RESÚMENES, DIAPOSITIVAS Y OTROS DOCUMENTOS DE COMUNICACIONES EN ENCUENTROS CIENTÍFICOS II

Recopilación 2013 - 2018

Eleonora Catsigeras

Instituto de Matemática y Estadística
“Prof. Ing. Rafael Laguardia” (IMERL)
Facultad de Ingeniería
Universidad de la República

Autores	Título de la comunicación / Evento	Año	
CATSIGERAS, E	*Abundant continuous dynamical systems with infinite entropy ergodic measures* Talk in Dynamics Days Latin America and the Caribbean.	2018	1
CALDERÓN, A; CATSIGERAS, E; GUIRAUD, P	*Spectral decomposition of piecewise contracting dynamics on the interval.* Poster presented by A. Calderón in International Congress of Mathematicians	2018	2
LORENZO, P; BARRIOS, M; CUBRÍA, F. CATSIGERAS, E	*Dynamics of Cooperative neuronal networks depending on their associated graphs.* Poster presented by P. Lorenzo in International Congress of Mathematicians.	2018	3
CATSIGERAS, E	*Generic C0 maps of the interval: ergodic and pseudo-physical measures.* Talk in the MathAmSud Meeting "PhySeCo", Montevideo, Uruguay	2017	4
CATSIGERAS, E	*Empiric stochastic stability of physical and pseudo-physical measures- T*alk in the Séminaire Dynamique, Arithmétique, Combinatoire, Institute de Mathématiques, Université Aix-Marseille, Luminy, France	2017	5
CATSIGERAS, E	*Stochastic Perturbations of Piecewise Continuous Maps-* Talk in the conference "New Trends in One-dimensional Dynamics Celebrating the 70th. Anniversary of Welington de Melo" , Rio de Janeiro , Brazil	2016	6

CATSIGERAS, E	*Lebesgue Essential Exponent and Positive Entropy of Diffeomorphisms with Dominated Splitting* Seminário de Teoria Ergódica do IMPA , Río de Janeiro, Brazil	2016	7
CATSIGERAS, E	*Exponente Lebesgue-esencial y entropía positiva en difeomorfismos C1 con splitting dominado.* Seminario Dinámica Porteña, Valparaíso, Chile	2016	8
RUBIDO, N.; CABEZA, C; CATSIGERAS, E; MARTÍ, A ; GREBOGI, C.; BAPTISTA, M.S.	*Transmission of Energy in Complex Networks* XIV Latin American Workshop on Nonlinear Phenomena (LAWNP) , Cartagena de Indias, Colombia	2015	9
CATSIGERAS, E TIAN, X	*Conditions for Positive Entropy of Diffeomorphisms with Dominated Splitting.* Talk in The Dynamical Systems, Ergodic Theory and Probability Conference Dedicated to the Memory of Nikolai Chernov, Birmingham, U.S.A.	2015	10
CATSIGERAS, E	*Sincronización de eventos en circuitos neuronales predominantemente excitatorios* Presentación oral en el Seminario de Investigadores de Física de la Facultad de Ciencias, UdelaR, Montevideo.	2015	11
MEYRONEINC, A CATSIGERAS, E GUIRAUD, P. UGALDE, E	*On the Asymptotic Properties of Piecewise Contracting Maps* EquaDiff 2015 , Lyon (Francia) , 2015. Resumen	2015	12
CATSIGERAS, E; TIAN, XUETING	*Condiciones suficientes de difeos C1 con splitting dominado para entropía positiva* Presentación oral en el Seminario de Sistemas Dinámicos , Montevideo	2015	13

CATSIGERAS, E PASSEGGI, A	*Teoría Ergódica* Curso dictado en la XVIII Escuela Venezolana de Matemática, EMALCA Venezuela	2015	14
NAVIA, R CATSIGERAS, E ALEMÁN, L	*Racionalidad y Cambio Conceptual en Ciencias y Filosofía* - Grupo de Trabajo VI Jornada de Investigación- Jornadas Académicas de la FHCE 2015, Montevideo.	2015	15
CATSIGERAS, E	*Matemática, Racionalidad y Relevancia* Presentación oral en el Seminario Interdisciplinario de Racionalidad y Filosofía de las Ciencias. Fac. Humanidades y Ciencias de la Educación, UdelaR.	2015	16
CATSIGERAS, E	*La Matemática al ingreso universitario: ¿cómo transforma y es transformado un investigador matemático cuando enseña a ingresantes?* Presentación oral en las II Jornadas de Investigación en Educación Superior , Montevideo. Artículo presentado publicado en CD-Rom.	2015	17
CATSIGERAS, E	*Entropía topológica e hiperbolicidad parcial.* Presentación oral en el Seminario de Investigación Matemática, IVIC, Altos de Pipes, Venezuela	2015	18
CATSIGERAS, E	*Bifurcaciones de Adición de Período en Sistemas Dinámicos Continuos a Trozos.*Presentación oral en las VIII Jornadas de Ingeniería Matemática, Montevideo	2015	19
CATSIGERAS, E	*La Enseñanza Basada en la Persona.* Presentación oral en *Las Tutorías Didácticas como Estrategias de Apoyo a Docentes.* Encuentro de Docentes de FING, Montevideo	2015	20

CATSIGERAS, E	*Redes Neuronales Cooperativas Acopladas por Impulsos.* Presentación oral en el Seminario de Mecánica Estadística y Física No Lineal, Fac. Ciencias, UdelaR.	2015	21
CATSIGERAS, E	*Atractores estadísticos y medidas pseudo-físicas* Presentación en el Seminario de Sistemas Dinámicos, Montevideo .	2014	22
CATSIGERAS, E BLASINA, L LOUREIRO, S MÍGUEZ, M	*Deserción en curso de Matemática al ingreso a la Universidad: ¿conocimientos previos o vínculos humanos?* Artículo y poster presentados en IV Conferencia Latinoamericana sobre el Abandono en la Educación Superior , Medellín (Colombia)	2014	23
CATSIGERAS, E	*Subcircuits with predominant excitatory pulsed interactions: Mathematical sufficient conditions to Synchronize Spikes* (ABSTRACT) , Talk in the International Symposium of Neurons, Circuits and Sytems , Montevideo	2014	24
CATSIGERAS, E.	*Sobre ciertas redes de unidades dinámicas acopladas por impulsos.* Talk in the IV Coloquio Uruguayo de Matemática, Montevideo, diciembre 2013.	2013	25

1

Abundant continuous dynamical systems with infinite entropy ergodic measures

Talk in Dynamics Days Latin America and the Caribbean.

Eleonora Catsigeras

2015

Submitted July 31, 2018 to:

Dynamics Days LAC 2018
Punta del Este, Uruguay
November 26 to 30, 2018

Abundant continuous dynamical systems with infinite-entropy ergodic measures.

Eleonora Catsigeras[1]

Abstract

We prove that the ergodic continuous dynamical systems with infinite entropy are abundant, i.e. “generic”in the sense of Baire cathegory classification. This is a joint work with Serge Troubetzkoy.

MSC 2010: Primary: 37A35; Secondary: 28D20, 37A05

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[1] Instituto de Matemática y Estadística Rafael Laguardia (IMERL), Universidad de la República, Uruguay. E-mail: eleonora@fing.edu.uy

Abundant continuous dynamical systems with infinite entropy

Eleonora Catsigeras

Universidad de la República
Montevideo, Uruguay

`eleonora@fing.edu.uy`

in a joint work with
Serge Troubetzkoy

Dynamics Days - LAC 2018
Punta del Este, November 26-30, 2018

X = compact phase space of finite dimension $m \geq 1$
$C^0(X)$ = space of all the deterministic continuous dynamical systems
$f : X \mapsto X$. Recurrent relation

$$x_{n+1} = f(x_n)$$

Definition

A phenomenon or property P is **GENERIC OR TYPICAL** if the family of systems that exhibit it contains a countable intersection of OPEN AND DENSE families in $C^0(X)$.

- **OPEN:** if a systems exhibits P then it still exhibits P after ANY small perturbation of its parameters.
- **DENSE:** if a system does not exhibit P then it will exhibit P after SOME small perturbation of its parameters.

If a phenomenon P is generic or typical, then the family of systems that does not exhibit P is MEAGER, and the family of systems that does exhibit P is ABUNDANT (in the sense of Baire Cathegory Theory).

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- Phenomenon: **INFINITE METRIC ENTROPY:**

Infinite velocity in which the expected value of the probabilistic information quantity of the system increases.

- CAN a deterministic system have infinite metric entropy?

No, if the system is differentiable.
Yes, if the system is continuous but non differentiable.

- HOW FREQUENTLY a continuous non differentiable system has infinite metric entropy?

Theorem 1. (Infinite metric entropy ergodic measures)

Generic maps $f \in C^0(X)$ have ergodic measures μ such that

$$h_\mu(f) = +\infty$$

INFINITE METRIC ENTROPY IS GENERIC OR TYPICAL

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Route of the proof of Theorem 1.

- In the box $[0,1]^m$: CONSTRUCT a nonempty G_δ-family $\mathcal{H} \subset C^0([0,1]^m)$ of continuous maps $h : [0,1]^m \mapsto [0,1]^m$, which we call **MODELS**.

Main Lemma

Any model h has an ergodic measure ν such that $h_\nu(h) = +\infty$.

- In the compact phase space X of finite dimension, for a map $f \in C^0(X)$, DEFINE

PERIODIC SHRINKING BOX $K \subset M$.

Lemma 1

Generic $f \in C^0(X)$ has some periodic shrinking box K.

Lemma 2

Generic $f \in C^0(X)$ has some periodic shrinking box K such that the return map $f^p|_K : K \mapsto \text{int}K$ is conjugated to some model map h.

- END DE PROOF: Joining Lemma 2 and Main Lemma conclude that generic $f \in C^0(X)$ has an ergodic measure with infinite entropy.

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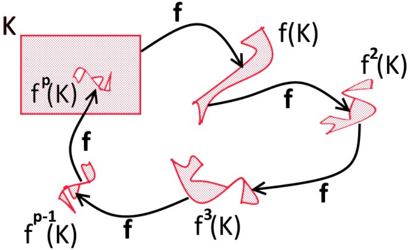
Let $f \in C^0(X)$.

Definition

Periodic shrinking box with period p is a compact set $K \subset M$ homeormorphic to $[0,1]^m$, such that

- $K, f(K), \ldots, f^j(K), \ldots, f^{p-1}(K)$ are pairwise disjoint,
- $f^p(K) \subset$ interior(K),
- diam$(f^j(K)) <$ diam(K) for all $j \geq 1$.



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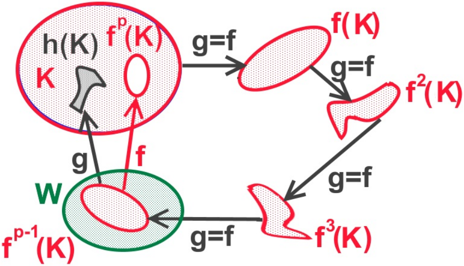
Lemma 2

For a generic map $f \in C^0(M)$ there exists a periodic shrinking box K such that the return map $f^p|_K$ coincides, up to the conjugacy that transforms K onto the cube $D^m := [0,1]^m$, with a model map $h : D^m \mapsto D^m$.

Proof



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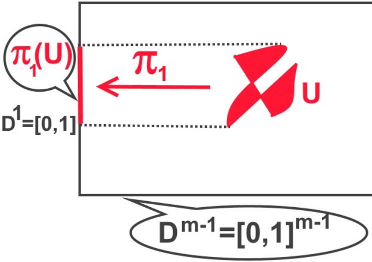
Construction of the MODEL maps in the cube $D^m := [0,1]^m$.

Step 1: THE PROJECTION π_1 of D^m onto the interval $[0,1]$

$\pi_1 : D^m \mapsto D^1 := [0,1]$ is the following **projection**

$$\pi_1(x_1, x_2, \ldots, x_{m-1}, x_m) := x_m \in [0,1].$$



Construction of the MODEL maps in the cube $D^m := [0,1]^m$.

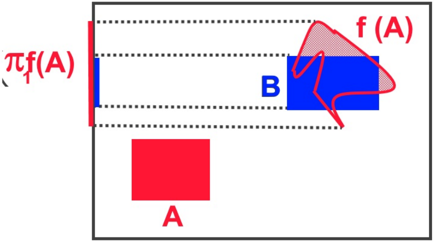
Step 2: The $\pi_1 f$-covering relation between boxes

Let A, B be two boxes in the interior of D^m. Let $f \in C^0(D^m)$.

Definition

A $\pi_1 f$**-covers B**; $\quad A \rightarrow_{\pi_1 f} B$ if

- interior$(f(A)) \cap B \neq \emptyset$,
- interior$(\pi_1 f(A)) \supset \pi_1 B$.



Construction of the MODEL maps in the cube $D^m := [0,1]^m$.

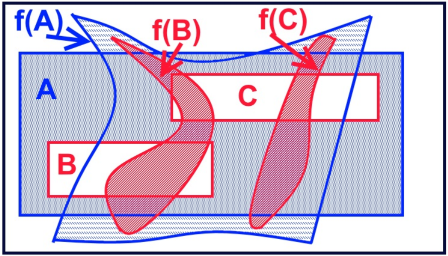
Step 3: ATOMS OF GENERATION 0 and 1.

Let A, B, C be three boxes in the interior of D^m such that

- $B, C \subset$ interior(A), $B \cap C = \emptyset$,
- $B \to_{\pi_1 f} A$, $C \to_{\pi_1 f} A$.

Definition

If so, we call **A the atom of generation 0**,
and **B, C the two atoms of generation 1**.



REMARK: The above condition is **OPEN** in $C^0(D^m)$. The **same** boxes A, B, C are also atoms of gen. 0 and 1 resp. $\forall\ g$ near enough f.

Construction of the MODEL maps in the cube $D^m := [0,1]^m$.

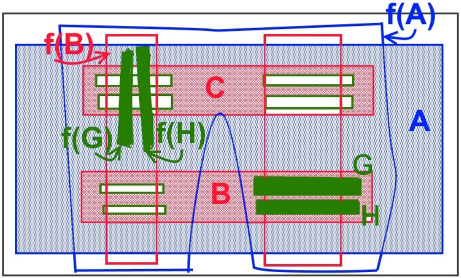
Step 5: ATOMS OF GENERATION n

By induction on $n \geq 1$:

- $\mathcal{A}_n$ is a finite collection of exactly $2^{n(n+1)/2}$ pairwise disjoint compact boxes such that, for an adequate collection of pairs (B, C) of atoms of gen. $n-1$, there exists exactly two different boxes in $G, H \in \mathcal{A}_n$ such that
- $G, H \subset \text{int}(B)$ • $G \mapsto_{\pi_1 f} C$, $H \mapsto_{\pi_1 f} C$.

Definition

If so, the boxes of $\mathcal{A}_n$ are called **the atoms of generation n**.



Construction of the MODEL maps in the cube $D^m := [0,1]^m$.

Final step: Definition of the MODEL

Definition

We call a map $f \in C^0(D^m)$ **a MODEL** if there exists a sequence

$$\mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n, \dots$$

of finite collections $\mathcal{A}_n$ of pairwise disjoint boxes such that:

- For all $n \geq 0$, the boxes of $\mathcal{A}_n$ satisfy the definition of being atoms of generation n for f.
- $\lim_{n \to +\infty} \max_{A \in \mathcal{A}n} \text{diam}(A) = 0$.

Main Lemma

If $f \in C^0(D^m)$ is a model then it has an ergodic measure ν such that $h_\nu(f) = +\infty$.

Route of the proof.

- DEFINITION: The Λ-**set** is

$$\Lambda := \bigcap_{n\geq 0} \bigcup_{A\in\mathcal{A}_n} A.$$

- Λ is a Cantor set.
- Λ is f-invariant: $\quad f(\Lambda) = \Lambda$.
- The Borel σ-algebra in Λ is generated by the atoms $A \cap \Lambda$.

- CONSTRUCT the pre-measure ν on Λ:

$$\nu(A\cap\Lambda) = \frac{1}{\#\mathcal{A}_n} = \frac{1}{2^{n(n+1)/2}} \qquad \forall\ A \in \mathcal{A}_n, \ \ \forall\ n \geq 0.$$

- The above pre-measure defines a unique Borel probability measure ν supported on Λ.
- ν is f-invariant and ergodic.

- Compute the **metric entropy of** ν and check that $h_\nu(f) = +\infty$.

Conclusions and further results: • Infinite metric entropy measures do not exist if $f \in C^{\text{Lips}}(M)$ because $h_{top}(f) < +\infty$.

• $h_{top}(f) = +\infty$ for generic $f \in C^0(M)$
(Yano, Inv. Math. 1980).

$\Rightarrow$ (1): *For all $K > 0$ there exists f-invariant μ_K such that* $h_{\mu_K}(f) \geq K$.

• (1) $\not\Rightarrow$ $\exists$ μ such that $h_\mu(f) = +\infty$, because the metric entropy function is not upper semi-continuous.

• $h_{top}(f) = +\infty$ also for generic $f \in C^{\text{Hölder}}(M)$
(de Faria - Hazard - Tresser, ArXiv 2017).

• Does Theorem 1 also hold in $C^{\text{Hölder}}(M)$?
• If dim$(M) \geq 2$, does Theorem 1 also hold in Homeo(M)?
• Theorem 1 is false in Homeo(M) if M is only a compact metric space but not a manifold (Akin-Glasner-Weiss, Trans. AMS, 2008).

2

Spectral decomposition of piecewise contracting dynamics on the interval.

Poster presented by A. Calderón in International Congress of Mathematicians.

Alfredo Calderón, Eleonora Catsigeras, Pierre Guiraud

2018

N°

Spectral decomposition of piecewise contracting dynamics on the interval

Alfredo Calderón
Centro de Investigación y Modelamiento de Fenómenos Aleatorios Valparaíso, Universidad de Valparaíso, Chile
Doctoral Program Consortium PUCV-USM-UV
`alfr.calderon@gmail.com`

Eleonora Catsigeras
Instituto de Matemática y Estadística Rafael Laguardia,
Universidad de la República, Montevideo, Uruguay
`eleonora@fing.edu.uy`

Pierre Guiraud
Centro de Investigación y Modelamiento de Fenómenos
Aleatorios Valparaíso, Universidad de Valparaíso, Chile
`pierre.guiraud@uv.cl`

Abstract. We study the topological attractors of injective piecewise contracting maps on a compact interval with any finite number $N \geq 2$ of continuity pieces. We prove the existence of a "spectral decomposition" of the attractor into a finite number of transitive components that are either periodic orbits or Cantor sets. In the non-generic case, we prove that some orbits accumulate at both sides of the discontinuities points, and that this phenomenon generates the transitive Cantor sets of the attractor. **Keywords:** Interval map, Piecewise contraction, Minimal Cantor sets. **MSC 2010:** 37E05 – 47H09 – 54H20.

Definitions and Spectral Decomposition Theorem

Let $X \subset \mathbb{R}$ be a compact interval. A map $f : X \to X$ is a *piecewise contracting interval map with* $N \geq 2$ *continuity pieces* and *contracting rate* $\lambda \in (0,1)$ (in short (N,λ)-PCIM) if there exists a pairwise disjoint open subintervals collection $X_1, X_2, \dots, X_N$ such that

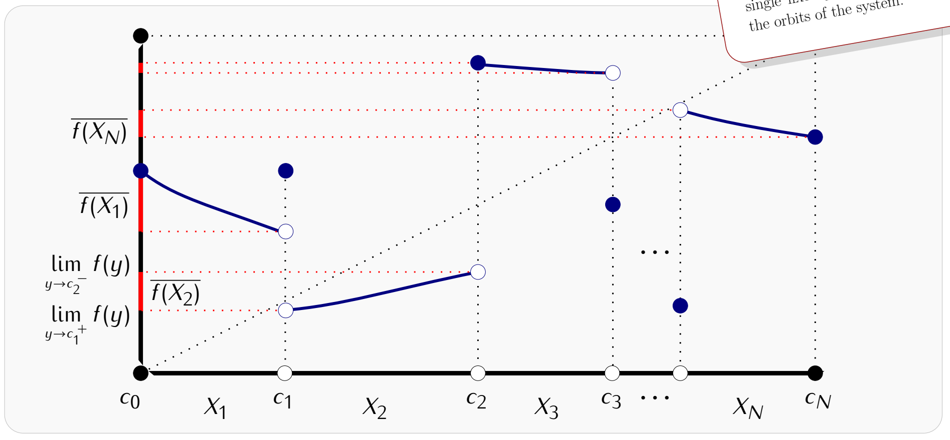
$$X = \bigcup_{1 \leq i \leq N} \overline{X_i} \qquad \text{and} \qquad |f(x) - f(y)| \leq \lambda\,|x-y| \quad \forall\, x,y \in X_i,\ \ \forall\, i \in \{1,2,\dots,N\}.$$

Also, the map f is supposed to be discontinuous (with unavoidable discontinuities) at the points of the set

$$\Delta := \bigcup_{i \neq j} \partial X_i \cap \partial X_j =: \{c_1, \dots, c_{N-1}\}.$$

When $N = 1$, the map f is a simple contraction. Thus, there exists a single fixed point that attracts all the orbits of the system.

Graphic of an (N,λ)-PCIM.



The *attractor* of f is defined by the following equalities:

$$\Lambda := \bigcap_{n \geq 1} \Lambda_n \qquad \text{where} \quad \Lambda_1 := \overline{f(X \setminus \Delta)} \quad \text{and} \quad \Lambda_{n+1} := \overline{f(\Lambda_n \setminus \Delta)} \quad \forall\, n \geq 1.$$

We say that f satisfies the *separation property* if $f|_{X_i}$ is injective for each i and

$$\overline{f(X_i)} \cap \overline{f(X_i)} = \emptyset \qquad \forall\, i,j \in \{1,\dots,N-1\} \ :\ \ i \neq j.$$

Besides, let D be the set of lateral limits of f at their points of discontinuity, i.e.

$$D := \left\{ \lim_{y \to c_i^-} f(y) : 1 \leq i \leq N-1 \right\} \cup \left\{ \lim_{y \to c_i^+} f(y) : 1 \leq i \leq N-1 \right\} \cup \left\{ f(c_0), f(c_N) \right\}.$$

We are interested in the orbits that do not intersect the set Δ of discontinuities. In this way, we may disregard how f is defined in Δ. Precisely, we construct the set

$$\widetilde{X} := \bigcap_{n \geq 0} f^{-n}(X \setminus \Delta).$$

If f satisfies the separation property, then $\widetilde{X}$ is dense in X.

