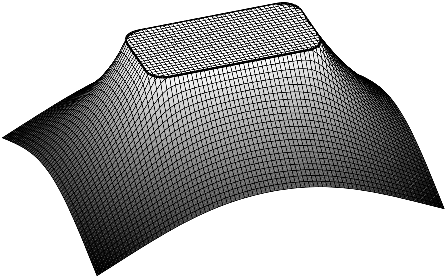
The proposed function ψ is defined as the solution of:

$$\begin{aligned} \Delta\psi(x) &= 0 && \text{in } \Omega^*\,, \\ \psi(x) &= 0 && \text{on } \Gamma^*\,, \\ \int_{\Gamma^*} \nabla\psi(x)\cdot\nu\, d\Gamma &= -1\,. \end{aligned} \tag{44}$$

Choosing a real negative value ψ_0, the geometric constraints are:

$$\psi(x_j(\mathbf{u}_c)) - \psi_0 \leq 0 \quad \forall j\,. \tag{45}$$

Geometric Constraints



Function ψ

Outline

Nonlinear Optimization Problem

To solve the discretized Inverse Problem we use the FDIPA algorithm. Given the following nonlinear optimization problem:

- find $\mathbf{x} \in \mathbb{R}^n$ such that:

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) \\ \text{subject to:} & \mathbf{g}(\mathbf{x}) \geq 0 \\ & \mathbf{h}(\mathbf{x}) = 0 \end{array} \tag{46}$$

- Feasible region:

$$\Omega = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{g}(\mathbf{x}) \geq 0,\ \mathbf{h}(\mathbf{x}) = 0\} \tag{47}$$

- $\mathbf{x}^*$ is a local minimum if exist $\mathcal{N}(\mathbf{x}^*)$ such that:

$$f(\mathbf{x}) \geq f(\mathbf{x}^*)\,, \quad \forall \mathbf{x} \in \Omega \cap \mathcal{N}(\mathbf{x}^*) \tag{48}$$

Karush-Kuhn-Tucker

- we assume the LICQ: for all $\mathbf{x} \in \Omega$:

$$\{\nabla \mathbf{g}_i(\mathbf{x}) \mid \mathbf{g}_i(\mathbf{x}) = 0, \nabla \mathbf{h}_i(\mathbf{x}) \mid i \in \{1, .., p\}\} \quad \text{is l.i.}$$

- **Karush-Kuhn-Tucker theorem**:

$$\nabla f(\mathbf{x}^*) - \sum_{i=1}^{m} \lambda_i \nabla \mathbf{g}_i(\mathbf{x}^*) - \sum_{i=1}^{p} \mu_i \nabla \mathbf{h}_i(\mathbf{x}^*) = 0 \qquad (49)$$

$$\mathbf{g}_i(\mathbf{x}^*)\lambda_i = 0 \qquad (50)$$

$$\mathbf{h}(\mathbf{x}^*) = 0 \qquad (51)$$

$$\mathbf{g}(\mathbf{x}^*) \geq 0 \qquad (52)$$

$$\boldsymbol{\lambda} \geq 0 \qquad (53)$$

Outline

FDIPA Algorithm

Herskovits (1998).

- FDIPA generates a sequence $\{\mathbf{x}_k\}_{k\in\mathbb{N}} \subset \Delta$:

$$\Delta = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{g}(\mathbf{x}) \geq 0,\ \mathbf{h}(\mathbf{x}) \geq 0\} \tag{54}$$

- The value of the potential function $\phi_{\mathbf{c}}(\mathbf{x})$ is reduced at each iteration:

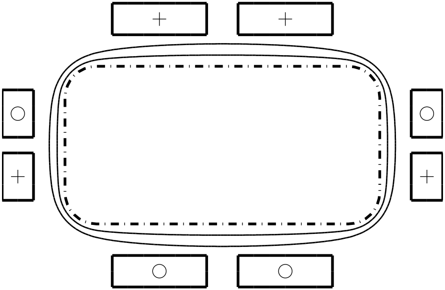
$$\phi_{\mathbf{c}}(\mathbf{x}) = f(\mathbf{x}) + \sum_{i=1}^{p} \mathbf{c}_i |\mathbf{h}_i(\mathbf{x})| \tag{55}$$

- **THEOREM**: FDIPA has global convergence to KKT points of the optimization problem (Herskovits, 1998).

Outline

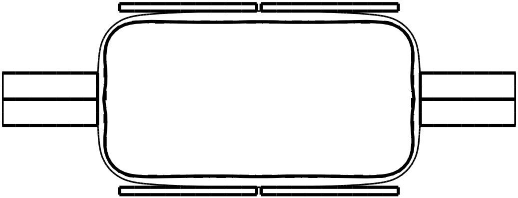
Examples

Initial Configuration



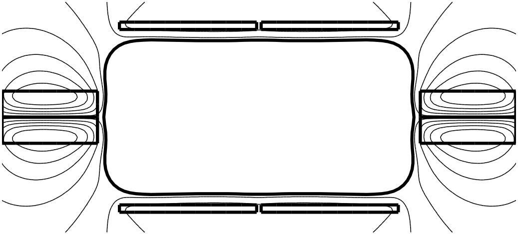
Examples

Result



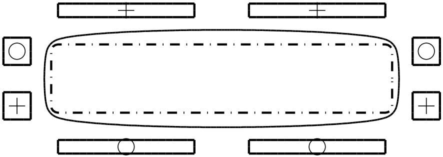
Examples

Flux lines of the Magnetic field



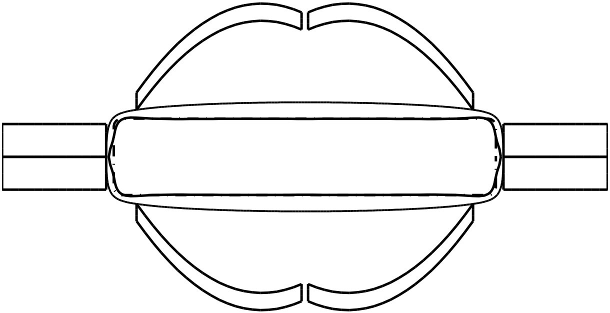
Examples

Initial Configuration



Examples

Result



Examples

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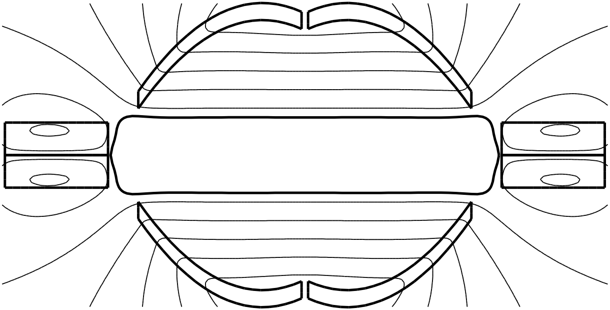

Outline

Conclusions

- A numerical method for designing suitable inductors for Electromagnetic Casting was proposed.
- We also have shown how to consider geometric constraints that prevent the inductors from penetrating the liquid metal.
- Some presented examples show that the proposed technique is effective to design suitable inductors.

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Conclusions

Further works:

- Consider a solution method for finding good initial configurations by means of topology optimization techniques.
- Consider the case of low frequencies of the electric current.

Thank You !

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