Dinámica de una red neuronal cooperativa.

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RESUMEN

Con el título “Dinámica de una red neuronal cooperativa” se hizo una presentación de standt y póster en el evento *Ingeniería de Muestra de la* Facultad de Ingeniería de la Univ. de la República, del 19 al 21 de octubre de 2017.

En este documento se incluye el póster presentado en el evento.

Dynamics of cooperative neuronal networks depending on their associated graphs

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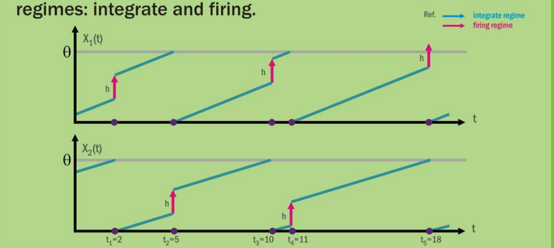
Basic definitions

A *neuronal network* N describes the dynamics of a set of N neurons that interact.
The *configuration of the network* is given by a vector
$X(t)=(X_1(t),...,X_N(t))$.

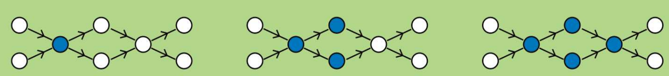
The dynamic evolution of the system will be given by two regimes: integrate and firing.



Let θ_i indicate the *threshold* of the neuron *i*. When $X_i(t)$ reaches θ_i, the neuron fires, and interacts with a neuron *j* by h_{ij}, when $h_{ij} > 0$. We will also take into consideration the *parameters* θ, *h* and *n* such that $\theta \geq \theta_i$, $h \leq h_{ij}$ for all *i*, *j*, and $nh \geq \theta \geq (n-1)h$.

Neuronal network's *associated graph*: It is a graph with the set of neurons as vertices and an edge from *i* to *j* if $h_{ij} > 0$.

At the instant t_m of the m[th] fire, we define *J(m)* as the set of neurons that fire in that instant. If $\#J(m)=N$, we say that the network reaches *grand coalition*.



It is worth noting that there are neurons that reach their threshold naturally, and others that reach it due to the interaction with the network.

Strongly Connected Graph

We say that a graph is *strongly connected* if given two vertices, there exists an oriented path from one to the other.

Theorem : In a neuronal network with strongly connected associated graph, every neuron fires, actually, infinitely many times.

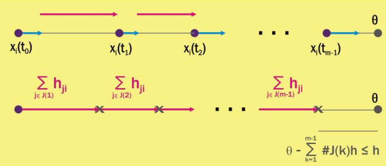
The key to this proof is the property of that if a neuron fires $\lceil(\theta/h)\rceil^d$ times, then every neuron at distance d fires at least once in that period.
That, adding to the fact that in a strongly connected graph there is always an oriented path with a length of at most N, gives the result.

Complete graphs K_N

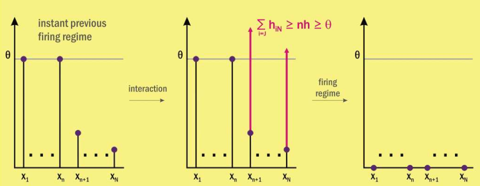
In this subsection we will work with networks with associated graph K_N

Theorem (*Large Cooperativity Principle*)**: If $N \geq 2n-1$ the network reaches grand coalition.**

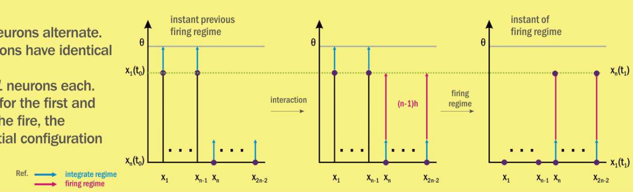
Let $a_s = \#J(s)$ and *m* such that $\sum_{s=1}^{m-1} a_s < n$ and $\sum_{s=1}^{m} a_s \geq n$.
Then, we have that $\left(\bigcup_{s=1}^{m-1} J(s)\right)^c \subseteq J(m)$, hence,
$\#J(m) \geq N-(n-1) \geq 2n-1-(n-1) = n$, so $\#J(m) = N$.



We will use the following property: if $\#J \geq n$, then $\#J = N$.

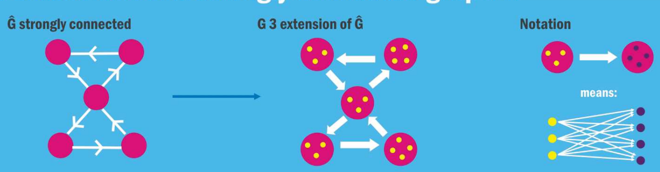


The previous bound is optimal:
If $N = 2n-2$ we can obtain a situation in which firing neurons alternate.
Example: Let us consider a network in which the neurons have identical parameters and dynamics.
Then, we will consider two groups of neurons, with *n-1* neurons each.
The following graphics shows the initial configuration for the first and second group of neurons respectively. Besides, after the fire, the configuration for the first group turns out to be the initial configuration for the second group and viceversa.



Extensions

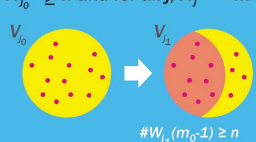
ß extensions of a strongly connected graph $\hat{G}$



Theorem: A neuronal network with associated graph G which is a ß extension of a strongly connected graph $\hat{G}$, will reach grand coalition.

Steps of the proof:

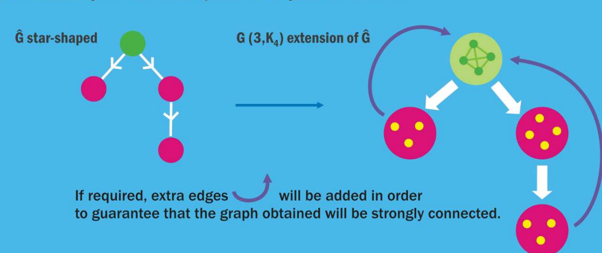
- If all the neurons in V_j fire at a certain moment, then any neuron in the network fires at that moment because of the structure of the graph *G*. In fact, if *n* of the neurons in V_j fire we obtain the same result.
- Let A_j^m be the amount of fires produced in V_j until the instant of the m[th] fire and $W_j(m)$ the subset of neurons in V_j that have not fired until the mentioned instant.
- $\sum_j A_j^m = \sum_{i=1}^{m} \#J(i) \geq m$ then, we can assure that there exist m_0 and j_0 such that $A_{j_0}^{m_0} \geq n$ and for all *j*, $A_j^{m_0-1} < n$.



- We have that $J(m_0) \cap V_{j_1} \supseteq W_{j_1}(m_0-1)$, so $\#(J(m_0) \cap V_{j_1}) \geq \#W_{j_1}(m_0-1) \geq n$, which implies that the network reaches grand coalition.

(γ,K) extensions of a star-shaped graph $\hat{G}$

$\hat{G}$ is a star-shaped graph if it has a distinguished vertex which can be connected by an oriented path to any other vertex.



Theorem: A neuronal network with associated graph G which is a (γ,K_N) extension of a star-shaped graph $\hat{G}$ with $N \geq 2n-1$ vertices, will reach grand coalition.

Steps of the proof:

- If all the neurons in *K* fires at a certain moment, then any neuron in the network fires at that moment because of the structure of the graph $\hat{G}$.
- An argument similar to the one used in the proof of the large cooperativity principle can be applied in order to prove that there exists a moment in which all the neurons in *K* fires.