We say that $A \subset X$ is *pseudo-invariant* if for any $x \in A$ we have

$$\lim_{y \to x^-} f(y) \in A \qquad \text{or} \qquad \lim_{y \to x^+} f(y) \in A.$$

It is easy to prove that sets such as $\omega(x)$ (whenever $x \in \widetilde{X}$) or the attractor Λ are pseudo-invariant.

Note that if $A \subset X$ is pseudo-invariant, then $f(x) \in A$ for any $x \in A \setminus \Delta$ and $A \cap \widetilde{X}$ is invariant. The following is the main result of our work:

Theorem (Spectral Decomposition)

Suppose that f satisfies the separation property and $D \subset \widetilde{X}$, then there exist two natural numbers N_1 and N_2 satisfying $1 \leq N_1 + N_2 \leq 2N$ and such that the attractor Λ of f can be decomposed as follows:

$$\Lambda = \left(\bigcup_{i=1}^{N_1} \mathcal{O}_i \right) \cup \left(\bigcup_{i=1}^{N_2} K_i \right),$$

where $\mathcal{O}_1, \mathcal{O}_2, \dots, \mathcal{O}_{N_1} \subset \widetilde{X}$ are pairwise different periodic orbits and $K_1, K_2, \dots, K_{N_2}$ are transitive pseudo-invariant Cantor sets of X. Moreover, for any $x \in \widetilde{X}$, either there exists $i \in \{1,\dots,N_1\}$ such that $\omega(x) = \mathcal{O}_i$ or there exists $i \in \{1,\dots,N_2\}$ such that $\omega(x) = K_i$.

If f is piecewise increasing, then $1 \leq N_1 + N_2 \leq N$.

Short state of art

With respect to periodic attractors, if $\Lambda \cap \Delta = \emptyset$ is known that the asymptotic dynamics is supported by a finite number of periodic orbits (this is also valid in higher dimensions – see [2]). Furthermore, it is known that this behavior is Lebesgue-generic and that generically $N_1 \leq N$ (first proved in the injective case [4, 5] and later in the general case [6], always in one dimension).

With respect to non-generic attractors is known that, for the particular case $N = 2$, the ω-limit set of all point is either a periodic orbit or a Cantor set (see [3]).

Our work generalizes in a certain sense these previous results (we use some complexity tools for this type of systems. The complexity of the dynamics has been also studied in [1]).

Orbits classification and route of the proof

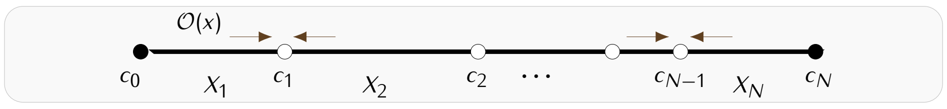
Finally, we say that $c_i \in \Delta$ is *left-right recurrently visited* by the orbit of $x \in \widetilde{X}$, if there exists two strictly increasing sequences $\{t_j\}_{j\geq 0}$ and $\{s_j\}_{j \geq 0}$ of natural numbers such that

$$f^{t_j}(x) \in X_i \ \text{ and } \ f^{s_j}(x) \in X_{i+1} \quad \forall\, j \geq 0, \qquad \text{and} \qquad c_i = \lim_{j\to\infty} f^{t_j}(x) = \lim_{j \to \infty} f^{s_j}(x).$$

We denote by $\Delta_{lr}(x) \subset \Delta$ the set of discontinuity points that are lr-recurrently visited by the orbit of x, and define

$$\Delta_{lr} := \bigcup_{x \in \widetilde{X}} \Delta_{lr}(x).$$

Graphic situation of the affirmation: $c_1, c_{N-1} \in \Delta_{lr}(x)$.



Suppose that $f : X \to X$ is a (N,λ)-PCIM that satisfies the separation property and that $D \subset \widetilde{X}$.

Lemma 1

The attractor Λ of f can be decomposed as follows:

$$\Lambda = \bigcup_{d \in D} \omega(d). \tag{1}$$

Theorem 2 (periodic ω-limits)

Let $x \in \widetilde{X}$ be such that $\Delta_{lr}(x) = \emptyset$, then $\omega(x)$ is a periodic orbit contained in $\widetilde{X}$.

Theorem 3 (Cantor ω-limits)

Let $x \in \widetilde{X}$ be such that $\Delta_{lr}(x) \neq \emptyset$, then $\omega(x)$ is a Cantor set and $\omega(x) = \overline{\mathcal{O}(y)}$ for all $y \in \omega(x) \cap \widetilde{X}$.

Spectral Decomposition Theorem proof. Applying Theorems 2 and 3 to the points of D, we rewrite (1) as follows:

$$\Lambda = \bigcup_{d \in D} \omega(d) = \left(\bigcup_{i=1}^{N_1} \mathcal{O}_i \right) \cup \left(\bigcup_{i=1}^{N_2} K_i \right), \tag{2}$$

where $\mathcal{O}_1, \mathcal{O}_2, \dots, \mathcal{O}_{N_1} \subset \widetilde{X}$ are pairwise different periodic orbits, $K_1, K_2, \dots, K_{N_2}$ are transitive pseudo-invariant Cantor sets, and $N_1 + N_2 \leq \#D = 2N$. Besides, each periodic orbit $\mathcal{O}_i$, and each Cantor sets K_i, is the ω-limit set of some point $d \in D$.
Now, let us prove that the ω-limit set of any other point in $\widetilde{X}$ also coincides, either with one periodic orbit $\mathcal{O}_i$, or with one Cantor set K_i. First, recall that the ω-limit set $\omega(x)$ of any point $x \in \widetilde{X}$ satisfies $\omega(x) \cap \widetilde{X} \neq \emptyset$. Then, there exists $y \in \omega(x) \cap \widetilde{X}$. Since $\omega(x) \subset \Lambda$, from Lemma 1 we deduce that there exists $d \in D$ such that $y \in \omega(d)$, so $y \in \omega(x) \cap \omega(d) \cap \widetilde{X}$. Besides, $x, d \in \widetilde{X}$, so we can apply Theorems 2 and 3 to deduce that both $\omega(x)$ and $\omega(d)$ are transitive sets. Therefore,

$$\overline{\mathcal{O}(y)} = \omega(x) = \omega(d).$$

This proves that $\omega(x)$ coincides with some set of the decomposition (2). We conclude that, for any $x \in \widetilde{X}$, either there exists $i \in \{1,\dots,N_1\}$ such that $\omega(x) = \mathcal{O}_i$, or there exists $i \in \{1,\dots,N_2\}$ such that $\omega(x) = K_i$, ending the proof. ■

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Acknowledgments

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3

Dynamics of Cooperative neuronal networks depending on their associated graphs.
Poster presented by P. Lorenzo in International Congress of Mathematicians.

LORENZO, P; BARRIOS, M;
CUBRÍA, F.
CATSIGERAS, E

2018

Dynamics of cooperative neuronal networks depending on their associated graphs

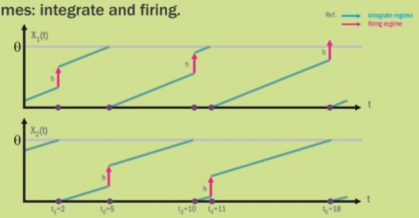
Lorenzo, P. | Barrios, M. | Cubría, F. | Advisor: Catsígeras, E.

UNIVERSIDAD DE LA REPÚBLICA URUGUAY

PONTIFICIA UNIVERSIDAD CATOLICA DE VALPARAISO

Basic definitions

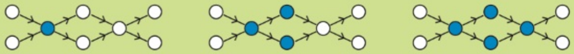
A *neuronal network* N describes the dynamics of a set of N neurons that interact.
The *configuration of the network* is given by a vector
$$X(t)=(X_1(t),\dots,X_N(t)).$$
The dynamic evolution of the system will be given by two regimes: integrate and firing.



Let θ_i indicate the *threshold* of the neuron i. When $X_i(t)$ reaches θ_i, the neuron fires, and interacts with a neuron j by h_{ij}, when $h_{ij} > 0$. We will also take into consideration the *parameters* θ, h and n such that $\theta \geq \theta_i$, $h \leq h_{ij}$ for all i, j, and $nh \geq \theta \geq (n-1)h$.

Neuronal network's *associated graph*: It is a graph with the set of neurons as vertices and an edge from i to j if $h_{ij} > 0$.

At the instant t_m of the mth fire, we define $J(m)$ as the set of neurons that fire in that instant. If $\#J(m)=N$, we say that the network reaches *grand coalition*.



It is worth noting that there are neurons that reach their threshold naturally, and others that reach it due to the interaction with the network.

Strongly Connected Graph

We say that a graph is *strongly connected* if given two vertices, there exists an oriented path from one to the other.

Theorem : In a neuronal network with strongly connected associated graph, every neuron fires, actually, infinitely many times.

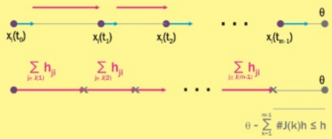
The key to this proof is the property of that if a neuron fires $\lceil(\theta/h)\rceil^d$ times, then every neuron at distance d fires at least once in that period.
That, adding to the fact that in a strongly connected graph there is always an oriented path with a length of at most N, gives the result.

Complete graphs K_N

In this subsection we will work with networks with associated graph K_N

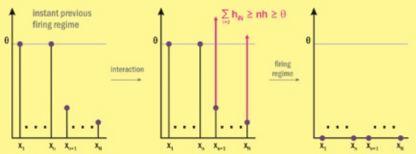
Theorem *(Large Cooperativity Principle)*: **If $N \geq 2n-1$ the network reaches grand coalition.**

Let $a_s = \#J(s)$ and m such that $\sum_{s=1}^{m-1} a_s < n$ and $\sum_{s=1}^{m} a_s \geq n$.
Then, we have that $\left(\bigcup_{s=1}^{m-1} J(s)\right)^c \subseteq J(m)$, hence,
$\#J(m) \geq N-(n-1) \geq 2n-1-(n-1) = n$, so $\#J(m) = N$.

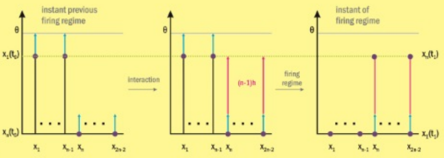


$$\theta - \sum_{k=1}^{m-1} \#J(k)h \leq h$$

We will use the following property: if $\#J \geq n$, then $\#J = N$.

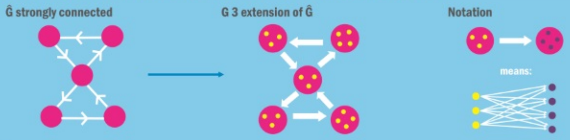


The previous bound is optimal:
If $N = 2n-2$ we can obtain a situation in which firing neurons alternate.
Example: Let us consider a network in which the neurons have identical parameters and dynamics.
Then, we will consider two groups of neurons, with $n-1$ neurons each.
The following graphics shows the initial configuration for the first and second group of neurons respectively. Besides, after the fire, the configuration for the first group turns out to be the initial configuration for the second group and viceversa.



Extensions

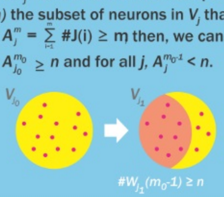
ß extensions of a strongly connected graph $\hat{G}$



Theorem: A neuronal network with associated graph G which is a ß extension of a strongly connected graph $\hat{G}$, will reach grand coalition if ß > 2n-1.

Steps of the proof:

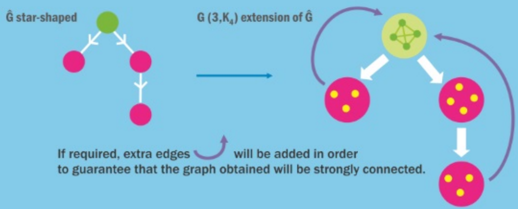
- If all the neurons in V_j fire at a certain moment, then any neuron in the network fires at that moment because of the structure of the graph G. In fact, if n of the neurons in V_j fire we obtain the same result.
- Let A_j^m be the amount of fires produced in V_j until the instant of the mth fire and $W_j(m)$ the subset of neurons in V_j that have not fired until the mentioned instant.
- $\sum_j A_j^m = \sum_{i=1}^{m} \#J(i) \geq m$ then, we can assure that there exist m_0 and j_0 such that $A_{j_0}^{m_0} \geq n$ and for all j, $A_j^{m_0-1} < n$.



- We have that $J(m_0) \cap V_{j_1} \supseteq W_{j_1}(m_0-1)$, so $\#(J(m_0) \cap V_{j_1}) \geq \#W_{j_1}(m_0-1) \geq n$, which implies that the network reaches grand coalition.

(γ,K) extensions of a star-shaped graph $\hat{G}$

$\hat{G}$ is a star-shaped graph if it has a distinguished vertex which can be connected by an oriented path to any other vertex.



If required, extra edges will be added in order to guarantee that the graph obtained will be strongly connected.

Theorem: A neuronal network with associated graph G which is a (n,K_N) extension of a star-shaped graph $\hat{G}$ with N ≥ 2n-1 vertices, will reach grand coalition.

Steps of the proof:

- If all the neurons in K fires at a certain moment, then any neuron in the network fires at that moment because of the structure of the graph $\hat{G}$.
- An argument similar to the one used in the proof of the large cooperativity principle can be applied in order to prove that there exists a moment in which all the neurons in K fires.

4

Generic C0 maps of the interval: ergodic and pseudo-physical measures. Talk in the MathAmSud Meeting "PhySeCo", Montevideo, Uruguay

Eleonora Catsigeras

2017

MathAmSud Meeting “Physeco”.
Montevideo, from 11 to 13 December, 2017

Generic C^0 maps of the interval: ergodic and pseudo-physical measures

Eleonora Catsigeras

Abstract

We study the ergodic properties of generic C^0 maps of the interval I. We prove that all ergodic measures are pseudo-physical, and that any pseudo-physical measure is in the weak*-limit of the set of ergodic measures. Nevertheless, we also prove that the set of pseudo-physical measures is meager in the space of all invariant measures.

This is a joint work with Serge Troubetzkoy.

I compact interval with nonempty interior, $f \in C(I)$.
$\mathcal{P}$: Borel probability measures on I - weak*-topology.
$\mathcal{P}_f \subset \mathcal{P}$: f-invariant prob. measures.

Definitions:

- *Empiric probabilities:* $\sigma_{n,x} := \sum_{j=0}^{n-1} \delta_{f^j(x)}; \quad x \in I, \quad n \geq 1.$
- *P-omega-limit of $x \in I$:*

$$p\omega(x) \colon = \{\mu \in \mathcal{P} : \lim \sigma_{n_j,x} = \mu \text{ for some } n_j \to +\infty\}$$

- *Physical measure μ* if $\text{Leb}\Big(\big\{x \in I : p\omega(x) = \{\mu\}\big\}\Big) > 0.$
- *ϵ-weak basin of statistical attraction of $\mu \in \mathcal{P}_f$:*

$$A_\epsilon(\mu) = \{x \in I : \text{dist}(p\omega(x), \mu) < \epsilon\}.$$

- *Pseudo-physical measure μ* if $\text{Leb}\Big(A_\epsilon(\mu)\Big) > 0 \ \forall \ \epsilon > 0.$

Theorem Abdenur-Anderson (CMP, 2013)
C^0-generically, f has not physical measures, and for Lebesgue a.e. $x \in I$ there exists a (unique) measure $\mu_x \in \mathcal{P}_f$ such that $p\omega(x) = \{\mu_x\}$.

Our first result:

Theorem

C^0-generically:
• Any ergodic measure is pseudo-physical.
• Any pseudo-physical measure is in the closure of the ergodic measures, as well as in the closure of atomic measures.
• The subspace of pseudo-physical measures is a topologically meager subset of $\mathcal{P}_f$.

SKETCH OF THE PROOF: on the board

Our results on the entropy of C^0-generic systems on the interval:

Theorem

For C^0-generic $f\colon I \mapsto I$,

- *The metric entropy function $\mu \in \mathcal{P}_f \to h_\mu(f)$ is everywhere neither upper semi-continuous nor lower semi-continuous.*
- *There exists non countably infinitely many pseudo-physical measures μ that are atomic, hence $h_\mu(f) = 0$*
- *For any natural number $m \geq 1$, there exists infinitely many pseudo-physical measures μ for which $h_\mu(f) = \log m$. Hence, the topological entropy is infinite.*
- *There exists infinitely many pseudo-physical measures μ for which $h_\mu(f) = \infty$.*

SKETCH OF THE PROOF (on the board)

DEFINITION Periodic shrinking interval period $p \geq 1$

nonempty open interval $I \subset I_0$ such that

- $\{f^j(I)\}_{0 \leq j \leq p-1}$ pairwise disjoint.
- $Leb(f^j(I)) < Leb(I)$ $\forall\, 1 \leq j \leq p$
- $\boxed{f^p(I) \subset I}$

Eventually periodic shrink. interval

nonempty open interval $J \subset I_0$ such that

- $\exists \begin{cases} n_0 \geq 0 \\ I \subset I_0 \end{cases}$ such that $Leb(f^j(J)) < Leb(J)$ $\forall\, 1 \leq j \leq n_0$, I periodic shrinking interval $\boxed{f^{n_0}(J) \subset I}$

LEMMA 1 C^0-generically

Leb-a.e. $x \in I_0$ belongs to a sequence $\{I_q\}_{q \geq 1}$ of eventually periodic shrink. intervals or periodic

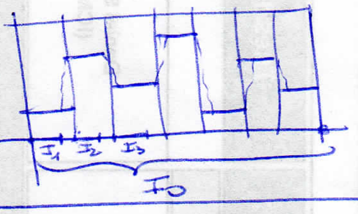
I_q with length $(I_q) < \frac{1}{q}$

Route of the proof: $\mathcal{S}_{q,k} := \{ f \in C(I_0)$ such that $\exists \{I_1, I_2, \ldots, I_q\}$

- $leb(I_i) < \frac{1}{q}$
- I_i shrinking interval
- $leb(I \setminus \bigcup_{i=1}^{k} I_{i}) < 1/k \}$

- $\mathcal{S}_{q,k}$ open in $C(I_0)$
- $\mathcal{S}_{q,k}$ dense in $C(I_0)$

$\bigcap_{k \geq 1} \bigcap_{q \geq 1} \mathcal{S}_{q,k}$ generic.

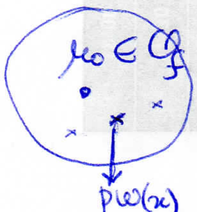


COROLLARY – C^0 generically f is Lebesgue a.e non expansive in the future

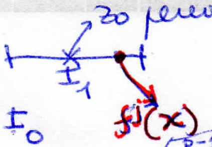
Def. leb a.e. $x \in I_0$, $\forall \alpha > 0$ $Leb(\{ y \in I_0 ;\ dist(f^n(y), f^n(x)) < \alpha \ \forall n \geq 0 \}) > 0$

COROLLARY $\boxed{\mathcal{O}_f \subset \overline{Per_f} = \overline{\mathcal{E}_f}}$

Koscielniak – Mazur – Oprocha – Pilarczyk (DCDS – 2014).



$p\omega(x)$
For leb >0 set of $x \in I_0$



I_1 periodic shrinking interval

$\bigcup_{j=0}^{p-1} f^j(\overline{I_1}) \supset supp\, \mu_x$ $\bigcup_{j=0}^{p-1} f^j(\overline{I_1}) \supset orb(z_0)$

DEFINITION Shrinking ~~periodic~~ atomic MEASURES (supported on periodic orbits)

$\nu_0 \in Shr_q Per_f$ if $\cdot\ supp(\nu_0) \subset \bigcup_{j=1}^{p} f^j(I)$ where I periodic shrinking interval (of period p)

$\cdot\ length(I) < \frac{1}{q}$

DEFINITION ε–Aprox shrinking atomic Measure

$\nu \in A_\varepsilon Shr_q Per_f$ if $\exists\, \nu_0 \in Shr_q Per_f$ such that $dist(\nu, \nu_0) < \varepsilon$

COROLLARY of LEMMA 1

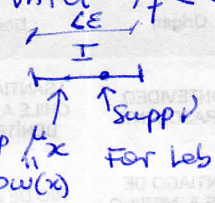
C^0-generically

$$\boxed{\bigcap_{\varepsilon>0} \bigcap_{q\geq 1} A_\varepsilon Shr_q Per_f \subset \mathcal{O}_f}$$

Proof:

$\mu \in A_\varepsilon Shr_q Per_f$

$\nu \in Shr_q Per_f$ with $1/q < \varepsilon$



$supp\, \mu_x = p\omega(x)$ For Leb a.e $x \in I$.

$\Rightarrow \mu \in \mathcal{O}_f$

~~Plan of the proof~~ ~~$\mathcal{U}_{q,r}$:~~

LEMMA 2

C^0-generically

$$\boxed{Per_f \subset \bigcap_{\varepsilon>0} \bigcap_{q\geq 1} A_\varepsilon Shr_q Per_f}$$

Route of the proof

$\mathcal{U}_{q,r} := \{ f \in C(I_0) : \exists \{ U_1, U_2, \ldots, U_h \}$

- $\mathcal{U}_{q,r}$ open in $C(I_0)$
- $\mathcal{U}_{q,r}$ dense in $C(I_0)$
- $\bigcap_{r\geq 1} \bigcap_{q\geq 1} \mathcal{U}_{q,r}$ generic

$\cdot\ Leb(U_i) < \frac{1}{q}$

$\cdot\ U_i \supset I_i$ periodic shrinking interval period that divides r.

