The Learning of Recursive Algorithms from a Psychogenetic Perspective

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Outline

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Context

- Teaching functional programming and discrete mathematics
- Mathematics education and computer science education
- Theoretical framework
- Research about the learning of the concept of recursion
- An example of pedagogical application of the theroy

Theoretical framework

Jean Piaget's Genetic Epistemology

This theory provides satisfactory and detailed explanations regarding the central questions of the process of constructing knowledge, founded in wide and deep empirical works.

- the general law of cognition
  - the interaction between the subject and the environment
  - the grasp of consciousness
  - from instrumental to conceptual knowledge

Applying principles from the epistemological theory to the learning-teaching of the concept of recursion.

The concept of recursion

The learning of the concept of recursive algorithms as school subject

- primitive and course-of-values recursion
- design and formalize recursive solutions to problems

Educators community shows consensus on

- fundamental concept
- the existence of serious difficulties in teaching-learning the concept
- due to the lack of day-to-day situations which can help in understanding recursion

The concept of recursion

Alternatively, the motivation of our approach arises from:

> the observation of the existence of day-to-day situations in which people use recursive methods to solve problems or perform tasks

In the sense that in people's descriptions of their methods it appears:

- *I do the same*
- *now I know how to do*
- *remaining parts of the object (of the same type)*

The relation between the structure and the method characterizes recursion.

Examples: searching, sorting, games, arithmetic operations, algorithms used in high school mathematics courses.

The starting premise

Extending results from *La Formation des Raisonnements Recurrentiels* by Piaget and collaborators.

The premise

> the source of thinking which permits the design recursive solutions to problems lies in elemental forms of reasoning arising from students' comprehension of *the relations between the elements to which their actions are applied* when attempting to solve instances of problems.

Starting point to the development of the empirical work.

Empirical work

The students

- 13 students from High School/University and several orientations
- minimal influence of previous instruction
- no evaluation

The tools

- individual interviews
- collective classes
- mathematics (and programming languages)

Empirical work

General goal

To learn about

- the transformation by the students of instrumental knowledge into conceptual one
- the construction by the students of the correspondence between the conceptual knowledge and its formalization in mathematics

Specific goal

To learn about the influence of the knowledge about the inductive structure on the design of the recursive method

The example

Counting the a's of the words of a language given by

- ab is a word
- if * is a word then a*a is a word
- if * is a word then b*b is a word
- closure

ab, aaba, babb, aaabaa, baabaab, aaaabaaa, baaabaab, ...

Where do the obstacles lie?

The structure

Level 1: the relation between the initial element "ab" and any word $(ab \rightarrow w_n\ )$

- Write some words of the language.
- Can you determine of these sequences which are words of the language and which are not?
- With which rule did you form this word?

With the first, the second and the third rule

The structure

Level 2: the relation between any word and the next one $(w_{n-1} \rightarrow w_n)$

- In this word, which would the little symbol be?
- A student said that rules 2 and 3 say that a word is formed from a word formed before. Do you agree?
- Which is the previous word of this word then?

The students confuse the previous word with the initial word "ab"

The method

Level 3b: the relation between any word and the previous one $(w_n \rightarrow w_{n-1})$

- How many a's does this word have?
- And the little symbol?
- In any word, if we know how many a's the little symbol has, can we determine how many a's the word has?
- How? Write it down please.

The clauses of the inductive cases are easily defined

The method

Level 3a: the relation between any word and the initial one ($w_n \rightarrow ab$)

- Determine the a's of abaababa by using only what you have written.

R: We count the ones from the little symbol.
Q: Very good, come on.
R: One, two, ...
Q: NO, NO, using what you wrote.
R: I can't.
Q: Why not? Count the a's from the little symbol using what you wrote.

In the following lies the essence of recursion.

The method

R: Oh! (Thinking) Yes, again, we do it with *, yes, yes, yes, I understand now, in this case, if I cross this out, I get it this word, then it will be + 2 ...

a(baabab)a

Q: Write it, please.
R: Sure, the ones of the little symbol + 2 ... I mark the little symbol again and it will have the same number because it is rule 3, I mark the little symbol again and now the rule I applied that was rule 2. So, it will be 2 +, and then I cannot decompose "ab" any more.

Now, she discovers that her algorithm is incomplete.

The method

Q: But how many a's does it have?
R: One.
Q: So, what do we do now?
R: $+\ 1$.

- What is missing in what you wrote so as to be able to use it until the end?

She completes her algorithm with the base case

The greatest obstacle is the passage from $ab \rightarrow w_n$ to $w_{n-1} \rightarrow w_n$

The formalization

Synthesis of students' formulations

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nr-of-a's-of any word:
     if the rule is 2, it is 2 + nr-of-a's-of *
     if the rule is 3, it is nr-of-a's-of *
     if the rule is 1, it is 1
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A mathematical function definition is derived:

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nr-of-as : Set-of-words -> N
nr-of-as (word) =
    if word = ab then 1
     else if word = a * a then 2 + nr-of-as (*)
           else if word = b * b then nr-of-as (*)
```

Conclusions

From the analysis of the responses of the students evidence about the relevance of the construction of the concept of the structure as the source of forms of thought that facilitate reasoning on recursive methods arises.

- the students have difficulties in understanding the implication in the structure. The main obstacle is the passage from $ab \rightarrow w_n$ to $w_{n-1} \rightarrow w_n$
- the students quite easily succeeded in constructing the inverse relationships $w_n \rightarrow w_{n-1}$ and $w_n \rightarrow ab$. Consequently they correctly formulated the clauses for the inductive cases and the base case of the algorithm respectively.

Further work

- the psychogenesis of
  - inductive structures
  - proof by induction
- the stage of implementing
- teaching-learning recursion
- other learning theories