$\cdot\ \bigcup_{i=1}^{h} U_i \supset \{ x = f^r(x) \}$

COROLLARY OF LEMMAS 2 $\overline{Per_f} \subset \mathcal{O}_f$

~~END OF THE~~

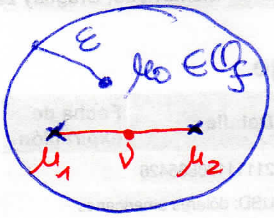
END OF THE PROOF OF THEOREM 1

we have proved: $\mathcal{O}_f \subset \overline{\mathrm{Per}_f} = \overline{\mathcal{E}_f}$ $\Big\}$

$\overline{\mathrm{Per}_f} \subset \mathcal{O}_f$

$$\boxed{\mathcal{O}_f = \overline{\mathrm{Per}_f} = \overline{\mathcal{E}_f}} \;;\Rightarrow\; \boxed{\mathcal{E}_f \subset \mathcal{O}_f}$$

Now let us prove that $\mathcal{O}_f$ is megger in $\mathcal{P}_f$ (closed with empty interior)



not isolated in $\mathcal{O}_f$

$\exists\, \mu_1, \mu_2 \in \bigcap_q \mathrm{Shr}_q \mathrm{Per}_f$

$\mu_1 \neq \mu_2$

I_1, I_2 $\left\{\begin{array}{l} \text{periodic} \\ \text{shrinking intervals} \\ \text{lengths} < 1/q \\ \text{orbits} \text{ ~~pa~~ at distance} > 0. \end{array}\right.$

let $\nu = \lambda\mu_1 + (1-\lambda)\mu_2$ with $0<\lambda<1$

we assert $A_\delta(\nu) = \emptyset$ for some $\delta > 0$ hence $\nu \notin \mathcal{O}_f$

Let $\varphi : I_0 \to [0,1]$ continuous $\varphi(f^p(I_1)) = 1$

$\varphi(I_1^{\text{compl}}) = 0$

By contradiction assume $\exists y \in A_\delta(\nu)$: $\mathrm{dist}(p\omega(y), \nu) < \delta$

$$\left| \frac{1}{n} \sum_{j=0}^{n-1} \varphi(f^j(y)) - \int \varphi\, d\nu \right| < \textcircled{\delta}$$

$\Downarrow$ $0 < \lambda/p < 1$ $\left(\text{si elijo } \delta < \frac{\lambda}{2p}\right.$

$\Rightarrow \exists\, n_0 \geq 1$ such that $\varphi(f^{n_0}(y)) > 0$

$f^{n_0}(y) \in I_1 \Rightarrow f^{n_0+p}(y) \in f^p(I_1)$

$$\varphi(f^{n_0+jp}(y)) = 1 \quad \forall j$$

$$\lim_{n\to+\infty} \frac{1}{n} \sum_{j=0}^{n-1} \varphi(f^{n_0+jp}(y)) = \frac{1}{p} \qquad \left(\frac{1}{p} - \frac{\lambda}{p}\right) < \delta$$

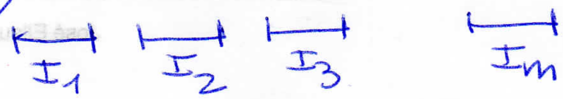
Absurdo si elijo $\delta < \frac{1-\lambda}{p}$.

THEOREM 2 c)

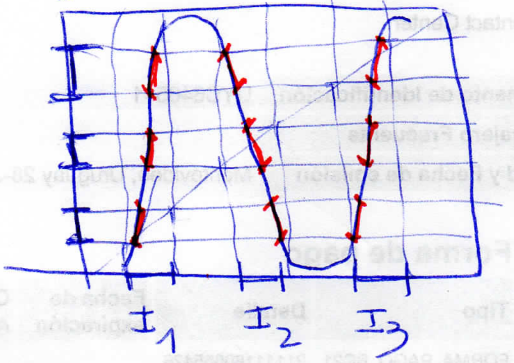
$\boxed{\text{PSEUDO-PHYSICAL MEASURE } h_\mu(f) = \log m}$

- $\boxed{\text{DEFINITION}}$ "m-horseshoe"

pairwise disjoint nonempty open intervals



such that $\operatorname{int}(f(I_i)) \supset I_1 \cup I_2 \cup \dots \cup I_m$



Atoms:

gen. 1 $A_1 = \{I_1, I_2, \dots, I_m\}$

gen. 2 $A_2 = \{I_{ij} \;\; 1 \le i,j \le m\}$

we choose ~~one~~ intervals I_{ij}

such that $I_{ij} \subset I_i$

$\operatorname{int} f(I_{ij}) \supset I_j$

$$\Lambda_n := \bigcup_{A \in \mathcal{A}_n} A$$

Λ set $\boxed{\Lambda := \bigcap_{n \ge 1} \Lambda_n}$

Def. C^0-hyperbolic m-horseshoe

$\boxed{\max_{A \in \mathcal{A}_n} \operatorname{length}(A) < \lambda^n}$ for ~~[illegible]~~ a constant $0 < \lambda < 1$

⇒ Λ is a Cantor set

- Itinerary $\theta(x)$ of $x \in \Lambda$

$\theta : \Lambda \longrightarrow \{1, 2, \dots, m\}^{\mathbb{N}}$ homeomorfismo conjugacion con shift.

Bernoulli measure μ ergodic ~~erg~~, $h_\mu(f) = \log m$

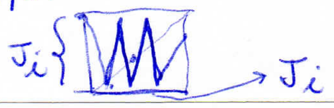
$\boxed{\text{LEMMA 3}}$ C^0-generically for any point $x_0 = f(x_0)$ and for any $q \in \mathbb{N}^+$

∃ an m-horseshoe ~~[illegible]~~ contained in $\left[x_0, x_0 + \frac{1}{q}\right]$.

Proof $\mathcal{B}_{q,m} := \{f \in C(I_0)$ such that $\exists \{J_1, J_2, \dots, J_q\}$

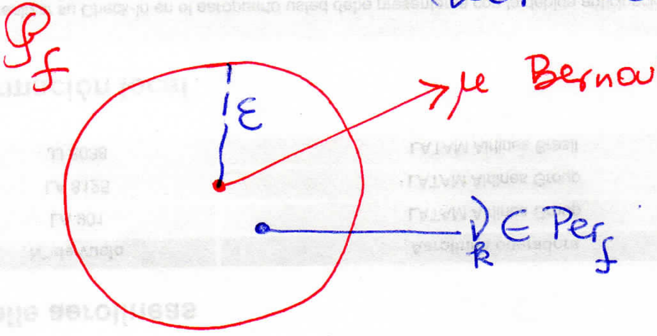
- Leb$(J_i) < 1/q$
- $J_i \supset C^0$-hyperbolic m-horseshoe
- $(\bigcup J_i)$ covers the fixed points of f

- $\mathcal{B}_{q,m}$ is G_δ in $C^0(I_0)$
- $\mathcal{B}_{q,m}$ is dense in $C(I_0)$



⑤

Corollary The entropy function is neither upper semi-continuous nor lower semi-continuous.



$\to \mu$ Bernoulli $\in \mathcal{E}_f \subset \mathcal{O}_f$

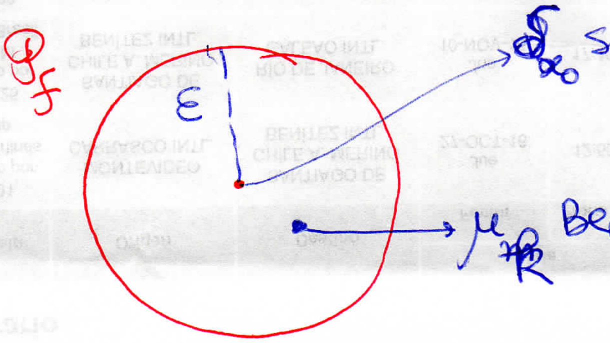
$\to \nu_R \in Per_f$

$h_{\nu_R}(f) = 0$

$\nu_R \to \mu$

$h_\mu(f) = \log m > 0$

$\boxed{h_\cdot(f) \text{ is NOT LOWER Semi-Cont.}}$



$\to \delta_{x_0}$ supported on a fixed point $x_0 = f(x_0)$

$\delta_{x_0} \in Per_f \subset \mathcal{O}_f$

$\to \mu_R$ Bernoulli $h_{\mu_R} = \log m$

$\mu_R \to \delta_{x_0}$

$h_{\delta_{x_0}}(f) = 0$

$\boxed{h_\cdot(f) \text{ is NOT UPPER Semi-Cont.}}$

THEOREM 2 d.

$\boxed{\text{PSEUDO-PHYSICAL MEASURE} \cdot h_\mu(f) = +\infty}$

- One constructs "An atom doubling cascade"
- Defines ~~[illegible]~~ $\mu \in \mathcal{P}_f$ μ is equidistributed in the atoms of ~~all~~ each generation n

$$\# A_n = \frac{n(n+1)}{2}$$

- Prove μ is ergodic
- Compute applying definition the metric entropy $h_\mu(f) = +\infty$

5

*Empiric stochastic stability of physical and pseudo-physical measures- T*alk in the Séminaire Dynamique, Arithmétique, Combinatoire, Institute de Mathématiques, Université Aix-Marseille, Luminy, France

Eleonora Catsigeras

2017

Empiric stochastic stability of physical and pseudo-physical measures.

Eleonora Catsigeras

ABSTRACT

We define the empiric stochastic stability of an invariant probability measure by adapting to the finite-time scenario, the classical definition of stochastic stability. We prove that, for any continuous system, an invariant measure is empirically stochastically stable if and only if it is physical. We define also the empiric stochastic stability of a weak*-compact set of invariant measures, instead of a single measure. Even if the system has no physical measure, we show that it still has minimal empirically stochastically stable sets of measures. Besides, we prove that such sets are necessarily composed by pseudo-physical measures. Finally, we apply the results to the one-dimensional C^1-expanding case, to conclude that the empirically stochastically measures or sets of measures satisfy Pesin Entropy Formula.

INSTITUT DE MATHÉMATIQUES
de MARSEILLE

Séminaire Dynamique, Arithmétique, Combinatoire (Ernest)

par Drappeau Sary, Guillon Pierre, Lozingot Eric, Merlet Glenn - publié le 1er février 2014, mis à jour le 26 octobre 2016 à 10h31min

Agenda

Archives | **À venir** Avec résumé | Sans résumé

Séminaire

Mardi 12 septembre 11:00-12:00 - Eleonora Catsigeras - Universidad de la República. Uruguay
Empiric stochastic stability of physical and pseudo-physical measures

---

groupe de travail

Manifestation scientifique

Ajouter un événement Modifier l'article

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Descriptif

Nature	Séminaire
Intitulé	Dynamique, Arithmétique, Combinatoire (Ernest)
Responsables	Sary Drappeau & Pierre Guillon
Équipe de rattachement	Géométrie, Dynamique, Arithmétique, Combinatoire et leurs interactions (GDAC)
Fréquence	Hebdomadaire
Jour-Horaire	Mardi. 11h05-12h
Lieu	Luminy, salle des séminaires 304-306 (accès)
Lien	-

- **Analyse Appliquée (AA)**
- **Analyse et Géométrie**
- **Arithmétique et Théorie de l'Information (ATI)**
- **Dynamique, Arithmétique, Combinatoire (Ernest)**
- **Géométrie, Dynamique et Topologie (GDT)**
- **Géométrie Complexe**
- **Logique et Interactions**
- **Mathématiques, Évolution, Biologie (MEB)**
- **Modèles Probabilistes pour l'Évolution (MPE)**
- **MABioS**
- **Probabilités et Statistique**
- **Processus Déterminantaux (ERC IChaos)**

6

Stochastic Perturbations of Piecewise Continuous Maps- Talk in the conference "New Trends in One-dimensional Dynamics Celebrating the 70th. Anniversary of Welington de Melo" , Rio de Janeiro , Brazil

Eleonora Catsigeras

2016

New Trends in Onedimensional Dynamics
Celebrating the 70^{th} anniversary of Welington de Melo

Rio de Janeiro, November 14 - 18, 2016

Title: ***Stochastic perturbations of piecewise continuous maps.***

Author: *Eleonora Catsigeras*

Abstract.

We consider a piecewise continuous map $f : X \mapsto X$ with a finite number of continuity pieces on a compact metric space X. We study the ergodic properties of the stochastic dynamical system (X, fP_ϵ), obtained by adding at each iterate of f a noise of level $\epsilon > 0$, namely, a stochastic perturbation of f with family $P_\epsilon = P_\epsilon(x, \cdot)$ of transition probabilities, supported on the ball $B_\epsilon(f(x))$ for each $x \in X$.

We construct a transfer operator $\mathcal{L}^*_\epsilon$ in the space of probability measures, whose fixed points are the stationary measures of (X, f, P_ϵ). Under mild hypothesis on the transition probabilities, we prove the existence and finitude of ergodic stationary probability measures μ. We also prove that for each ergodic μ there exists a unique maximal period $p \geq 1$ and a $\mathcal{L}^*_\epsilon$-periodic probability measure ν with period p, such that $\mu = (1/p)\sum_{j=0}^{p-1} \mathcal{L}^{*j}_\epsilon \nu$. Finally, we prove that the ergodic periodic measures ν of maximal period are weakly mixing and also ergodic for all the multiples of $\mathcal{L}^{*p}_\epsilon$.

This is a joint work with Pierre Guiraud, Arnaldo Nogueira and Sandro Vaienti.

Stochastic Perturbations of Piecewise Continuous Maps

Eleonora Catsigeras

Universidad de la República
Montevideo, Uruguay

`eleonora@fing.edu.uy`

New Trends in Onedimensional Dynamics
Celebrating the 70th anniversary of Welington de Melo
IMPA, Rio de Janeiro, November 14-18, 2016

Notation:

X compact metric space, $\mathcal{A}$ is its Borel sigma-algebra.
$\mathcal{M}$ is the space of all the probability measures on $(X, \mathcal{A})$ endowed with the weak* topology;
$C_0(X, \mathbb{C})$ space of continuous functions $\varphi : X \mapsto \mathbb{C}$ with the sup norm;
L_∞ space of bounded measurable functions.

Definition

$f : X \mapsto X$ **is Piecewise Continuous** if there exists a "topological partition" $\{X_i\}_{1 \le i \le N}$ (i.e. X_i open, $X_i \cap X_j = \emptyset$ if $i \neq j$, and $\bigcup \overline{X}_i = X$)
and continuous maps $f_i : \overline{X}_i \mapsto X$ such that

$$f|_{X_i} = f_i.$$

Notation: $\Delta =$ set of discontinuity points of f

Definition

A Stochastic System by perturbation of f with noise of level $\epsilon > 0$ is (X, f, P_ϵ) where

$$P_\epsilon = \{P_{\epsilon,i}(x,\cdot)\}_{x\in\overline{X}_i} 1 \leq i \leq N$$

is the *family of transition probabilities* $P_{\epsilon,i}(x,\cdot) \in \mathcal{M}$ *of the stochastic process* $\{x_n\}_{n\geq 0}$, i.e.
$P_{\epsilon,i}(x, A) = \text{prob.}\big(\{x_{n+1} \in A \,|\, x_n = x\}\big)$ for all $x \in \overline{X}_i$, and

$$\bullet \text{supp}(P_{\epsilon,i}(x,\cdot)) = \overline{B}_\epsilon(f_i(x)) \ \ \forall\ A \in \mathcal{A}, \ \ \forall\ x \in \overline{X}_i.$$

Hypothesis on the noise: • $x \in \overline{X}_i \ \mapsto\ P_{\epsilon,i}(x,\cdot) \in \mathcal{M}$ is continuous.

- $P_{\epsilon,i}(x, \Delta) = 0 \ \ \forall\ x.$
- If $A \subset \overline{B}_\epsilon(f_i(x)) \cap \overline{B}_\epsilon(f_j(x'))$, then $P_{\epsilon,i}(x, A) = 0$ if and only if $P_{\epsilon,j}(x', A) = 0$.

Definition

The transfer operator $\mathcal{L} : L_\infty \mapsto L_\infty$ is

$$(\mathcal{L}\varphi)(x) := \int \varphi(y)\, P_{\epsilon,i}(x, dy) \quad \forall\, x \in X_i$$

The dual transfer operator $\mathcal{L}^* : \mathcal{M} \mapsto \mathcal{M}$ is

$$\int \varphi\, d(\mathcal{L}^*\mu) := \int (\mathcal{L}\varphi\, d\mu \ \ \forall\, \mu \in \mathcal{M}.$$

A stationary measure $\mu \in \mathcal{M}$ is a fixed point of $\mathcal{L}^*$.
A periodic measure μ **with period** p is a fixed point of $\mathcal{L}^{*p}$ for a minimum natural value of $p \geq 1$.
A measure μ **of period** p **is ergodic** if for any $\mathcal{L}^p$-invariant set A (i.e. $\mathcal{L}^p\chi_A = \chi_A$ μ-a.e.) either $\mu(A) = 1$ or $\mu(A) = 0$.

Theorem

(Existence and finitude of ergodic stationary measures)
The set $\mathcal{E}$ of stationary ergodic measures for the stochastic system (X, f, P_ϵ) is nonempty and finite.

SKETCH OF THE PROOF (on the board)

Theorem

(Periodic ergodic measures of maximal period)
For each ergodic stationary measure μ there exists a unique maximal period $p \geq 1$ and a periodic and ergodic probability measure ν with period p, such that

$$\mu = \frac{1}{p}\nu + \mathcal{L}^*\nu + \ldots + \mathcal{L}^{*p-1}.$$

Besides, the ergodic periodic measures ν of maximal period are weakly mixing and also ergodic for all the multiples of $\mathcal{L}^{*p}$.

SKETCH OF THE PROOF (on the board)

Definition

A piecewise continuous map $f : X \mapsto X$ **is piecewise contracting** if there exists a constant $0 < \lambda < 1$ such that, for any continuity piece X_i of f:

$$\mathsf{dist}(f_i(x), f_i(x')) \leq \lambda \cdot \mathsf{dist}(x, x') \ \ \forall\, x, x' \in X_i.$$

A piecewise contracting map **is typically periodic** if its attractor Λ does not intersect the set Δ of discontinuity points. (In such a case, the attractor Λ is composed by a finite number of periodic orbits).

Theorem

If f is piecewise contracting and typically periodic, then, for all the stochastic perturbations of f with noise level $\epsilon > 0$ small enough, there exists an invertible correspondence ξ between the family of ergodic periodic measures with maximal period for the transfer operator $\mathcal{L}^*$, and the periodic orbits of the attractor of the deterministic system.
Besides,

$$\text{period}(\nu) = \text{period}(\xi(\nu)), \quad \text{and} \quad \xi(\nu) \in \text{supp}(\nu) \subset \overline{B}_{\epsilon/(1-\lambda)}(\xi(\nu)).$$

SKETCH OF THE PROOF (on the board)

7

Lebesgue Essential Exponent and Positive Entropy of Diffeomorphisms with Dominated Splitting

Seminário de Teoria Ergódica do IMPA , Río de Janeiro, Brazil

Eleonora Catsigeras

2016

Lebesgue-essential exponent and positive entropy of C^1 diffeomorphisms with dominated splitting.

Eleonora Catsigeras[1]

Seminário de Teoría Ergódica do IMPA, Río de Janeiro, July 1st, 2016

Abstract

We consider a C^1 diffeomorphism f on a compact manifold with dominated splitting. From an example of Gourmelon and Potrie, it is known that the topological entropy of f may be zero. Here, we will show sufficient conditions for the topological entropy be positive. We define the Lebesgue-essential exponent taking the asymptotic exponential rate of local variation of the Lebesgue measure, either to the future or to the past. We prove that if the Lebesgue-essential exponent is not very negative, then the topological entropy is positive. As a corollary, if the Lebesgue measure is f-invariant, or if it is non invariant but recurrent"(we will define this concept), then the topological entropy is positive.

This is a joint work with Xueting Tian.

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[1] Instituto de Matemática y Estadística Prof. Rafael Laguardia (IMERL), Universidad de la República, Uruguay

8

Exponente Lebesgue-esencial y entropía positiva en difeomorfismos C1 con splitting dominado. Seminario Dinámica Porteña, Valparaíso, Chile

Eleonora Catsigeras

2016

Exponente Lebesgue-esencial y entropía positiva en difeomorfismos C^1 con splitting dominado.

Eleonora Catsigeras

Universidad de la República, Uruguay

Viernes 29 de abril
17:00 hrs. Sala 2-2.
Instituto de Matemática, PUCV

Sea f un difeomorfismo de clase C^1 con splitting dominado en una variedad compacta. A partir de un ejemplo de Gourmelon y Potrie, se sabe que la entropía topológica de f puede ser cero. Aquí expondremos condiciones suficientes para que la entropía sea positiva. Estas condiciones consideran la variación de la medida de Lebesgue en la variedad, al iterar el difeomorfismo. Definimos el exponente “Lebesgue-esencial” como la tasa exponencial asintótica de crecimiento (o decrecimiento si fuera negativa) de la medida de Lebesgue. Probamos que si el exponente Lebesgue-esencial hacia el futuro o hacia el pasado no es muy negativo, entonces la entropía topológica de f es positiva. Como caso particular, si la medida de Lebesgue es invariante, o si es una medida no invariante pero “recurrente” (según definición que introduciremos), entonces la entropía topológica de f es positiva. Este es un trabajo conjunto con Xueting Tian.

CONTACT

CARLOS H. VASQUEZ
email: carlos.vasquez@pucv.cl
phone: (+56 32) 2274011

webpage: http://ima.ucv.cl/seminarios/dinamica-portena/

PLACE AND TIME

FRIDAY 16:00 - 18:00, ROOM 2-2
Instituto de Matemáticas
Pontificia Universidad Católica de Valparaíso

Blanco Viel 596, Cerro Barón, Valparaíso, Chile.

9

Transmission of Energy in Complex Networks
XIV Latin American Workshop on Nonlinear Phenomena (LAWNP) , Cartagena de Indias, Colombia

RUBIDO, N.; CABEZA, C; CATSIGERAS, E; MARTÍ, A ; GREBOGI, C.; BAPTISTA, M.S.

2015

Matheron and Marsily (Is transport in porous media always diffusive? A counterexample, Water Resources Research, Vol. 16. No. 5, 1980), where the continuous case was studied. This type of model has important applications in research as to the contaminant propagation in aquifers and oil extraction processes, where it is difficult to make a good prediction of the characteristics of the medium by using (for example) conventional methods like the injection and recollection of tracers (inert particles carried by diffusive-advective flow). We show a solution for the MSD in this discrete form, aided by numeric simulations we compare the analytical solution with numerical data. We verify that in this case the MSD from the walkers goes like t^a, where $a = 3/2$, showing that the advection and diffusion coupling in this type of medium is also a super-diffusive dynamics.

An information theoretical approach to characterize the neural dynamics

Fernando Montani[1], *Román Baravalle*[2], *Lisandro Montangie*[2], Osvaldo A. Rosso[2]

[1] Instituto de Física de Líquidos y Sistemas Biológicos (IFLYSIB), Universidad Nacional de La Plata, La Plata, Argentina
[2] Instituto de Física, Universidade Federal de Alagoas, Maceió, AL, Brazil

Neurons tend to fire a spike when they are near a bifurcation from resting to spiking activity. Many possible ionic mechanisms can be accounted for as the source of spike generation; moreover the biophysics and the dynamics behind it can be usually described through the phase diagram membrane voltage versus the activation variable of the ionic channel. We present an information theoretical approach to accurately distinguish the most fundamental properties of neurophysiological neurons that were previously described by Izhikevich considering the phase-space trajectory, using a time causal space: Statistical Complexity vs. Fisher Information vs. Shannon Entropy.

Transmission of energy in complex networks

Nicolás Rubido[1], *E. Catsigeras*[2], *C. Cabeza*[2], *A. C. Martí*[2], *C. Grebogi*[3], *M. S. Baptista*[3]

[1] Universidad de la República, Instituto de Física Facultad de Ciencias, Uruguay, and University of Aberdeen, King's College, Institute for Complex Systems and Mathematical Biology, Aberdeen, UK
[2] Universidad de la República, Instituto de Física Facultad de Ciencias, Montevideo, Uruguay
[3] University of Aberdeen, King's College, Institute for Complex Systems and Mathematical Biology, Aberdeen, UK

A fundamental motivation in Complexity Sciences is to understand and predict the implicit relationship between the structure and the behaviour of a complex system. The structure is a topological representation of how the units in the complex system are interconnected, i.e., the

system's connectivity skeleton. It constitutes the backbone of the system's interactions and it is formally described by the theory of graphs. Behaviour is a functional observable of the collective dynamics that the units in the network have. It shows how the dynamical properties of the complex system evolve and can be measured by a variety of different methodologies. The importance of understanding this relationship is because, in nature and society, both quantities are usually not simultaneously known. For example, in the brain, the behaviour can be measured (e.g., EEG and fMRI) but the exact structure is unknown. On the other hand, the structure of the modern power-grids are known, but the behaviour sometimes is unpredictable (e.g., black-outs and power drops). In this work I am going to focus on the derivation of explicit relationships between the structure and the dynamics of supply-demand networks, in particular, in the results we find for a phase-oscillator model of the power-grid transmission of energy known as the swing equations. Despite our particular choice, the results are unrestricted to power-grids and can be extended to other complex systems of interacting oscillators.

Persistent photoconductivity in $SrTiO_3$ single-crystalline fibers

Jorge-Enrique Rueda-P.[1], *J. E. F. S. Rodrigues*[2], *Antonio Carlos Hernandes*[2]

[1] Departamento de Física, Universidad de Pamplona, Pamplona, Colombia
[2] Instituto de Física de São Carlos, Universidade de São Paulo, SP, Brazil

A metastable increase of the dark conductivity caused by short illumination, known as persistent photoconductivity (PPC), has been observed in a variety of semiconductor materials. PPC has mainly been observed in III-V or II-V semiconductors and in crystalline organic semiconductors. Room temperature PPC has been observed in semiconductors such as GaN and GaInNAs, between other materials as polymorph of \alpha-sexithiophene. Recently, room temperature PPC has been observed in Verneuil-grown $SrTiO_3$ bulk single crystal. We have found persistent photoconductivity in a $SrTiO_3$ single-crystal fiber. The fiber was obtained using the Laser-Heat Pedestal Growth technique, and after the fiber had been annealed at high temperature with $SrCO_3$ powder in order to produce vacancy defects. When exposed to laser light of 450 nm wavelength at room temperature, the effect was observed for 24 hours after the light had been turned off.

Nonlinear transport in pump-ratchet hybrids

Nicolás Medina Sánchez, *Thomas Dittrich*

Departamento de Física, Universidad Nacional de Colombia, Bogotá, Colombia

Directed transport by means of nonlinear dynamics has been studied in two alternative frameworks, in periodically driven sawtooth potentials ("ratchets") and in the context of driven chaotic scattering ("pumps"). We here consider hybrids between these two types, spatially periodic po-

10

Conditions for Positive Entropy of Diffeomorphisms with Dominated Splitting. Talk in The Dynamical Systems, Ergodic Theory and Probability Conference Dedicated to the Memory of Nikolai Chernov, Birmingham, U.S.A.

Eleonora Catsigeras, Xueting Tian.

2015

THE DYNAMICAL SYSTEMS, ERGODIC THEORY, AND PROBABILITY CONFERENCE DEDICATED
TO THE MEMORY OF NIKOLAI CHERNOV
University of Alabama at Birmingham, May 18-20, 2015

CONDITIONS FOR POSITIVE ENTROPY OF DIFFEOMORPHISMS WITH DOMINATED SPLITTING

Eleonora Catsigeras (1)
in a joint work with Xueting Tian (2)

(1) Instituto de Matemática, Universidad de la República -URUGUAY eleonora@fing.edu.uy
(2) School of Mathematical Sciences, Fudan University, Shangai, CHINA xuetingtian@fudan.edu.cn

Parallel Session - Talk

Eleonora Catsigeras in a joint work with Xueting Tian

Title:

Conditions for positive entropy of diffeomorphisms with dominated splitting.

Abstract:

We find several conditions that imply positive entropy for C1 diffeomorphisms with dominated splitting. These conditions are related with Pesin's Formula of the Entropy which is satisfied by all the (always existing) SRB-like measures. This is a joint work with Xueting Tian.

Subject Area: 37D30; 37A35; 37B40

Lebesgue-essential exponents and positive entropy of C^1-diffeomorphisms with dominated splitting.

Eleonora Catsigeras* and Xueting Tian**

Dedicated to the Memory of Prof. Nikolai Chernov

Abstract

We study C^1-diffeomorphisms on compact manifolds that have global dominated splitting. We define the Lebesgue-essential exponents by considering the exponential rates according to which the differential of the Lebesgue measure asymptotically changes to the future and to the past. We find lower negative bounds of the Lebesgue-essential exponents that suffice for the topological entropy be positive. Finally, we prove some corollaries in particular cases: when a smooth measure is preserved, or, more generally, when the Lebesgue measure is recurrent.

Key words: Dominated Splitting; Entropy; Volume Change; SRB-like Measures
MSC 2010: 37D30; 37B40; 37D25; 37A35;

1 Introduction

Consider a compact and connected C^1-Riemannian manifold M without boundary and let $f \in \text{Diff}^1(M)$ be a C^1 diffeomorphism on M.

Definition 1.1. (Dominated Splitting) The diffeomorphism f has a *dominated splitting* $TM = E \oplus F$ if this splitting is defined in all the points of the tangent bundle, is continuous and non trivial (i.e. $\dim(E), \dim(F) \neq 0$), and there exists a constant $\alpha < 1$ such that

$$\|Df|_{E(x)}\| \cdot \|Df^{-1}|_{F(f(x))}\| \leq \alpha, \forall x \in M.$$

We call E and F the *dominated and dominating subbundles* respectively. We call α the *domination constant.*
Note: In the above definition the continuity of the splitting is a redundant condition [1].

Definition 1.1 is a generalization of uniform hyperbolicity and also of partial hyperbolicity. Since uniform and partial hyperbolic systems have positive topological entropy, we first pose the following question:

¿Have all C^1 diffeomorphisms with dominated splitting positive entropy?

The answer is negative. In fact, Gourmelon and Potrie [4] have recently constructed a counterexample on the torus $\mathbb{T}^2$. Nevertheless, it is known that the answer is positive under some additional restrictive hypothesis of f. For instance, if the dominated splitting is partially hyperbolic, then the topological entropy of f is positive, as proved by Saghin, Sun

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*Instituto de Matemática y Estadística "Rafael Laguardia" (IMERL), Facultad de Ingeniería, Universidad de la República. Uruguay. E-mail: eleonora@fing.edu.uy

** School of Mathematical Sciences, Fudan University, Shanghai 200433, People's Republic of China. E-mail: xuetingtian@fudan.edu.cn; tianxt@amss.ac.cn

and Vargas in [5]. Also, if f has a dominated splitting and preserves the Lebesgue measure (or a finite measure that is absolutely continuous with respect to Lebesgue), then it necessarily has positive entropy (see Corollary 3.4 at the end of this paper). This latter result is also a consequence of a Pesin-like formula for the metric entropy in the C^1-context, proved in [6].

To generalize the results of positive topological entropy for diffeomophisms with dominated splitting that do not necessarily preserve the Lebesgue measure and are not necessarily partially hyperbolic, we will focus on the assymptotic exponential derivative of the Lebesgue measure when applying f or f^{-1}. We will find a relation among those Lebesgue-essential exponents and the positiveness of the topological entropy. To do so, we introduce the following definition:

Definition 1.2. (Lebesgue-essential exponents)

For any point $x \in M$, we define the Lebesgue exponents $\lambda^+(x)$ and $\lambda^-(x)$ at x, to the future and the past respectively, by

$$\lambda^+(x) := \limsup_{n\to+\infty} \frac{1}{n} \log |\det Df^n_x|, \qquad \lambda^-(x) := \limsup_{n\to+\infty} \frac{1}{n} \log |\det Df^{-n}_x|.$$

Now, we define the *Lebesgue-essential exponents* λ^+_{ess} and λ^-_{ess}, to the future and the past respectively, by:

$$\lambda^+_{ess} := \text{Leb-ess sup } \lambda^+(x), \quad \lambda^-_{ess} := \text{Leb-ess sup } \lambda^-(x),$$

where Leb-ess sup $u(x)$ denotes the essential supremum with respect to the Lebesgue measure of the measurable real function u.

We are ready to state the main result of this paper:

Theorem 1. *Let $f \in \text{Diff}^1(M)$ have a dominated splitting $TM = E \oplus F$, where E and F are the dominated and dominating sub-bundles respectively. Let $0 < \alpha < 1$ be the domination constant. If*

$$\lambda^+_{ess} > -\dim(E) \log \alpha^{-1} \quad \text{or} \quad \lambda^-_{ess} > -\dim(F) \log \alpha^{-1}, \tag{1}$$

then the topological entropy of f is positive.

In Section 2 we will prove Theorem 1, and in Section 3 we will state and prove its corollaries.

2 Proof of Theorem 1.

To prove Theorem 1, we will construct an f-invariant probability measure with positive metric entropy. Thus, applying the variational principle, this construction implies that the topological entropy of f is positive, as wanted. The construction of such a probability measure will be based on the theory of pseudo-physical or SRB-like measures for C^1 maps, which was introduced in [3]. We will apply a result in [2] (generalizing a theorem in [6]): it provides a Pesin-like formula for the entropy to all the pseudo-physical or SRB-like measures of any $f \in \text{Diff}^1(M)$ with dominated splitting.

In the sequel, we denote by $\mathcal{M}$ the space of all the Borel probability measures on M endowed with the weak*-topology. We denote by $\mathcal{M}_f$ the set of f-invariant measures contained in $\mathcal{M}$. Recall that that $\mathcal{M}$ and $\mathcal{M}_f \subset \mathcal{M}$ are nonempty, weak*-compact, sequentially compact, and convex metric spaces.

Definition 2.1. (p-omega limit of x) For any point $x \in M$ we construct the sequence $\{\sigma_{f,n}(x)\}_{n \geq 1} \subset \mathcal{M}$ of *empiric probabilities* along the finite pieces of the future orbit of x, by

$$\sigma_{f,n}(x) := \frac{1}{n} \sum_{j=0}^{n-1} \delta_{f^j(x)}, \tag{2}$$

where, δ_y denotes the Dirac-delta probability measure supported on $y \in M$.

We define the *the p-omega limit* $p\omega_f(x) \subset \mathcal{M}_f$ by:

$$p\omega_f(x) := \{\rho \in \mathcal{M} \colon \exists\, n_j \to +\infty \text{ such that } \lim_{j \to +\infty} \sigma_{f,n_j}(x) = \rho\}.$$

Now, we fix a metric dist* in $\mathcal{P}$ that endows the weak*-topology, and recall the following definition taken from [3]:

Definition 2.2. (Pseudo-physical or SRB-like measures) Fix $\mu \in \mathcal{M}_f$ and $\epsilon > 0$. We define the *basin* $B_\epsilon(\mu)$ *of* $\epsilon-$*weak attraction of* μ by:

$$B_\epsilon(\mu) := \{x \in M \colon \ \text{dist}^*(p\omega_f(x), \mu) < \epsilon\}.$$

We call the probability measure μ *pseudo-physical or SRB-like* for f if

$$\text{Leb}(B_\epsilon(\mu)) > 0 \ \ \forall\, \epsilon > 0.$$

The following previous results are taken from [3], [2] and [6]:

Theorem 2.3. (C.-Enrich [3])

For any continuous map $f : M \mapsto M$ the set of pseudo-physical probability measures is nonempty, weak*-compact, does not depends of the chosen weak* metric, and contains $p\omega_f(x)$ for Lebesgue almost all $x \in M$.

Theorem 2.4. (Pesin-like formula, C.-Cerminara-Enrich [2] and Sun-T. [6])

If $f \in$ Diff$^1(M)$ has a dominated splitting $TM = E \oplus F$, where E and F are the dominated and dominating sub-bundles respectively, and if $\mu \in \mathcal{M}_f$ is pseudo-physical, then

$$h_\mu(f) \geq \int \sum_{i=1}^{\dim(F)} \chi_i(x) d\mu = \int \log|\det Df|_F|\, d\mu, \tag{3}$$

where $\chi_1 \geq \chi_2 \cdots \geq \chi_{\dim(M)}$ denote the Lyapunov exponents defined μ-a.e.

We are ready to start the proof of Theorem 1:

Let $f \in \text{Diff}^1(M)$ have a dominated splitting $TM = E \oplus F$, where E and F are the dominated and dominating sub-bundles respectively. Define

$$\lambda^F_{ess} := \ \text{Leb-ess}\sup\ \lambda^F(x),$$

where

$$\lambda^F(x) := \limsup_{n \to +\infty} \frac{1}{n} \log|\det Df^n_x|_{F(x)}|.$$

Lemma 2.5. *If $\lambda^F_{ess} > 0$ then the topological entropy of f is positive.*

Proof. Consider the set $A := \{x \in M : \limsup_{n\to+\infty} \frac{1}{n} \log|\det Df^n_x|_{F(x)}| > 0\}$. From the condition $\lambda^F_{ess} > 0$, we deduce that Leb(A) > 0. Denote by $\mathcal{P}_f \subset \mathcal{M}_f$ the non-empty set of physical-like measures for f, and apply Theorem 2.3:

$$p\omega_f(x) \subset \mathcal{P}_f \ \text{ Leb.- a.e.} x \in M.$$

Choose and fix a point $x \in A$ such that $p\omega_f(x) \subset \mathcal{P}_f$, and fix a sequence $n_j \to +\infty$ such that

$$\lim_{j\to+\infty} \frac{1}{n_j} \log|\det Df^{n_j}_x|_{F(x)}| = a > 0. \tag{4}$$

By choosing an adequate subsequence, there exists $\mu \in \mathcal{M}_f$ such that:

$$\lim_{j\to+\infty} \sigma_{f,n_j} = \mu \in \mathcal{M}_f. \tag{5}$$

After Definition 2.1, $\mu \in p\omega_f(x) \subset \mathcal{P}_f$. So, applying Theorem 2.4:

$$h_\mu(f) \geq \int \psi\, d\mu, \ \text{ where } \psi := \log|\det Df|_F|. \tag{6}$$

By the definition of the weak* topology in $\mathcal{P}$ (since ψ is a continuous real function), and from equalities (5) and (4), we deduce:

$$\int \psi\, d\mu = \lim_{j\to+\infty} \int \psi\, d\sigma_{f,n_j}(x) = \lim_{j\to+\infty} \frac{1}{n_j} \sum_{i=0}^{n_j-1} \psi(f^i(x)) =$$

$$\lim_{j\to+\infty} \frac{1}{n_j} \sum_{i=0}^{n_j-1} \log|\det Df_{f^i(x)}|_{F(f^i(x))}| = \lim_{j\to+\infty} \frac{1}{n_j} \log|\det Df^{n_j}_x|_{F(x)} = a > 0. \tag{7}$$

Joining inequalities (6) and (7) we obtain $h_\mu(f) > 0$ as wanted. ☐

End of the proof of Theorem 1

Proof. By hypothesis $\lambda^+_{ess} > -\dim(E)\log(\alpha^{-1})$ or $\lambda^-_{ess} > -\dim(F)\log(\alpha^{-1})$. It is not restrictive to assume that the first inequality holds. If not we would apply the same proof with the second inequality instead of the first one, and with f^{-1}, F, E in the place of f, E, F respectively.

Arguing as in the proof of Lemma 2.5 there exists a point $x \in M$, a sequence $n_j \to +\infty$, and a physical-like probability measure μ such that

$$\lim_{j\to+\infty} \frac{1}{n_j} \log|\det Df^{n_j}_x| = b > -\dim(E)\log\alpha^{-1}, \tag{8}$$

$$h_\mu(f) \geq \int \log|\det Df_F|\, d\mu, \tag{9}$$

$$\int |\det Df|\, d\mu = \lim_{j\to+\infty} \frac{1}{n_j} \sum_{i=0}^{n_j-1} |\det Df_{f^i(x)}| = \lim_{j\to+\infty} \frac{1}{n_j} \log|\det Df^{n_j}_x| = b. \tag{10}$$

Since $E \oplus F = TM$ is a Df-invariant splitting and μ is an f-invariant measure, applying Oseledets Theorem we obtain:

$$\int \log|\det Df|\,d\mu = \int \sum_{k=1}^{\dim M} \chi_k\,d\mu = \int \sum_{k=1}^{\dim F} \chi_k\,d\mu + \int \sum_{k=\dim F+1}^{\dim M} \chi_k\,d\mu =$$

$$\int \log|\det Df|_F|\,d\mu + \int \log|\det Df|_E|\,d\mu.$$

Thus,

$$\int \log|\det Df|_F|\,d\mu = \int \log|\det Df|\,d\mu - \int \log|\det Df|_E|\,d\mu \tag{11}$$

Besides, applying the definition of dominated splitting, we obtain:

$$\log|\det Df_x|_{E(x)}| \leq \dim(E)\log\|Df_x|_{E(x)}\| \leq \dim(E)\cdot\log\Big(\frac{\alpha}{\|Df^{-1}_{f(x)}|_{F(f(x))}\|}\Big) \leq$$

$$-\dim(E)\,\log\alpha^{-1} + \dim(E)\cdot\frac{\log|\det Df_x|_{F(x)}|}{\dim(F)}. \tag{12}$$

We recall that the dimensions of E and F are constant, because these sub-bundles are continuous and the manifold M is connected. Joining equality (11) with inequality (12):

$$\Big(1+\frac{\dim(E)}{\dim(F)}\Big)\int \log|\det Df|_F|\,d\mu \geq \int \log|\det Df|\,d\mu + \dim(E)\log\alpha^{-1}. \tag{13}$$

Finally, from inequalities (8), (9), (10) and (13) we conclude:

$$\Big(1+\frac{\dim(E)}{\dim(F)}\Big)h_\mu(f) \geq \Big(1+\frac{\dim(E)}{\dim(F)}\Big)\int \log|\det Df|_F| \geq \log|\det Df|\,d\mu + \dim(E)\log\alpha^{-1}$$

$$= b + \dim(E)\log\alpha^{-1} > -\dim(E)\log\alpha^{-1} + \dim(E)\log\alpha^{-1} = 0.$$

So, we conclude that $h_\mu(f) > 0$ as wanted. □

3 Corollaries.

In this section we apply Theorem 1 to some particular cases:

Corollary 3.1. *If $f \in \mathit{Diff}^1(M)$ has a dominated splitting and preserves the Lebesgue measure, then the topological entropy of f is positive.*

Proof. In fact, since $|\det Df| = 1$, the Lebesgue-essential exponents are zero. Thus, the condition 1 holds; hence Theorem 1 implies $h_{\text{top}}(f) > 0$. □

We will generalize Corollary 3.1 to cases for which the Lebesgue measure is not f-invariant, but the Lebesgue-essential exponents are still zero. To so do, we need the following definition:

Definition 3.2. (Recurrent measures)

Let $\rho \in \mathcal{M}$ (i.e. *rho* is a non necessarily invariant, Borel probability measure on M). We call ρ *a recurrent measure* if there exists a real number $0 < \delta < 1$ such that for any measurable set $B \subset M$, if $\rho(B) \geq 1-\delta$, then there exists $n_j \to +\infty$ such that

$$B \cap f^{n_j}(B) \neq \emptyset \quad \forall\, j \in \mathbb{N}.$$

Note that, due to Poincaré Lemma any f-invariant measure is recurrent (in such a case, δ can be arbitrarily chosen in the open interval $(0,1)$).

In the sequel, we denote by Leb the Lebesgue measure on M after a rescaling to make it a probability measure.

Corollary 3.3. *If $f \in \mathrm{Diff}^1(M)$ has a dominated splitting and the Lebesgue probability measure is recurrent, then the topological entropy of f is positive.*

Proof. Applying Theorem 1, it is enough to prove that $\lambda^+_{ess} \geq 0$ or $\lambda^-_{ess} \geq 0$. Arguing by contradiction, and recalling Definition 1.2, assume that there exists a real number $-a < 0$ such that the $\mathrm{Leb}(A) = 1$, where

$$A := \Big\{x \in M : \ \limsup_{n \to +\infty} \frac{\log|\det Df^n_x|}{n} < -a, \quad \limsup_{n \to +\infty} \frac{\log|\det Df^{-n}_x|}{n} < -a\Big\}.$$

For any natural number $N \geq 1$ and define

$$A_N := \Big\{x \in M : \ \frac{\log|\det Df^n_x|}{n} < -a, \quad \frac{\log|\det Df^{-n}_x|}{n} < -a \ \ \forall\, n \geq N\Big\}.$$

We have $A_N \subset A_{N+1}$ and $A = \bigcup_{N=1}^{+\infty} A_N$. So, $\lim_{N\to+\infty} \mathrm{Leb}(A_N) = \mathrm{Leb}(A) = 1$. So, for any given $0 < \delta < 1$ there exists $N \geq 1$ such that

$$\mathrm{Leb}(A_N) > 1 - \delta.$$

Consider any measurable set $C \subset A_N$. We obtain:

$$\mathrm{Leb}(C \cap f^n(A_N)) = \int_{x \in f^{-n}(C)\cap A_N} |\det Df^n_x|\, d\mathrm{Leb}(x) \leq e^{-na} \cdot \mathrm{Leb}(f^{-n}(C) \cap A_N) \quad \forall\, n \geq N.$$

$$\mathrm{Leb}(C \cap f^{-n}(A_N)) = \int_{x \in f^{n}(C)\cap A_N} |\det Df^{-n}_x|\, d\mathrm{Leb}(x) \leq e^{-na} \cdot \mathrm{Leb}(f^{n}(C) \cap A_N) \quad \forall\, n \geq N.$$

In particular, applying the above inequalities to $C_1 := A_N \cap f^n(A_N)$ and $C_2 := A_N \cap f^{-n}(A_N)$, we deduce $\mathrm{Leb}(C_1) \leq e^{-na} \cdot \mathrm{Leb}(C_2)$, $\mathrm{Leb}(C_2) \leq e^{-na} \cdot \mathrm{Leb}(C_1)$ $\forall\, n \geq N$. Thus, $\mathrm{Leb}(C_1) = \mathrm{Leb}(C_2) = 0$; hence

$$\mathrm{Leb}(A_N \cap f^n(A_N)) = \mathrm{Leb}(A_N \cap f^{-n}(A_N)) = 0 \quad \forall\, n \geq N. \tag{14}$$

Now, construct the measurable set $B := A_N \setminus \Big(\bigcup_{n=N}^{+\infty} f^n(A_N)\Big)$. From equalities (14) we obtain $\mathrm{Leb}(B) = \mathrm{Leb}(A_N) > 1 - \delta$. And by construction of B we obtain $B \cap f^n(B) = \emptyset$ $\forall\, n \geq N$. Since the above assertions hold for any $0 < \delta < 1$, we conclude that Leb is not recurrent, contradicting the hypothesis. ☐

Finally, we state and prove the following consequence of Theorem 1. It is a generalization of Corollary 3.1.

Corollary 3.4. *If $f \in \mathrm{Diff}^1(M)$ has a dominated splitting and preserves a smooth probability measure ρ (i.e. $\rho \ll$ Leb), then the topological entropy of f is positive.*

Proof. By hypothesis $\rho \ll \text{Leb}$. Let us prove that there exists $0 < \delta < 1$ such that $\rho(C) < 1$ for any measurable set $C \subset M$ such that $\text{Leb}(C) < \delta$. In fact, arguing by contradiction, if the latter assertion were false, then for any natural number $n \geq 2$ there would exist $C_n \subset M$ such that $\text{Leb}(C_n) < 1/n$ and $\rho(C_n) = 1$. So, taking $A = \bigcap_{n=1}^{+\infty} C_n$ we would obtain $\rho(A) = 1$ and $\text{Leb}(A) = 0$, which contradicts the hypothesis $\rho \ll \text{Leb}$. So, we have proved the existence of a real number $0 < \delta < 1$ satisfying the assertion at the beginning.

Take any measurable set $B \subset M$ such that $\text{Leb}(B) > 1 - \delta$. Thus $\text{Leb}(M \setminus B) < \delta$. By construction of δ we obtain $\rho(M \setminus B) < 1$; hence $\rho(B) > 0$. But besides $\rho \in \mathcal{M}_f$. Thus, from Poincaré Recurrence Lemma we deduce that there exists infinitely many future iterates of B that intersect B. In brief, we have proved that for any measurable set B such that $\text{Leb}(B) > 1 - \delta$ there exists $n_j \to +\infty$ such that $B \bigcap f^{n_j}(B) \neq \emptyset$. Applying Definition 3.2, the Lebesgue measure is recurrent. Finally, from Corollary 3.3, we conclude that the topological entropy of f is positive. □

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11

Sincronización de eventos en circuitos neuronales predominantemente excitatorios

Presentación oral en el Seminario de Investigadores de Física de la Facultad de Ciencias, UdelaR, Montevideo.

Eleonora Catsigeras

2015

Subcircuitos predominantemente excitatorios: condiciones matemáticas suficientes para la sincronización de eventos

Eleonora Catsigeras

IMERL - Fac. Ingeniería
Universidad de la República

`eleonora@fing.edu.uy`

Seminario de Física
Instituto de Física de la Facultad de Ciencias
Universidad de la República
Montevideo, 19 de noviembre de 2015.

Seminario de Física
Facultad de Ciencias
Universidad de la República

Presentación oral
jueves 19 de noviembre de 2015

Eleonora Catsigeras

Título: Sincronización de eventos en subcircuitos neuronales predominantemente excitatorios

Resumen:

Se consideran modelos matemáticos simplificados de subcircuitos neuronales con conexiones sinápticas predominantemente excitatorias.

Se demostrará ciertas desigualdades matemáticas que vinculan algunos de los parámetros del subcircuito como condiciones
suficientes para la presencia recurrente de sincronización de los disparos de todas las neuronas del subcircuito.

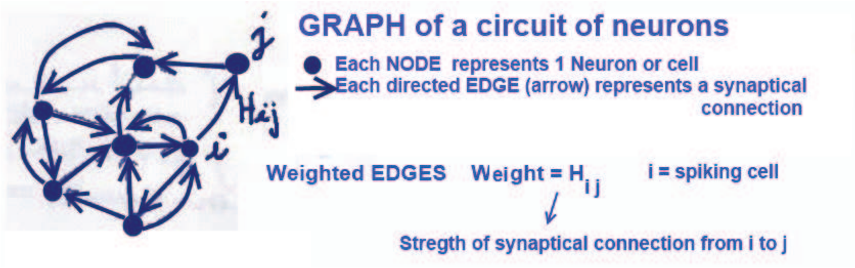
Las desigualdades matemáticas encontradas involucran los siguientes parámetros del subcircuito:

cantidad de neuronas excitatorias e inhibitorias en el subcircuito,
configuración topológica del grafo de conexiones sinápticas,
mínimo peso positivo de las excitaciones sinápticas,
y máximo peso negativo de las inhibiciones sinápticas en el subcircuito.

En sentido contrario, mostraremos que aunque las excitaciones sinápticas excitatorias sean predominantes, la sincronización de disparos en forma recurrente no se produce
si los circuitos considerados involucran relativamente pocas neuronas, en relación con la mínima interacción positiva.

ORGANIZATION OF THIS TALK

- Object of study: A mathematical system modeling a circuit of "neurons".
(Abstract – General – Simplified)
- Questions to research: Quantitative and qualitative dynamics of the mathematical system.
- Methodology of research: Logical deductive proofs (Rigorous proofs).
- Obtained results: Theorems 1 and 2 (their statements and their proofs).
- Mathematical proofs: Unfortunately not included in this talk, but they are the most enjoyable parts of the work.
- Conclusions: How to interpret the statements of Theorems 1 and 2 and their corollaries. They are necessarily true in the (simplified - general - abstract) mathematical model. But ¿do they necessarily hold in the real physical world?



GRAPH of a circuit of neurons

● Each NODE represents 1 Neuron or cell

→ Each directed EDGE (arrow) represents a synaptical connection

Weighted EDGES Weight = H_{ij} i = spiking cell

↓

Stregth of synaptical connection from i to j

Definition

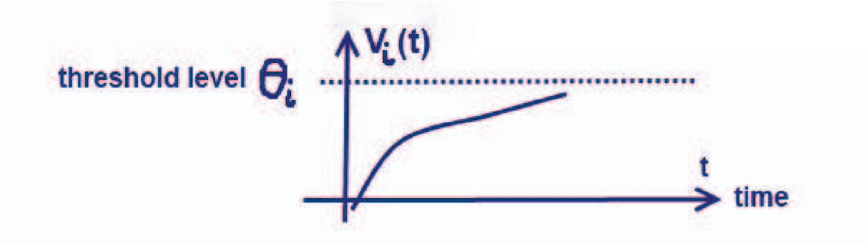
The cell i (node i) is EXCITATORY if $H_{ij} \geq 0 \quad \forall\, j \neq i.$
The cell i (node i) is INHIBITORY if $H_{ij} \leq 0 \quad \forall\, j \neq i.$

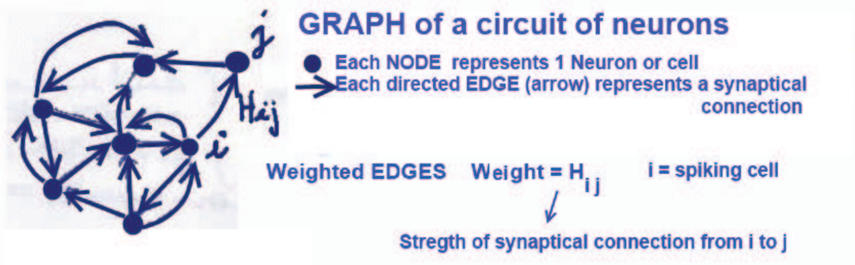
Interspike Regime

(One-dimensional neuron)
Governed by a 1-dimensional differential equation:

$$\frac{dV_i}{dt} = F_i(V_i), \text{ where } F_i > 0, \text{ while } V_i(t) \leq \theta_i$$

The value θ_i is the *THRESHOLD LEVEL*.





GRAPH of a circuit of neurons

- ● Each NODE represents 1 Neuron or cell
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Weighted EDGES Weight = H_{ij} i = spiking cell

↓

Stregth of synaptical connection from i to j

The cell i (node i) is excitatory if $H_{ij} \geq 0 \quad \forall\, j \neq i$.
The cell i (node i) is inhibitory if $H_{ij} \leq 0 \quad \forall\, j \neq i$.

Synaptical Rule

When neuron i spikes (at instant t_0), the potential V_j of neuron $j \neq i$ suffers and "instantaneous" jump:

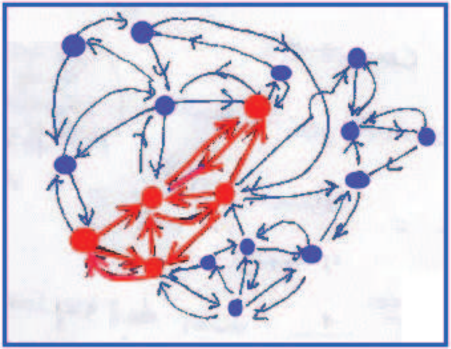
$$V_j(t_0) = V_j(t_0^-) + H_{i,j} \text{ if this number is } < \theta_j, \text{ or}$$

$$V_j(t_0) = 0 \text{ otherwise, and if so, also } j \text{ spikes at instant } t_0.$$

This math. model of the synaptical actions is SIMPLIFIED so:

- Instantaneous jump in the postsynaptical potential V_j.
- The REFRACTORY PHENOMENON holds for the spiking cells.

Graph of a circuit containing the graph of a subcircuit (in red) which is called a SUBGRAPH



GRAPH (CIRCUIT)

SUBGRAPH (SUBCIRCUIT)

Parameters' Space

Which are the "parameters" of the mathematical system modeling the circuit of neurons? Their values of some of them are NOT numbers but FUNCTIONS or other non numerical MATHEMATICAL STRUCTURES.

For the relaxation and spiking regime of the neurons:

$$(m, F_1, F_2, F_3, \ldots, F_m, \theta_1, \theta_2, \theta_3, \ldots, \theta_m), \text{ where}$$

- m is the number of neurons in the circuit
- F_i is the (vectorial) function at the second member of the system of differential equations $d\overline{x}_i/dt = F_i(\overline{x}_i)$ governing the relaxation regime of the neuron i.
- θ_i is a real number: the threshold level of neuron i.

For the synaptical connections:

$$(G,\ H_{1,2}, H_{1,3}, \ldots, H_{m-1,m}), \text{ where}$$

- G is a Graph Structure: the graph of the circuit, with m nodes $i \in \{1, 2, \ldots, m\}$ and directed and weighted edges $(i, j) :\ i \neq j$
- The weights $H_{i,j}$ of the edges of the graph G: they are (positive or negative or zero) real numbers.

Questions of Research: Dynamics of the Network (circuit or graph)

Which are the qualitative and quantitative mathematical properties that one may obtain, BY LOGICAL DEDUCTION, from the general mathematical model defined above?

EXAMPLES:

- Sequence of spiking instants
- ISI (interspike intervals) of a cell or of a subcircuit.
- Attractors and their basins - periodic orbits - limit cycles
- Synchronization of the spikes of several cells (periodic or non periodic synchronization)
- Waiting times until synchronization
- Recurrence

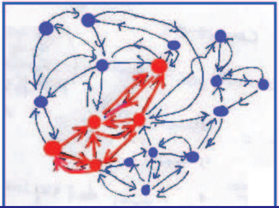
Definition

Recurrent synchronization of spikes in a subcircuit S
if there exists instants $t_1, t_2, \ldots, t_n, t_{n+1}, \ldots, \ldots$
such that at instant t_n all the cells of the subcircuit S spike simultaneously.

In Game Theory the phenomenon of synchronization is called “Grand Coalition”.

Remark: Between the simultaneous synchronizations at instants t_n and t_{n+1}, some neurons of the subcircuit may spike.

Sufficient mathematical conditions for recurrent synchronization of the spikes in a subcircuit.



GRAPH (CIRCUIT)

SUBGRAPH (SUBCIRCUIT)

Theorem 1

If

- *S is complete and excitatory: $H_{i,j} > 0 \quad \forall i \neq j$ in S,*
- *at least one cell in S is pacemaker,*
- *the number m of cells in S is large enough in relation to the mininum excitatory weight:*

$$\sqrt{m} \geq \frac{\max_{j \in S} \theta_j}{\min_{i \neq j \text{ in } S} H_{i,j}},$$

then

all the cells of the subcircuit S recurrently synchronize spikes while S does not receive inhibitions from the cells outside S.

Theorem 1

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then

all the cells of the subcircuit S recurrently synchronize spikes while S does not receive inhibitions from the cells outside S.

Remark This theorem holds:

- For any initial state of the cells
- Disregarding which are the functions F_i, and if they are similar or mutually very different, and which are the dimensions of vectorial states of the cells.
- No matter if the cells are mutually very different
- No matter if the interactions are mutually very different
- Disregarding how short or long are the refractory periods (but the refractory phenomenon must exist).

Theorem 1

If

- S *is complete and excitatory:* $H_{i,j} > 0 \ \ \forall i \neq j$ *in* S*,*
- *at least one cell in* S *is pacemaker,*
- *the number* m *of cells in* S *is large enough in relation to the mininum excitatory weight:*

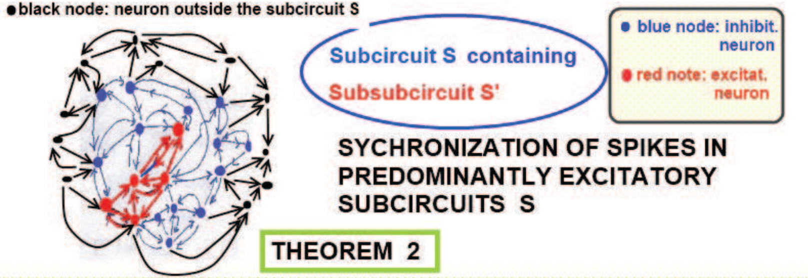
$$\sqrt{m} \ \geq \ \frac{\max_{j \in S} \theta_j}{\min_{i \neq j \text{ in } S} H_{i,j}},$$

then

all the cells of the subcircuit S *recurrently synchronize spikes while* S *does not receive inhibitions from the cells outside* S*.*

CHANGING THE CONNECTIONS OF THE SUBCIRCUIT S to be non complete, but still excitatory, provided the number of non null connections is large enough, still produce recurrent synchronization, if certain other mathematical conditions and inequalities hold (work in progress).

Some HISTORY:
1992 Mirollo-Strogatz
1996 Bottani



SYCHRONIZATION OF SPIKES IN PREDOMINANTLY EXCITATORY SUBCIRCUITS S

THEOREM 2

Theorem 2

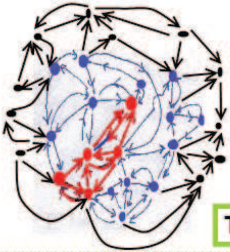
If the subcircuit S contains a sub-subcircuit S' such that

- *S' is complete and excitatory*
- *all the cells in $S \setminus S'$ are inhibitory*
- *at least one neuron of S' is a pacemaker*
- *the number m' of excitatory neurons in S' satisfies the following inequality:*

$$\sqrt{m'} \geq \frac{\left(\max_{j\in S}\theta_j\right) + \left(\max ISI_{S'}/\min_{j\in S} ISI_j\right)\cdot\left(\min_{j\in S\setminus S', i\in S'}|H_{j,i}|\right)}{\min_{i\neq j, i\in S', j\in S} H_{i,j}},$$

then all the cells of the subcircuit S recurrently synchronize spikes, while they do not receive inhibitions from the cells outside S.

● black node: neuron outside the subcircuit S



Subcircuit S containing

Subsubcircuit S'

● blue node: inhibit. neuron

● red note: excitat. neuron

SYCHRONIZATION OF SPIKES IN PREDOMINANTLY EXCITATORY SUBCIRCUITS S

THEOREM 2

CONCLUSIONS

1. To avoid recurrent synchr. of spikes in the subcircuit S' composed by excitatory cells connect the nodes of S' with edges coming from inhibitory cells, BUT:
2. If the inhibitory cells connected to S' are themselves excited by the cells of S, then
   - they do not avoid the recurrent synchronization of S'.
   - Worst, the inhibitory cells also synchronize spikes with the excitatory cells of S'.
   - So, due to the refractory phenomenon, the inhibitory cells spiking simultaneously with those of S' do not inhibit them.
3. Other cells outside S (in black in the figure), that are inhibited by S but not excited by S', may turn off (do not spike).

12

On the Asymptotic Properties of Piecewise Contracting Maps EquaDiff 2015 , Lyon (Francia) ,

MEYRONEINC, A; CATSIGERAS, E; GUIRAUD, P; UGALDE, E

2015.

On the Asymptotic Properties of Piecewise Contracting Maps

Arnaud Meyroneinc*[1], Eleonora Catsigeras[2], Pierre Guiraud[3], and Edgardo Ugalde[4]

[1]Departamento de Matematicas, Instituto Venezolano de Investigaciones Cientificas (IVIC) – Apartado 20632, Caracas 1020A, Venezuela

[2]Instituto de Matematica y Estadistica Rafael Laguardia, Universidad de la Republica (IMERL) – Montevideo, Uruguay

[3]Centro de Investigacion y Modelamiento de Fenomenos Aleatorios Valparaiso, Universidad de Valparaiso (CIMFAV) – Valpariaso, Chile

[4]Instituto de Fisica, Universidad Autonoma de San Luis Potosi – 78000 San Luis Potosi, Mexico

Abstract

We are interested in the phenomenology of the asymptotic dynamics of piecewise contracting maps. They appear for instance as Poincaré maps in the study of bifurcations of stable heteroclinic cycles of some C^1 (non-equivariant) vector fields on $\mathbb{R}^n$, in the characterization of Cherry flows on compact two-manifolds, or in the characterization of the asymptotic stability of some piecewise continuous vector fields. They can also be found as discrete time models of regulatory networks with thresholds. In either way, they naturally appear in the modelling of some biological systems (neural, genetic and ecological) and in engineering (electro-mechanical and switched arrival-server systems), where the number of variables can be significantly high.

We consider here a wide class of such maps, i.e. Lipschitz contracting when restricted to any piece of a finite and dense union of disjoint open pieces in a compact metric space X. We give sufficient conditions to ensure some general basic properties, such as the periodicity, the total disconnectedness or the zero Lebesgue measure (when $X \subset \mathbb{R}^n$, $n \geq 1$) of the attractor. These conditions show in particular that a non-periodic attractor necessarily contains discontinuities of the map. Under this hypothesis, we obtain numerous examples of attractors, ranging from finite to connected and chaotic, contrasting with the (quasi-)periodic asymptotic behaviours observed so far.

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*Speaker

sciencesconf.org:equadiff2015:64620

13

Condiciones suficientes de difeos C1 con splitting dominado para entropía positiva
Presentación oral en el Seminario de Sistemas Dinámicos , Montevideo

Eleonora Catsigeras, Xueting Tian

2015

Exposición en Seminario de Sistemas Dinámicos

5 de junio de 2015

Eleonora Catsigeras
Trabajo conjunto con Xueting Tian

Título:

Condiciones suficientes para que difeos C1 con splitting dominado tengan entropia positiva.

Resumen: Encontramos varias condiciones que, si son verificadas por la medida de Lebesgue (no necesariamente invariante) implican entropía positiva en difeos C1 con splitting dominado. Algunas de estas condiciones están relacionadas con una fórmula que acota inferiormente la entropía métrica para ciertas medidas invariantes, y que en el caso particular de que el splitting sea parcialmente hiperbólico, es la misma fórmula de Pesin, en el contexto C1, genéricamente no C1 más Hölder.

14

Teoría Ergódica
Curso dictado en la XVIII Escuela Venezolana de Matemática, EMALCA Venezuela

Alejandro Passeggi, Eleonora Catsigeras

2015

Asociación Matemática Venezolana
Escuela Venezolana de Matemáticas

XXVIII Escuela Venezolana de Matemáticas
Escuela de Matemática de América Latina y El Caribe - Venezuela 2015
Facultad de Ciencias, Universidad de Los Andes
Mérida, 30 de agosto al 4 de septiembre de 2015

- **Organización.**

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Julio Ramos, Universidad de Oriente.

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Neptalí Romero, Universidad Centroccidental Lisandro Alvarado.
Bladismir Ruíz, Universidad de Los Andes.
Carmen Judith Vanegas, Universidad Simón Bolívar.

- **Cursos para la XXVIII EVM y Emalca – Venezuela 2015**

CURSO I

Controlabilidad de ecuaciones de evolución semilineales.

Alexander Carrasco (Universidad Centroccidental Lisandro Alvarado, Venezuela), Hugo Leiva (Universidad de Los Andes, Venezuela) y Jahnett Uzcátegui (Universidad de Los Andes, Venezuela).

MOTIVACIÓN Y OBJETIVOS:

El objetivo de este curso es introducir de manera rápida y elegante a estudiantes de los últimos semestres de la licenciatura en matemática, estudiantes de maestrias y doctorados en temas afines e investigadores en ciencias aplicadas al fascinante mundo de la teoría matemática de los sistemas de control. En tal sentido, concentraremos nuestra estudio en los sistemas de control gobernados por ecuaciones de evolución; es decir, ecuaciones que involucran a una función desconocida con sus derivadas. Una vez definido el concepto de controlabilidad, probaremos que este es equivalente a que cierto operador lineal o semilineal, dependiendo de la ecuación, tienen rango

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Curso III

Teoría Ergódica.

Eleonora Catsigeras y Alejandro Passeggi (Universidad de La República, Uruguay).

Motivación y Objetivos:

Introducir las definiciones y teoremas básicos de la Teoría Ergódica, exponer algunos tópicos avanzados, y plantear algunos problemas abiertos de la Teoría Ergódica de los Sistemas Dinámicos determinísticos a tiempo discreto. Es un curso de matemática pura. Los resultados básicos que se estudiarán son aplicables en su mayoría a cualquier sistema dinámico determinístico continuo, con énfasis en las dinámicas que evolucionan en espacios o variedades de dimensión finita, incluyendo aquellas que son diferenciables. El curso requiere conocimientos de Análisis Real (teoría de la medida e integración abstracta en espacios de medida) e Introducción a la Topología. Otros conocimientos previos recomendados aunque no excluyentes: Geometría Diferenciable y Riemanniana, Introducción a los Sistemas Dinámicos.

Contenido:

1. Fundamentos de la Teoría Ergódica
   - Existencia de medidas invariantes.
   - Lema de Poincaré medible y de recurrencia topológica.
   - Teorema ergódico de Birkhoff. Teorema ergódico subaditivo de Kingman (solo enunciado).
   - Ergodicidad. Teorema integral de descomposición ergódica (solo enunciado).
2. Teoría Ergódica de atractores de sistemas dinámicos continuos
   - Atractor topológico y atractor de Milnor (definición y ejemplos).
   - Atractor ergódico de Pugh-Shub, medidas SRB o físicas (definiciones y ejemplos).
   - Atractor estadístico de Ilyashenko y medidas SRB-like (definiciones, ejemplos y teorema de existencia)
3. Teoría Ergódica de atractores de sistemas dinámicos diferenciables
   - Puntos regulares,exponentes de Liapunov, Teorema de Oseledets (enunciado general, demostración en dimensión 1).

- Región de Pesin. Subvariedades invariantes. Continuidad absoluta de foliaciones invariantes (solo enunciados).
- Relaciones entre medidas SRB y continuidad absoluta de medidas condicionales (solo enunciados y ejemplos).
- Teorema de Sinai-Ruelle-Bowen: existencia de medidas físicas (SRB) para atractores unif, hiperbólicos (enunciados, ruta de la demostración y planteo de algunos de los problemas abiertos relacionados).

4. ENTROPÍA Y FÓRMULA DE PESIN

- Entropía métrica y topológica. definiciones, interpretación, propiedades y ejemplos.
- Expansividad. Definición y ejemplos. Principios variacionales de la entropía (solo enunciados).
- Desigualdad de Ruelle para la entropía métrica (solo enunciado).
- Fórmula de Pesin para la entropía métrica. Mapas expansores (definiciones y ejemplos).
- Relación entre fórmula de Pesin, los EQ (estados de equilibrio respecto al potencial $-det\, Df$) y las medidas SRB (enunciados, demostración de alguno de los resultados en dimensión 1 y planteo de algunos de los problemas abiertos relacionados).

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CURSO IV

Teoría de Hipergrupos, problemas de Sturm-Liouville y polinomios ortogonales.

Yamilet Quintana (Universidad Simón Bolívar, Venezuela).

15

Racionalidad y Cambio Conceptual en Ciencias y Filosofía - Grupo de Trabajo VI Jornada de Investigación- Jornadas Académicas de la FHCE 2015, Montevideo.

NAVIA, R CATSIGERAS, E ALEMÁN, L

2015

Grupos de Trabajo aprobados para las VI Jornadas de Investigación, V Jornadas de Extensión y IV Encuentro de Egresados y Maestrandos.

GT	Título
1	**Del organismo al registro: arqueología de restos orgánicos** Federica Moreno Rudolph, Gonzalo Figueiro Lastreto
2	**Etnobotánica** Gregorio Tabakián, Juan Martín Dabezies, Juan Scuro
3	**Refugio y derechos humanos: diversas miradas sobre los movimientos forzados de población** Pilar Uriarte Bálsamo y Natalia Montealegre Alegria
4	**Salud - ambiente. Abordaje interdisciplinar** Betty Francia Ramos y Paola Rava Delepiane
5	**Educación Social, acreditación de saberes y mundo del trabajo** Dalton Rodriguez, Silvana Herou y Alberto Blanco
6	**Estudios en docencia: Rutinas y rupturas** Carmen Caamaño, Cristina Heuguerot y Mariela Lembo
7	**Etnografía e investigación en educación** Felipe Stevenazzi y Mabel Zeballos
8	**Experiencias escolares y convivencia: desigualdades, sociabilidad y construcción de ciudadanía en los países del Cono Sur** Nilia Viscardi Etchart y Pedro Núñez
9	**Formación en la Integralidad: intercambios metodológicos, pedagógicos y de trayectorias formativas** Eugenia Villarmarzo, Melina Romero y Felipe Stevenazzi
10	**Identificación, psicoanálisis y enseñanza** Ana María Fernández Caraballo y Joaquín Venturini
11	**Lecturas contemporáneas de Montaigne como filósofo y pedagogo** Andrea Díaz Genis, Robert Calabria y Enrique Puchet
12	**Políticas, prácticas educativas y sujetos de la educación** Pablo Martinis López, Clarisa Flous Lesca y Héctor Altamirano Martínez
13	**Elucidaciones matemáticas** José Seoane, Fernanda Pallares y Miguel Molina
14	**Estética y Filosofía del arte** Mónica Herrera y Fernando Suárez
15	**Filosofía de la Ciencia de la Computación** Alejandro Chmiel y Guillermo Nigro
16	**Heterogeneidades expresivas** José Seoane y Alejandro Chmiel
17	**Ian Hacking: una mirada a las ciencias sociales** María Laura Martínez y Matías Osta
18	**Ideas, ejemplaridad y forma. Desde Platón a la escolástica** Francisco O'Reilly y Andrea Carriquiry
19	**La formación humana para la vida buena** Andrea Dïaz Genis
20	**Problemas de Metafísica y de Metafilosofía de la Metafísica** Luciano Silva y Robert Calabria
21	**Racionalidad y cambio conceptual en matemática, arquitectura y filosofía (Grupo interdisciplinario)** Ricardo Navia, Eleonora Catsigeras y Laura Alemán

22 **Teoría lógica y práctica argumentales**
José Seoane y Ignacio Vilaró
23 **Verdad y significado en la filosofía moderna y contemporánea**
Carlos Caorsi, Ricardo Navia y Ronald Teliz
24 **La representación léxica de la temporalidad en español**
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Hekatherina Delgado y José Stagnaro
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Hebert Benítez Pezzolano
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34 **Literatura greco-latina**
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Cristina Pippolo Griego
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Claudia Pérez
37 **Leer a Cervantes en el Cuarto Centenario**
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Leticia Hornos Weisz, Alma Bolón y Pablo Rocca Pesce
39 **Problemas del archivo literario/ Problemas de la literatura**
Pablo Rocca, Alejandro Gortázar e Ignacio Dansilio
40 **¿Participar?... ¿Para qué? Experiencias y reflexiones en torno a la incidencia real de las prácticas participativas**
Deborah Techera Díaz, Lylieth Varela Fagúndez y Eliana Lotti Vigna
41 **Agua y Cultura. Paisajes, derecho humano y valorización de las aguas**
Javier Taks Donas
42 **Circulación de saberes: propuestas y demandas entre la universidad y las escuelas. Análisis de casos**
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Nicolás Duffau, Wilson González y Pablo Ferreira
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Yamandú Acosta, Carmen Beramendi y Lelio Nicolás Guigou
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47 **Cuerpo, poder y sociedad**
Marina Cardozo, Lourdes Peruchena y Andrea Salvo
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Alfredo Falero y Rossana Campodónico
49 **Fútbol, Cultura y Sociedad II**
Juan Luzuriaga Contrera y Andrés Morales Álvarez

50 **Izquierdas y memorias. Experiencias en la segunda mitad del siglo XX**
Carla Larrobla, Jimena Alonso y Magdalena Figueredo

51 **La ‘vuelta’. El retorno de los exiliados de la última dictadura, 1983- 1987**
Rossana Passeggi Baron y María de los Ángeles Fein

52 **La historiografía en los Estados de la Cuenca del Plata (siglos XIX y XX). Configuraciones, itinerarios, intercambios**
Dante Turcatti y Tomás Sansón Corbo

53 **Migraciones al Cono Sur. Recursos heurísticos, metodología e historiografía de las migraciones al Uruguay y la región (1870-1940)**
Dante Turcatti y Juan Andrés Bresciano

54 **Museología en el ámbito uruguayo. Presentación de experiencias de enseñanza, investigación y extensión**
Raquel Pontet y Carla Bica

55 **Uruguay y América Latina durante la Guerra Fría**
Magdalena Broquetas, Aldo Marchesi y Roberto García

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Eduardo Piazza

57 **Estudios de la danza**
Elisa Pérez Buchelli, Lucía Naser y Lucía Yáñez

58 **Humanidades en la universidad: modelos y presente universitario**
Ricardo Viscardi Capo y Alma Bolón

59 **Pensamiento y utopía de Pedro Figari a 100 años de la experiencia de la Escuela Nacional de Artes y Oficios (1915-1917)**
Pablo Thiago Rocca, Antonio Romano y Aníbal Corti

60 **Pensar las juventudes uruguayas y latinoamericanas. Abordajes de las ciencias sociales e interdisciplinarias**
Marcelo Rossal y Luisina Castelli Rodríguez

61 **Izquierdas, sindicatos y trabajadores en América Latina en el S. XX**
Rodolfo Porrini y Agustín Juncal

16

Matemática, Racionalidad y Relevancia

Presentación oral en el Seminario Interdisciplinario de Racionalidad y Filosofía de las Ciencias. Fac. Humanidades y Ciencias de la Educación, UdelaR.

Eleonora Catsigeras

2015

Jornadas Académicas de la Facultad de Humanidades y Ciencias de la Educación
VI Jornadas de Investigación
Octubre de 2015

Resumen para exposición oral
Grupo de Trabajo Nº 21
"Racionalidad y cambio conceptual en matemática, arquitectura y filosofía (Grupo interdisciplinario"

Fecha: 26 de junio de 2015
Enviado a: http://www.fhuce.edu.uy/index.php/presentacion-de-resumenes-de-ponencias-jornadas-2015

Título: Matemática, racionalidad y relevancia.

Autor: Eleonora Catsigeras

Resumen:

Se exploran algunos criterios
racionales de "relevancia" formal
e informal aplicados hoy en día en
la matemática.
¿Cuándo un teorema y su demostración
son considerados relevantes?

La discusión paradigmática
se desarrolla en forma escalonada
desde las concepciones racionales formales
de relevancia matemática,
hasta las racionales informales.

Entre las formales, se revisan los
criterios de relevancia lógica,
consistencia, no redundancia y
no trivialidad.

En un escalón intermedio
se exploran los criterios racionales
de relevancia relativa al contexto matemático
y la vinculada a la aplicabilidad del
resultado matemático a otras ciencias
y al desarrollo tecnológico.

Finalmente, entre los
criterios informales de relevancia racional en matemática,
se discute la relevancia estética y
la relativa a la sociología de la ciencia.

A lo largo de la exposición,

se ilustra la discusión con ejemplos concretos
que muestran cómo inciden y se aplican
estos criterios racionales de relevancia
en la investigación de la matemática.

17

La Matemática al ingreso universitario: ¿cómo transforma y es transformado un investigador matemático cuando enseña a ingresantes?

Presentación oral en las II Jornadas de Investigación en Educación Superior , Montevideo.
Artículo presentado publicado en CD-Rom.

Eleonora Catsigeras

2015

LA MATEMÁTICA AL INGRESO UNIVERSITARIO. ¿CÓMO TRANSFORMA Y ES TRANSFORMADO UN INVESTIGADOR MATEMÁTICO CUANDO ENSEÑA A INGRESANTES?

Eje Temático 2: Transformación de la enseñanza.

CATSIGERAS GARCÍA, Eleonora
Instituto de Matemática (IMERL),
Fac.de Ingeniería, Univ.de la República-URUGUAY
E-mail:eleonora@fing.edu.uy

Resumen. En este trabajo se investiga el problema de “Falta-de -Base -en -Matemáticas” de los estudiantes ingresantes a nuestra facultad. Se enfoca la investigación en estas preguntas:

- ¿Qué transformaciones puede producir en los estudiantes al ingreso, una reflexión filosófica sobre la matemática en sí misma, su creación y su enseñanza, por parte del docente-matemático-investigador?
- ¿Cómo es el proceso paulatino de transformación en el modo de aprender de los estudiantes y en el modo en el que el matemático va transformando sus concepciones y prácticas de enseñanza?

El marco teórico que antecede está determinado en tres campos: matemática, filosofía y educación. Se basa, entre otros, en:

- la revisión filosófica de Sriraman-English,(2010),
- la presentación de Dwyer,(2014) sobre su experiencia en universidades estadounidenses,
- la investigación teórico-experimental de Huikkola-Kirsi-Pohjolainen,(2012) al ingreso en universidades finlandesas.
- experiencias realizadas en cursos de matemática en nuestra facultad desde 2004.

La metodología de investigación teórica consiste en estudiar los fundamentos epistemológicos subyacentes en el aprendizaje-enseñanza de la matemática. La metodología experimental es “estudio de caso” en un curso de cálculo (modalidad diferenciada) al ingreso.

El análisis es conceptual-cualitativo, no numérico ni estadístico.

Algunas conclusiones preliminares son:

- La “Falta-de-Base-en-Matemáticas” tiene dos componentes. La primera es la falta de información y práctica sobre contenidos previos de Matemática. La segunda es desconocer qué es la Matemática (ontológica y epistemológicamente), y como consecuencia, qué es aprenderla y qué signitica enseñarla. Requiere *activar un proceso complejo de transformaciones* en las concepciones y prácticas de aprendizaje-enseñanza, *tanto en estudiantes como en docentes.*
- Las interacciones personales (docente-estudiante,estudiante-estudiante,docente-docente), individualizadas y en una multiplicidad de instancias, son necesarias para activar ese proceso de transformación.
- Las transformaciones son paulatinas y a largo plazo. Convencimiento y compromiso de estudiantes y docentes son imprescindibles para la transformación y no son fácilmente transferibles.
- Cada docente, aún el más experimentado, necesita recrear su propio proceso de transformación cada vez que reinicia un curso para ingresantes.
- Algunas teorías centradas en el constructivismo social y ontológico no funcionan bien en la enseñanza de la matemática.
- Los tiempos dedicados por un matemático-docente a la investigación y a la enseñanza compiten antagónicamente entre sí cuando los contenidos que enseña (por ejemplo en curso al ingreso) son disjuntos con sus investigaciones. Solución: la *investigación* en la enseñanza-aprendizaje al ingreso debería evaluarse también como *actividad de investigación del docente-matemático*, y no solo como parte de su tareas corrientes de enseñanza.

Referencias:

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Huikkola,M.,Silius,K.,Pohjolainen, S.(2008). Clustering and achievement of engineering students based on their attitudes, orientations, motivations and intentions. *WSEAS Trans.Adv.in Engineering Education*,Vol.5,Nº5,342--354.

Sriraman,B.,English,L.(2010). Surveying Theories and Philosophies of Mathematics Education.-In *Theories of Mathematics Education-Seeking New Frontiers,*7--32. Springer doi:10.1007/978-3-642-00742-2

Palabras Clave: *Matemáticas, Cálculo, Epistemología, Enseñanza de las Ciencias.*

18

Entropía topológica e hiperbolicidad parcial.
Presentación oral en el Seminario de Investigación Matemática, IVIC, Altos de Pipes, Venezuela

Eleonora Catsigeras

2015

Seminario de Matemática
Instituto Venezolano de Investigaciones Científicas (IVIC)

CONFERENCIA

16 de septiembre de 2015

Eleonora Catsigeras (trabajo conjunto con Xueting Tian)

Título:

Difeos C1 con "splitting dominado": condiciones suficientes para la positividad de su entropía topológica.

Resumen: Nos enfocamos en los difeomorfismos de clase C1 con "splitting dominado" en variedades compactas riemannianas de dimensión finita. Recientemente Gourmelon y Potrie contruyeron un ejemplo que muestra que la entropía topológica puede ser nula.

En este trabajo buscamos condiciones sufientes para que dichos difeos posean entropía positiva.
Encontramos varias de estas condiciones. Ellas relacionan el difeomorfismo con la medida de Lebesgue (no necesariamente invariante). Estas condiciones se cumplen como caso muy particular (no C1 genérico) cuando f es conservativo, pero también se cumplen en casos más generales en que f no preserva ninguna medida absolutamente continua respecto a la de Lebesgue.

Algunas de estas condiciones suficientes están relacionadas con una desigualdad que acota inferiormente la entropía métrica para ciertas medidas invariantes que llamamos "pseudofísicas" o "SRB-like".

En el caso particular de que el splitting sea parcialmente hiperbólico, esa desigualdad es una igualdad, la cual generaliza la conocida fórmula de Pesin para la entropía a un contexto de regularidad C1 y para las medidas pseudo-físicas o "SRB-like", en vez del contexto más restringido de los difeomorfismos C1 más Hölder, y en sustitución de las medidas físicas o SRB cuando estas no existen.

M compact manifold

$f \in \text{Diff}^1(M$.

DEFINITION DOMINATED SPLITTING (D.S.)

$TM = E \oplus F$ Df-invariant CONTINUOUS sub-bundles

Such that $\| Df_x|_{E_x} \| \cdot \| Df^{-1}_{f(x)}|_{F(f(x))} \| \le \sigma^{-1} < 1 \quad \forall x \in M$

$\sigma > 1$ CONSTANT.

Question

When a diffeo with D.S has positive entropy?

1st) NOT ALWAYS Gourmelon-Potrie Example in $\mathbb{T}^2$

2nd) If DS is partially hyperbolic : $\chi|_F \geq 0$ and $\chi|_E \leq 0$.

Saghin-Sun-Vargas

3rd) If Leb satisfies some conditions.

THEOREM 1 If f has D.S and Leb is recurrent (in particular if Leb $\in \mathcal{P}_f$) then $h_f > 0$

Def $\rho \in \mathcal{P}$ is "RECURRENT" if $\exists\ 0 < \delta < 1$ such that

$\rho(B) > 1 - \delta \Rightarrow f^{n_j}(B) \cap B \neq \emptyset$ for some $n_j \to +\infty$.

THEOREM 1 If f has D.S and ~~Leb is recurrent~~ then $h_f > 0$
(in particular if Leb $\in \mathcal{P}_f$)

Def $\rho \in \mathcal{P}$ is "RECURRENT" if $\exists\ 0 < \delta < 1$ such that
$\rho(B) > 1 - \delta \Rightarrow f^{n_j}(B) \cap B \neq \emptyset$ for some $n_j \to +\infty$.

THEOREM 2 If f has D.S and if at least ONE of the following holds then $h_f > 0$:

$$\lambda_{ess}^{TM,f} > -(\dim E) \cdot \log \sigma \quad (1) \qquad \Big| \qquad \lambda_{ess}^{F,f} > 0 \quad (2)$$

$$\lambda_{ess}^{TM,f^{-1}} > -(\dim F) \cdot \log \sigma \qquad \Big| \qquad \lambda_{ess}^{E,f^{-1}} > 0$$

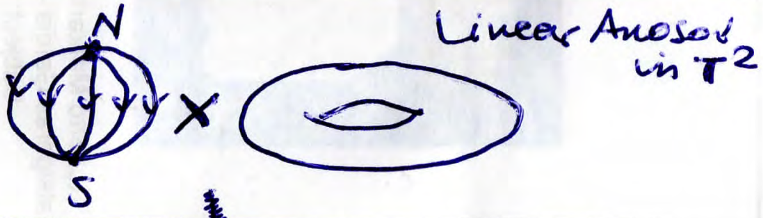
Def $\lambda^{TM,f}(x) := \limsup_{n \to +\infty} \frac{1}{n} \log |\det df^n_x|$

$\lambda^{TM,f}_{ess} :=$ Leb essent. sup of $\lambda^{TM,f}(x)$

$\lambda^{F,f}(x) := \limsup_{n \to +\infty} \frac{1}{n} \log \left| \det df^n_x \big|_{F_x} \right|$

$\lambda^{F,f}_{ess} =$ Leb ess. sup of $\lambda^{F,f}(x)$

Counterexample converse of theorem 1 is false in S^2



THE PROOFS

Theorem 1 is indeed a Corollary of Theorem 2 because

Lemma : if Leb is recurrent then $\lambda_{ess}^{TM,f} \geq 0$ or $\lambda_{ess}^{TM,f^{-1}} \geq 0$

THEOREM 1 If f has D.S and Leb is recurrent then $h_f > 0$

THEOREM 2 If f has D.S and if at least ONE of the following holds then $h_f > 0$:

$$\lambda_{ess}^{TM,f} > -(\dim E)\cdot\log\sigma \qquad \lambda_{ess}^{TM,f^{-1}} > -(\dim F)\cdot\log\sigma \qquad (1)$$

$$\lambda_{ess}^{F,f} > 0 \qquad \lambda_{ess}^{E,f^{-1}} > 0 \qquad (2)$$

Theorem 1 is indeed a Corollary of Theorem 2 because

Lemma: if Leb is recurrent then $\lambda_{ess}^{TM,f} \geq 0$ or $\lambda_{ess}^{TM,f^{-1}} \geq 0$

Proof of Lemma

By contradiction assume $\limsup_{n\to+\infty} \frac{1}{n} \log|\det df_x^n| < -\varepsilon$ and $\limsup_{n\to+\infty} \frac{1}{n} \log|\det df_x^{-n}| < -\varepsilon$ Leb a.e. x

$\exists N = N(x)$ Leb a.e. x such that $|\det df_x^n| < e^{-n\varepsilon}$, $|\det df_x^{-n}| < e^{-n\varepsilon}$ $\forall n \geq N = N(x)$

$A_N := \{x \in M : |\det df_x^n| < e^{-n\varepsilon}, |\det df_x^{-n}| < e^{-n\varepsilon} \ \forall n \geq N\}$

$A_N \subset A_{N+1}$ $\qquad \mathrm{Leb}(A_N) \xrightarrow[N\to+\infty]{} 1$ $\qquad \boxed{\mathrm{Leb}(A_N) > 1-\delta}$

$B \subset A_N$ $\qquad \mathrm{Leb}(A_N \cap f^n(B)) \leq e^{-n\varepsilon}\mathrm{Leb}(B)$; $\mathrm{Leb}(A_N \cap f^{-n}(B)) \leq e^{-n\varepsilon}\mathrm{Leb}(B)$

$B_1 = A_N \cap f^{+n}(A_N)$

$B_2 = A_N \cap f^{-n}(A_N)$

$B \subset A_N$

$B_1 = A_N \cap f^{+n}(A_N)$

$B_2 = A_N \cap f^{-n}(A_N)$

$$\mathrm{Leb}(A_N \cap f^{-n}(B_1)) = \mathrm{Leb}(\underbrace{A_N \cap f^{-n}(A_N)}_{B_2}) \leq e^{-n\varepsilon}\,\mathrm{Leb}(B_1$$

$$\mathrm{Leb}(\underbrace{A_N \cap f^{n}(B_2)}_{B_1}) \leq e^{-n\varepsilon}\,\mathrm{Leb}(B_2) < e^{-4n\varepsilon}$$

$$\boxed{\mathrm{Leb}(A_N \cap f^{n}(A_N)) = 0 \quad \forall n \geq N}$$

$\mathrm{Leb}(A_N) > 1-\delta$

$$C_N = A_N \setminus \bigcup_{n \geq N} f^{n}(A_N) \Rightarrow \mathrm{Leb}(C_N) = \mathrm{Leb}(A_N) > 1-\delta$$

↳ Leb no recurrent

$$C_N \cap f^{n}(C_N) = \emptyset \quad \forall n \geq N \longrightarrow$$

Proof of THEOREM 2 Ⓐ

Theorem 2 Ⓐ f with D.S.

If $\lambda^{F,f}_{ess} > 0$ then $h_f > 0$

where

$$\lambda^{F,f}(x) := \limsup_{n\to+\infty} \frac{1}{n} \log \left|\det df^n_x|_{F(x)}\right|$$

$$\lambda^{F,f}_{ess} = \text{Leb ess sup of } \lambda^{F,f}(x)$$

PREVIOUS DEFINITIONS

$x \in M$

$$p\omega(x) := \left\{ \nu \in \mathcal{P} : \frac{1}{n_j} \sum_{i=0}^{n_j - 1} \delta_{f^i(x)} \xrightarrow{\text{weak}^*} \nu \text{ for some } n_j \to +\infty \right\}$$

$p\omega(x)$ is $\overset{\text{weak}^*}{\text{compact}}$, nonempty $\subset \mathcal{P}_f$

Let $\mu \in \mathcal{P}$ BASIN OF STRONG STATISTICAL ATTRACTION OF μ is

$$B(\mu) = \{x \in M : p\omega(x) = \mu\}$$

BASIN OF ~~*~~ ε-WEAK STATISTICAL ATTRACTION of μ is

$$A_\varepsilon(\mu) = \{x \in M : \text{dist}^*(p\omega(x), \mu) < \varepsilon\}$$

Def μ is PHYSICAL OR SRB if $\text{Leb}(B(\mu)) > 0$

μ is PSEUDO-PHYSICAL OR SRB-like if $\text{Leb}(A_\varepsilon(\mu)) > 0 \ \forall \varepsilon > 0$

$$\left(\mathcal{O}_f := \{\mu \in \mathcal{P} : \mu \text{ is SRB-like}\}\right)$$

PREVIOUS RESULTS:

$\boxed{\text{Th.}}$ C-ENRICH Bull. Pol Acad. Sci 2011 If $f \in C^0(M)$

1) $\mathcal{G}_f$ is nonempty and weak* compact contained in $\mathcal{P}_f$

2) Leb a.e x $p\omega(x) \subset \mathcal{G}_f$

$\boxed{\text{Th.}}$ C-CERMINARA-ENRICH Erg Th. & DynSys 2015

If $f \in \text{Diff}^1(M)$ has D.S. then $\forall \mu \in \mathcal{G}_f$

$$h_\mu(f) \geq \int \log |\det df|_F| \, d\mu = \int \Big(\sum_{x_i \text{ of } df|_F} x_i \Big) d\mu.$$

PROOF OF THEOREM 2 A

$\lambda_{ess}^{F,f} > 0$ C $= \{ x \in M : \limsup_{n \to +\infty} \frac{1}{n} \log |\det df^n_x|_{F(x)}| > 0 \}$

$\hookrightarrow \text{Leb}(C) > 0$

Leb a.e $x \in C$ $p\omega(x) \subset \mathcal{G}_f$

choose $x \in C$ $n_j \to +\infty$

$$\boxed{\lim \frac{1}{n_j} \log |\det df^{n_j}_x| > 0}$$

$$\boxed{\lim^* \frac{1}{n_j} \sum_{i=0}^{n_j - 1} \delta_{f^i(x)} = \mu \in \mathcal{G}_f.}$$

$$h_\mu(f) \geq \int \log |\det df|_F| \, d\mu = \lim \frac{1}{n_j} \sum_{i=0}^{n_j-1} \log \Big| \det df_{f^i(x)}|_{F_{f^i(x)}} \Big| = \lim \frac{1}{n_j} \log |\det df^{n_j}_x| .$$

$$> 0.$$

$\square$

Proof of Theorem 2

Hip) $\underline{\lambda^{TM,f}_{ess} > -(\dim E)\log\sigma}$

$C = \{ x \in M : \limsup_{n\to+\infty} \frac{1}{n}\log|\det df^n_x| > -(\dim E)\log\sigma \}$

$\Rightarrow$ Leb $C > 0$ Leb a.e. $x \in C$: $p\omega(x) \in \mathcal{O}_f$

$n_j \to +\infty$ such that

(1) $\lim \frac{1}{n_j}\log|\det df^{n_j}_x| > -(\dim E)\log\sigma$

(2) $\lim^* \frac{1}{n_j}\sum_{i=0}^{n_j-1}\delta_{f^i(x)} = \mu \in \mathcal{O}_f$

C-Cerm. Enrich :

$h_\mu(f) \geq \int \log|\det df|_F|\,d\mu$ if >0 ✓

$$\boxed{h_\mu(f) \geq \int \log|\underset{\blacksquare}{\det}\, df|\,d\mu - \int \log|\det \underset{\blacktriangle}{df}|_E|\,d\mu \quad \text{if } \int \log|\det df|_F|\,d\mu \leq 0}$$

$$\boxed{\begin{array}{c} h_\mu(f) \geq \underbrace{\int \log|\det df|\,d\mu}_{\blacksquare} - \underbrace{\int \log|\det df|_E|\,d\mu}_{\blacktriangle} \\ \text{if } \int \log|\det df|_F|\,d\mu \leq 0 \end{array}}$$

$$\blacksquare = \int \log|\det df|\,d\underset{\in \mathcal{O}_f}{\mu} = \lim \int \log|\det df|\,d\,\frac{1}{n_j}\sum_{j=0}^{n_j-1}\delta_{f^i(x)} = \lim \frac{1}{n_j}\log|\det df^n_x|$$

$$\blacksquare > -(\dim E)\log\sigma$$

$$\blacktriangle = \int \log|\det df|_E|\,d\mu = \int \sum_{\substack{\text{Lyap exp}\\ \text{of } df|_E}} \chi_j\,d\mu < (\dim E)\left(\int \max_{\substack{\text{Lyap exp}\\ \text{of } df|_E}} \chi_j\,d\mu\right)$$

D.S. $\displaystyle \max_{\text{Lyap exp of } df|_E} \chi_j \leq \log\sigma^{-1} + \min_{\substack{\text{Lyap exp}\\ \text{of } df|_F}} \chi_i$ ±

$$\blacktriangle \leq -\dim E\log\sigma + \frac{1}{\dim F}\underbrace{\int \log|\det df|_F|\,d\mu}_{\leq 0} \leq -\dim E\log\sigma$$

$\square$

19

*Bifurcaciones de Adición de Período en Sistemas Dinámicos Continuos a Trozos.*Presentación oral en las VIII Jornadas de Ingeniería Matemática, Montevideo

Eleonora Catsigeras

2015

Facultad de Ingeniería - Montevideo - Uruguay

Maestria en Ingeniería Matemática

Menú

- Bienvenidos
- Programa
- Inscripción
- Organización
- Participantes inscriptos
- Contacto
- Jornadas Anteriores

Programa

El programa de las Jornadas incluye:

09:30 – 09:45: Presentación de la Maestría

9:45 – 10:30: Charla plenaria: Eleonora Catsígeras.

10:30 – 11:00: Pausa desayuno.

11:00 – 11:40: Charlas de estudiantes: Fernando Massa y Mario González

11:45 –12:25: Charla plenaria: Gabriel Usera.

12:25 – 12:55: Charla plenaria: Gregory Randall.

13:00 – 13:30: Charla plenaria: Sergio Nesmachnow.

13:30 – 15:30: Almuerzo.

15:30 – 16:30: Intercambio entre Estudiantes, Directores, y espacio de propuestas de Tesis.

16:30 – 17:30: Actividad social.

17:30: Regreso

Charlas

Sergio Nesmachnow

Título: “Planificación de eficiencia energética en sistemas computacionales”
Resumen: “La charla presenta los avances en la resolución de problemas de control y planificación de infraestructuras computacionales [centros de datos e infraestructuras de computación de alto desempeño] considerando eficiencia energética, mediante la aplicación de técnicas de inteligencia computacional”

Gabriel Usera
Titulo : "Simulación Numérica Multidinámica"
Resumen : Se presentaran las lineas de trabajo actuales del Grupo de Mecánica de los Fluidos Computacional del IMFIA, orientadas fundamentalmente a la simulación numérica de sistemas donde interaccionan distintos cuerpos, materiales y fenómenos, abordando una combinación de métodos como por ejemplo Volúmenes Finitos y Elementos Discretos.

Gregory Randall
Título: Algunas propuestas de temas para tesis de Maestría en trtamiento de imágenes por computadora
Resumen: el Departamento de Procesamiento de Señales del Instituto de Ingeniería Eléctrica ofrece varios temas de trabajo para estudiantes de la Maestría
en Ingeniería Matemática que quieran seguir una tesis y eventualmente un doctorado. En esta presentación presentaremos algunas de las propuestas
que se han presentado, acompañados de algunos antecedentes de trabajos previos realizados en el seno del grupo.

Eleonora Catsigeras

Título: Bifurcaciones de Adición de Período en Sistemas Dinámicos Continuos a Trozos.

Resumen: Algunos modelos simples de fenómenos dinámicos en la Física y la Neurociencia, entre otras ciencias, adoptan sistemas en varias variables reales que evolucionan en función del tiempo en forma autónoma y determinista, según ecuaciones diferenciales impulsivas. Estas ecuaciones, en muchos casos, pueden estudiarse como sistemas dinámicos por iterados de un mapa de retorno f (mapa de Poincaré) que es continuo a trozos. En el caso disipativo, se muestra que f es contractivo en cada trozo. Veremos algunos casos la dinámica de esos mapas contractivos a trozos presentan las llamadas bifurcaciones de adición de período. Este tipo de bifurcaciones no se producen en sistemas dinámicos por iterados de mapas continuos. Aún no están caracterizadas matemáticamente, pero aparecen en experimentos computacionales de diversos modelos en otras ciencias.

20

La Enseñanza Basada en la Persona.

Presentación oral en *Las Tutorías Didácticas como Estrategias de Apoyo a Docentes.*
Encuentro de Docentes de FING, Montevideo

Eleonora Catsigeras

2015

ENCUENTRO ENTRE DOCENTES DE FING

Los procesos de enseñanza y de aprendizaje en cursos con modalidad semipresencial en FIng

Presentan experiencias los docentes
Milton Vázquez y Mariana Pereira

Martes 1° de diciembre

Las tutorías didácticas en FIng como estrategias de apoyo a docentes

Presentan experiencias los docentes
Eleonora Catsígeras y Álvaro Giusto

Martes 8 de diciembre

Fechas: martes 1° y martes 8 de diciembre
Horario: 14:00 a 15:30 horas
Lugar: salón gris, número 727 (séptimo piso)

Organiza Unidad de Enseñanza Facultad de Ingeniería
Por consultas e inscripciones: uni_ens@fing.edu.uy

FACULTAD DE INGENIERIA

UNIDAD DE ENSEÑANZA
DE LA FACULTAD DE INGENIERÍA

Encuentro Docentes FIng - Tutorías Didácticas
8 de diciembre de 2015

ENSEÑANZA BASADA EN LA PERSONA

Experiencia en Cálculo 1 Anual 2014

Eleonora Catsigeras
eleonora@fing.edu.uy

Instituto de Matemática y Estadística "Rafael Laguardia" (IMERL)
Facultad de Ingeniería - Universidad de la República

Contexto Cálculo 1 al ingreso en FING

Enseñanza-aprendizaje de la Matemática al ingreso en la Facultad de Ingeniería

En particular: Cálculo Diferencial e Integral

¿"Falta-de-Base-En-Matemáticas"? ¿?

Alto índice de fracaso estudiantil en el 1er sem.

Estrategias de aprendizaje diversas o inexistentes

Masividad y diversidad

Estrategias de enseñanza uniformes ¿inadecuadas?

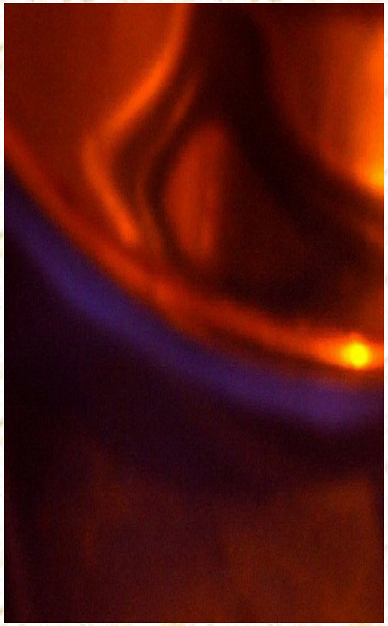
Cambio radical de objetivos y propósitos Sec. → Univ.

Gran desmotivación docente (salvo excepciones)

Tutoría Didáctica

TUTORÍA DIDÁCTICA (desde el punto de vista del docente)

- Asesoramiento desde punto de vista exterior al curso
- Presencia en clase
  Comentarios-discusión-observaciones inmediatas a la clase
- Encuestas y entrevistas estudiantes y docentes
- Reuniones de asesoramiento
- Evaluación continua de experiencia



Estrategias - Discusión

- La persona y su interacción social-académica (estudiantes-doc.)
- Ambiente
- Tipología de estudiantes
- Motivación subjetiva y colectiva (estudiantes y docentes)
- Fuerte dependencia entre estrategias y CONTENIDOS matem.

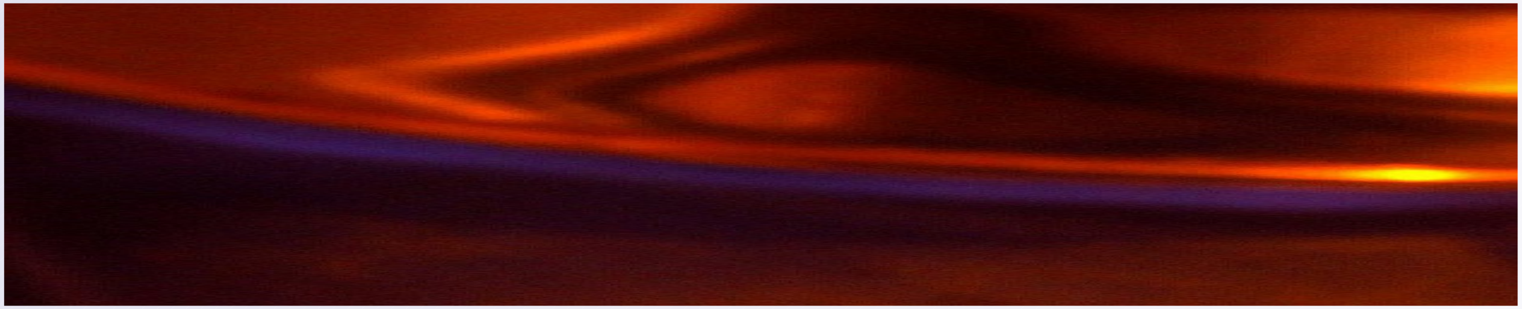
Estrategias - Discusión

Estudio en equipos relativamente pequeños

Interacción entre equipos

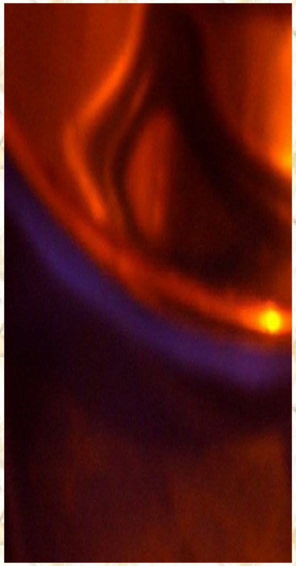
Clases expositivas magistrales: Minimización no, optimización sí

Dilución parcial de la diferencia entre “teórico” y “práctico”



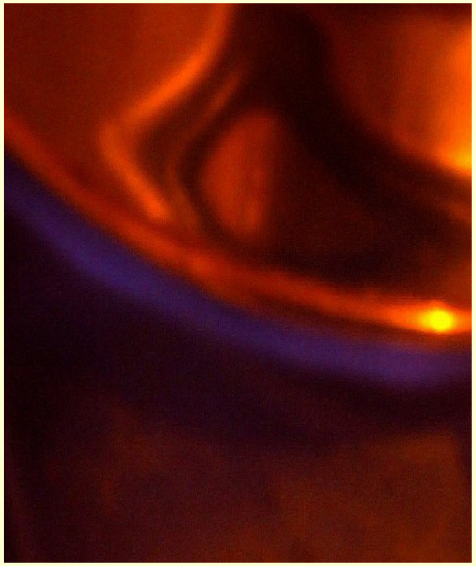
Estrategias - Discusión

- Evaluación continua en casi todas las clases, en equipo e individuales.
- Evaluaciones puntuales individuales a libro abierto.
- Integración de evaluaciones al proceso de aprendizaje - enseñanza



Estrategias - Discusión

- Clases de consulta sostenidas en equipos reducidos, frecuencia semanal (muy importantes)
- Lugar físico y ambiente de clases de consulta
- Visitas de estudiantes a institutos y entrevistas con ingenieros



Resultados cuantitativos

Total de Inscriptos al curso	123
Cantidad de estudiantes presentados a las instancias de evaluación continua con frecuencia aceptable durante el 1er semestre	85
Cantidad de estudiantes presentados al 1er parcial (mayo)	98
Cantidad de estudiantes presentados al 2do parcial (fin de junio)	81
Cantidad de estudiantes presentados a las instancias de evaluación continua con frecuencia aceptable durante2do. semestre-ver Comentario1	60
Cantidad de estudiantes presentados al 3er. parcial (octubre)	69
Cantidad de estudiantes presentados al 4to. parcial (fin de noviembre)	62
Cantidad de estudiantes que se presentaron todas las instancias de evaluación	57
Cantidad de estudiantes aprobados (con derecho a examen, y no exonerados)	16
Cantidad de estudiantes que exoneraron	55
Cantidad de estudiantes presentados al 1er. examen (diciembre de 2014)	5
Cantidad de estudiantes que aprobaron el 1er. examen	3

21

Redes Neuronales Cooperativas Acopladas por Impulsos.

Presentación oral en el Seminario de Mecánica Estadística y Física No Lineal, Fac. Ciencias, UdelaR.

Eleonora Catsigeras

2015

Redes Cooperativas Acopladas por Impulsos

Eleonora Catsigeras

IMERL - Fac. Ingeniería
Universidad de la República - Uruguay

`eleonora@fing.edu.uy`

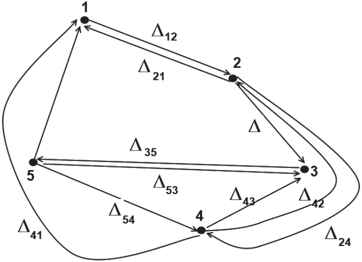
Presentación en el
Seminario de Física no Lineal
Facultad de Ciencias, Universidad de la República
Montevideo, 13 de julio de 2015

Objeto de estudio

Dinámica determinista de un red N
de $m \geq 2$ células (o neuronas: sub-sistemas dinámicos)
$i \in \{1, 2, \ldots, m\}$
acopladas por impulsos instantáneos.

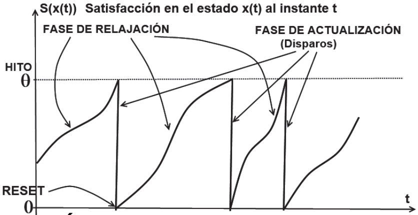
• CÉLULA O NEURONA i: subsistema dinámico autónomo
(*Dinámica libre de* i)

• ACOPLAMIENTOS $\Delta_{i,j}\ \forall\ (i,j) : i \neq j$



Dinámica libre de i:

Estado x_i de i en función del tiempo t
si i estuviera desacoplada de la red N. DOS FASES:



- **FASE DE RELAJACIÓN**

○ $\dot{x}_i = f_i(x_i)$ ecuación diferencial n-dim; flujo solución $x_i(t) = \Phi_i(x_i(0), t)$; $x_i \in X_i$

○ $S_i : X_i \mapsto [0, \theta_i]$ **función de satisfacción** ,

○ θ_i **goal ó hito ó threshold level**

$\frac{d}{dt}(S_i(x_i(t)) =< \bigtriangledown S_i, f_i \geq v_i > 0 \ \ \forall\ x_i \in S_i^{-1}[0, \theta_i)$

- **FASE DE ACTUALIZACIÓN - UPDATE RULE:**

○ $S_i(x_i(t^-)) = \theta_i \quad \Rightarrow \quad x_i(t) \in S_i^{-1}(0)$

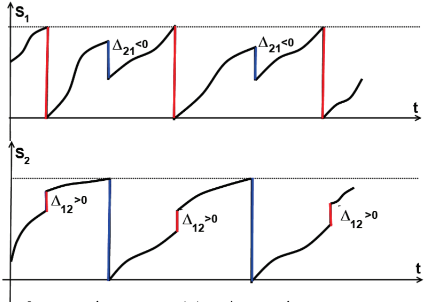
○ **Disparo o spike** Es la discontinuidad en el estado $x_i(t)$ que se produce cuando la variable de satisfacción S_i alcanza el hito θ_i: S_i se resetea a cero instantáneamente.

ACOPLAMIENTOS EN LA RED:

$$I(t) := \{1 \leq i \leq m \colon i \text{ dispara en el instante } t\}$$

REGLA DE INTERACCIONES INSTANTÁNEAS:

$$S(x_j(t)) := S(x_j(t^-)) + \sum_{i \in I(t),\ i \neq j} \Delta_{ij} \text{ si es } < \theta_j, \quad := 0 \text{ si es } \geq \theta_j.$$



Célula i es **Cooperativa:** $\Delta_{i,j} \geq 0\ \forall j \neq i$, $\Delta_{i,j} > 0$ para algún $j \neq i$.
Célula i es **Antagonista:** $\Delta_{i,j} \leq 0\ \forall j \neq i$, $\Delta_{i,j} < 0$ para algún $j \neq i$.
Principio de Dale (hipótesis) Cada célula o bien es cooperativa o bien es antagonista.

Red Cooperativa: Todas las células son cooperativas

Definiciones

Espacio (funcional) de parámetros de la red

$$\text{Param}(N) = \Big\{ \{(\phi_i, S_i, \theta_i)\}_{1 \leq i \leq m},\ \{\Delta_{i,j}\}_{1 \leq i,j \leq m,\ i \neq j} \} \Big\}$$

Topología C_0 en el espacio de parámetros.
Fenómeno robusto o persistente $\forall\ N$ que exhibe el fenómeno, existe un entorno V tal que toda red N' en ese entorno también exhibe el fenómeno.

Muerte de célula i en instante t_0
Para todo $t > t_0$ la célula i no dispara.
t_0 es el mínimo $t_0 \geq 0$ para el que ocurre lo anterior.

SINCRONIZACIÓN DE DISPAROS: existe $\{t_n\}_{n \geq 0}$
$0 < t_0 < t_1 < \ldots < t_n \to +\infty$ tal que $I(t_n) := \{1, \ldots, m\}$.
SINCRONIZACIÓN DE DISPAROS PERIÓDICA: existe $\{t_n^*\}_{n \geq 0}$ tal que $I(t_n^*) \neq \emptyset$ ($I(t_n^*)$ se llama n-ésimo **cluster**),
$I(t) = \emptyset$ si $t_n < t < t_{n+1}$, y existe $p \geq 1$ tal que

$$I(t_{hp}^*) = \{1, \ldots, m\}\ \forall\ h \in \mathbb{N}^+, \qquad I(t_n^*) = I(t_{np}^*)\ \forall\ n \in \mathbb{N}^*.$$

Características y ejemplos:
No hay reloj externo ni neuronas masters y otras slaves
Ejemplo de luciérnagas. Otros ejemplos en biología, neurociencias y ecología.
Ejemplos en redes en economía y otras ciencias sociales.

Resultados de sincronización

Teorema

1a) *Si la red es cooperativa con grafo completo, y si el número* m *de células es suficientemente grande, i.e.*

$$\sqrt{m} \geq \max \left\{ \sqrt{3}, \frac{\max_j \theta_j}{\min_{i \neq j} \Delta_{i,j}} + 1 \right\}$$

entonces la red **sincroniza disparos.** *El transitorio* T *entre un disparo simultáneo de todas las neuronas de la red y el siguiente, está acotado superiormente por* $T \leq \max_{1 \leq i \leq m} \dfrac{\theta_i}{v_i}$.

1b) *Si además la dinámica libre de cada neurona es tal que* $dS_i/dt = g_i(S_i)$*, entonces la sincronización de disparos es* **periódica con período** $p \geq 1$*: cada* p *disparos en la red, todas las neuronas disparan juntas. Hay* p **clusters diferentes** *entre un disparo de la red completa y el siguiente.*

1c) *Si además las neuronas son mutuamente similares, i.e.*

$$\frac{(\min_i \theta_i) \cdot (\min_i \min_{x_i \in X_i} < \nabla S_i, g_i >)}{(\max_i \theta_i) \cdot (\max_i \max_{x_i \in X_i} < \nabla S_i, g_i >)} > 1 - \frac{\min_{i \neq j} \Delta_{ij}}{\max_i \theta_i},$$

entonces $p = 1$*. Es decir, todas las neuronas disparan juntas cada vez que una de ellas dispara. Hay un solo cluster formado por todas las neuronas de la red.*

Demostración de sincronización

Hipótesis: $\sqrt{m} \geq \max\left\{\sqrt{3}, \dfrac{\max_j \theta_j}{\min_{i \neq j} \Delta_{i,j}} + 1\right\}$

Sean $t_1^* < t_2^* < \ldots < t_n^* < \ldots$ los instantes en que por lo menos una neurona de la red dispara.

Sea $K :=$ parte entera $\left(\dfrac{\max_j \theta_j}{\min_{i \neq j} \Delta_{i,j}}\right) + 1.$ (Obs: $K^2 < m$)

Afirmación A) A más tardar en el instante t_K ya dispararon todas las neuronas por lo menos una vez.

Afirmación B) Si en el instante t_n disparan por lo menos K neuronas simultáneamente, entonces disparan todas simultánemente.

Afirmaciones A) y B) $\Rightarrow$ existe un instante t_0 en que todas las neuronas disparan simultáneamente. De lo contrario la cantidad de neuronas sería menor que K^2 lo cual contradice la hipótesis. $\square$

Riesgo y factor de protección (Definiciones)

INTERFERENCIAS NEGATIVAS EXTERNAS A LA RED

Riesgo de muerte intrínseco de i, relativo a las otras neuronas de la red

$$R_i = \frac{\theta_i / v_i}{\max_{1 \leq j \leq m} (\theta_j / v_j)} \in (0,1]$$

Riesgo de muerte neto en red de i en el h-ésimo interspike interval $ISI_i^{(h)}$ de la neurona i:

$$R_i'^{(h)} = \frac{\max\{0,\ \theta_i - \sum_{j \in I(t): t \in ISI_i^{(h)}} \Delta_{j,i}\} \,/\, v_i}{\max_{1 \leq j \leq m} (\theta_j / v_j)} \in (0,1].$$

Factor de protección de la red a la neurona i en el h-ésimo interspike interval de i:

$$P_i^{(h)} = \min\left\{1,\ \frac{\sum_{j \in I(t): t \in ISI_i^{(h)}} \Delta_{j,i}}{\theta_i}\right\}.$$

Protección negativa si la red es antagonista.
Protección ≥ 0 si la red es cooperativa. Es 1 si $\sum_j \Delta_{j,i} \geq \theta_i$

Proposición:

$$R_i'^{(h)} = (1 - P_i^{(h)}) R_i \quad \text{es nulo cuando el factor de protección es } 1.$$

Resultados sobre riesgo y factor de protección

Teorema

2a) *(En las hipótesis del Teorema 1a (si la red es cooperativa de grafo completo y con suficiente cantidad de neuronas) entonces el factor de protección* $P_i^{(h)}$ *de cada neurona* i *en todo intervalo inter-spike* h*, es positivo. Luego el riesgo neto* R'_i *de muerte de la neurona* i *acoplada a la red, por interferencias negativas externas a la red, es menor estrictamente que el riesgo intrínseco* R_i *de la misma neurona si no estuviera acoplada a la red.*

2b) *Si además todas las neuronas son similares (hipótesis del Teorema 1b), entonces el factor de protección* P_i *de cada neurona es igual al máximo posible 100%, y su riesgo neto de muerte* R'_i *es el mínimo posible 0%.*

Cantidad de información (Definiciones)

$t_0^* < t_1^* \ldots < t_n^* <$ instantes en que por lo menos una neurona dispara.

- $I(t_n) = \{i : i$ dispara en el intante $t_n\} \neq \emptyset$.

Cantidad potencial de conjuntos diferentes en cada disparo: 2^m.

Cantidad potencial de información en cada disparo: $\log_2(2^m) = m$.

- **Spiking code (código de disparo):** Sucesión $\{I(t_n)\}_{n\geq 0}$. Depende de la condición inicial de todas las neuronas en la red.
- **Pattern de longitud finita $k \geq 1$:** Palabra de longitud k en el spiking code.

$$\pi_{n_0,k} = \Big(I(t_{n_0}), I(t_{n_0+1}), \ldots, I(t_{n_0+k-1}) \Big).$$ Depende del estado inicial.

- **Pattern recurrente de longitud $k \geq 1$:** palabra π_k de longitud k tal que existe algún estado inicial y una sucesión $n_j \to +\infty$ que lo realiza:

$$\pi_k = \pi_{n_j,k} \quad \forall\, j \in \mathbb{N}.$$

- $\Pi_k := \{\pi_k\}$ conjunto de todos los patterns recurrentes de longitud k.
- $\#\Pi_k$: cantidad de patterns recurrentes de longitud $k \geq 1$ diferentes que la red exhibe.
- **Cantidad de información** que la red puede procesar en forma recurrente:

$$H = \sup_{k\geq 0} \log \#\Pi_k.$$

Si $H = +\infty$ se define **entropía:** $h = \limsup_{k\to+\infty} \dfrac{\log \#\Pi_k}{k}$.

entropía = velocidad de crecimiento exponencial de la cantidad de información que la red puede procesar en forma recurrente.

Resultados sobre la cantidad de información

Teorema

2 a) *Si la red sincroniza periódicamente todos sus disparos con período* $p \geq 1$ *(por ej. en las hipótesis del teorema 1b), entonces* $H = \log_2 p < \dfrac{\log_2 m}{2}$.

2 b) *Si además las células son mutuamente similares, entonces* $H = \log_2 1 = 0$.

Conclusiones (redes cooperativas)

A) Cuando se maximiza el factor de protección de cada neurona (haciéndola 100% y logrando 0% de riesgo neto de muerte de cada una), se minimiza la cantidad de información H en la red, haciéndola nula.

B) Las redes cooperativas con suficiente cantidad de neuronas sincronizan y dan un factor de protección positivo a cada una de sus neuronas.
Si no son periódicas o si son periódicas con período p mayor que 1, entonces su cantidad de información total H(durante todo el tiempo futuro) es positiva. Si son periódicas, H es finita igual a $\log p$.

C) **Necesaria diversidad de células para tener cierta riqueza dinámica:**
Las redes cooperativas que pueden procesar una cantidad positiva de información están necesariamente compuestas por neuronas diversas, cuyas dinámicas intrínsecas difieren sensiblemente entre sí.

22

Atractores estadísticos y medidas pseudo-físicas

Presentación en el Seminario de Sistemas Dinámicos, Montevideo.

Eleonora Catsigeras

2014

ATRACTORES ESTADÍSTICOS Y MEDIDAS PSEUDO-FÍSICAS

Eleonora Catsigeras[1]

Presentación en el Seminario de Sistemas Dinámicos
16 de mayo de 2014

Resumen

Revisaremos la definición de atractor estadístico de Ilyashenko agregando la condición de minimalidad α-observable. Demostraremos la existencia de estos atractores, y su caracterización como el mínimo soporte compacto de todas las medidas pseudo-físicas de las órbitas en su cuenca de atracción estadística.

Finalmente, probaremos que, dado $\alpha > 0$ fijo, la variedad queda Lebesgue-c.t.p. partida en una cantidad finita de cuencas de atracción estadística de atractores de Ilyashenko α-observable minimales.

1. Introducción

La presentación sigue las diapositivas adjuntas. Su contenido corresponde a los resultados publicados en el artículo [6].

Más abajo incluimos las referencias bibliográficas citadas en dicho artículo.

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[1] Profesora Agregada del Instituto de Matemática y Estadística "Rafael Laguardia"(IMERL), Fac. de Ingeniería, Universidad de la República, Av. Herrera y Reissig 565, C.P. 11300, Montevideo, URUGUAY

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On Ilyashenko's statistical attractors

Eleonora Catsigeras

...

`eleonora@fing.edu.uy`

Seminario de Sistemas Dinámicos IMERL-CMAT
26 de julio de 2013
16 de mayo de 2014

$f: M \mapsto M$ Borel measurable map; M compact finite-dim. Riemannian manifold; m normalized Lebesgue measure $m(M) = 1$.

Definition

Ilyashenko's or Statistical Attractor is a compact set $\emptyset \neq K \subset M$ such that:

a) $m(B(K)) > 0$,

where the basin $B(K)$ of statistical attraction of K is

$$B(K) := \Big\{ x \in N : \quad \lim_{n \to +\infty} \frac{1}{n} \# \{ 0 \leq j \leq n-1 : \ \text{dist}(f^j(x), K) < \epsilon \} =$$

$$= 1 \quad \forall \, \epsilon > 0 \Big\}.$$

b) K is minimal with respect to its basin, i.e.:

$$\emptyset \neq K' \text{ compact } \subset K, \quad B(K') = B(K) \ m - \text{a.e.} \qquad \Rightarrow \qquad K' = K$$

COMMENTS: Global $m(B(K)) = 1$, existence; non global Karabacak-Ashwin; statistical attraction, physical measures, Milnor's attractors; examples Hu-Young, Quas, Eight, Bowen; non necessary f-invariance.

$0 < \alpha \leq 1; \quad \emptyset \neq K \subset M$ compact.

Definition

α-observability and α-obs. minimality
K is α-observable if $m(B(K)) \geq \alpha$.
K is minimally α-obs. if it is α-obs. and any proper compact subset of K is not α-obs.

Proposition

K is a Ilyashenko's statistical attractor if and only if it is a compact set minimally α-obs. for **some** *value of $\alpha > 0$.*

REMARKS Previously fixed $\alpha \in (0, 1]$: Ilyashenko's statistical attractor is not nec. α-observable, and if yes, then it is not nec. minimally α-obs. (Examples)
Maybe $\nexists$ minimal Stat. Attr. among all (Example)

Theorem

(Existence) *For any* $\alpha \in (0,1]$ *there exists an* α*-obs. minimal statistical attractor. If* $\alpha = 1$*, then it is unique.*

PROOF (on the board)

Recall Definition of SRB-like (or pseudo-physical) probability measure, and the theorem of their necessary existence (for any Borel measurable map $f : M \mapsto M$).

Theorem

(SRB-like characterization) *Any statistical attractor* K *is the minimal compact support of all the SRB-like probability measures of the map* $f|_{B(K)}$.
Conversely, if B *is a* m*-positive* f*-invariant set and if* K *is the minimal compact support of all the SRB-like measures of* $f|_{B(K)}$ *then* K *is a statistical attractor.*

PROOF (on the board)

Finite Lebesgue-decomposition of M : $\{B_1, B_2, \dots, B_k\}$ such that $m\big(M \setminus (\cup B_i)\big) = 0$ and $m(B_i \cap B_j) = 0$ for all $i \neq j$.

Theorem

Decomposition of the space *For any Borel measurable map $f : M \mapsto M$ and for any $\alpha \in (0, 1]$ there exists a* **finite** *Lebesgue-decomposition of M into the basins $B(K_i)$ of α_i-obs. minimal statistical attractors, with $\alpha_i = \alpha$ for all i except at most one.*

PROOF (on the board)

23

Deserción en curso de Matemática al ingreso a la Universidad: ¿conocimientos previos o vínculos humanos?

Artículo y poster presentados en IV Conferencia Latinoamericana sobre el Abandono en la Educación Superior , Medellín (Colombia)

CATSIGERAS, E; BLASINA, L; LOUREIRO, S; MÍGUEZ, M.

2014

DESERCIÓN EN CURSO DE MATEMÁTICA AL INGRESO A LA UNIVERSIDAD : ¿CONOCIMIENTOS PREVIOS O VÍNCULOS HUMANOS?

CATSÍGERAS, E.[1]; BLASINA, L.[2]; LOUREIRO, S.[2] y MÍGUEZ, M.[2]
[1] Instituto de Matemática y Estadística Rafael Laguardia
[2] Unidad de Enseñanza de Facultad de Ingeniería,
Universidad de la República – URUGUAY
mmiguez@fing.edu.uy

La Facultad de Ingeniería de la Universidad de la República ofrece desde el año 2005 una versión anual del curso "Cálculo 1" (Cal1) que tradicionalmente se imparte semestralmente. La propuesta se enfoca en la necesidad de un trabajo más profundo sobre los procesos educativos en el aula y las estrategias de aprendizaje de los estudiantes. Con este fin se realizaron ajustes en la presentación de los contenidos y sistemas de evaluación, y se incorporaron actividades que atienden a las dificultades de los estudiantes ingresantes: talleres sobre estrategias de aprendizaje, trabajo en grupo, modalidades de evaluación abiertas, etc. La inscripción a esta modalidad es opcional para los estudiantes que ingresan. En este trabajo se describe la experiencia del curso Cal1 que se viene desarrollando en 2014 en su modalidad anual, presentando algunos resultados preliminares.

Población que opta por Cal1: presenta diferencias con respecto al universo de los ingresantes, tanto en procedencia geográfica y en el tipo de institución en la que cursaron estudios secundarios, como en el resultado obtenido en la prueba diagnóstica (HDI) que se aplica al ingreso.

RESULTADOS Y ANÁLISIS

Sistemas de información utilizados:

- Herramienta Diagnóstica al Ingreso (HDI)
- Resultados académicos en pruebas parciales.
- Observaciones de clase realizadas por la Unidad de Enseñanza.
- Encuestas de opinión estudiantil.

RESULTADOS EN LAS PRUEBAS PARCIALES

•80 estudiantes se presentaron a las dos instancias de prueba durante el primer semestre del curso.

•35% ha superado el 60% del puntaje total (estarían exonerando)

•55% alcanzó entre el 25% y 59% de ese total (estarían aprobando el curso, ganando derecho a rendir examen)

•10% logra superar el 25% del puntaje total (tendrían que reprobar la asignatura)

80% indica que asiste a las 3 clases de teórico que se imparten semanalmente. (asistencia libre)

63% están al día con las tareas académicas.

78% de los estudiantes opina que la modalidad favorece la comprensión de la asignatura

59% indica que le resulta útil la modalidad de trabajo grupal.

"Me gusta la modalidad, aporta mucho aprendizaje, ...con trabajo grupal he comprendido los conceptos de manera eficiente.."

"Habría que incentivar a más estudiantes a cursar esta modalidad y cambiar las malas estadísticas que esta asignatura tienen en modalidad semestral…"

"Me gusta la modalidad, agradezco que se den estas oportunidades a los estudiantes que trabajamos y no tenemos buena base."

CONCLUSIONES

La población de estudiantes que opta por esta modalidad estaba a priori en riesgo de fracaso académico, de acuerdo a estudios realizados a partir de la HDI y de las características relevadas sobre la población desertora en la Facultad.

Los resultados en las evaluaciones obtenidos hasta el momento igualan o mejoran los resultados obtenidos en los cursos semestrales.

Se insiste en el enorme valor agregado relativo a los procedimientos y actitudes vinculados con el proceso de aprendizaje. Cuando lo que mueve al aprendizaje es el deseo de aprender (motivación intrínseca), sus efectos sobre los resultados obtenidos parecen ser más sólidos y consistentes que cuando el aprendizaje está movido por motivos externos.

La experiencia realizada viene resultando exitosa, mostrándose recomendable también para otros cursos.

24

Subcircuits with predominant excitatory pulsed interactions: Mathematical sufficient conditions to Synchronize Spikes (ABSTRACT) .

Talk in the International Symposium of Neurons, Circuits and Sytems , Montevideo

Eleonora Catsigeras

2014

SUBCIRCUITS WITH PREDOMINANT EXCITATORY PULSED INTERACTIONS: MATHEMATICAL SUFFICIENT CONDITIONS TO SYNCHRONIZE SPIKES

Eleonora Catsigeras[1]

Talk in the International Symposium on
Neurons, Circuits and Systems
Montevideo, Nov.30th – Dec.3rd, 2014

Abstract

We study a simplified mathematical model of neural subcircuits with synaptical interactions that are predominantly excitatory.

We prove that some inequalities relating parameters of the subcircuit in the model, are enough to produce the recurrent (but not necessarily periodic) synchronization of the spikes of all the neurons of the subcircuit.

The mathematical inequalities which are sufficienti conditions for recurrent syncrhonization take into account the following parameters of the subcircuit:

- Number of excitatory neurons of the subcircuit.
- The Graph mathematical Structure, which represents the oriented and weighted synaptical connections among neurons in the subcircuit.
- Minimum positive weight of the excitations in the subcircuit.
- Maximum absolute value of the negative weights of the inhibitions in the subcircuit.
- The maximum ISI (Interspike intervals) of the sub-subcircuit of excitatory neurons in the subcircuit.
- The minimum ISI of each inhibitory neuron in the subcircuit.

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[1] Instituto de Matemática y Estadística “Rafael Laguardia”(IMERL), Fac. de Ingeniería, Universidad de la República, Av. Herrera y Reissig 565, C.P. 11300, Montevideo, URUGUAY
E-mail: eleonora@fing.edu.uy

Subcircuits with predominant excitatory pulsed interactions: mathematical sufficient conditions to synchronize spikes

Eleonora Catsigeras

IMERL - Fac. Ingeniería
Universidad de la República - Uruguay

`eleonora@fing.edu.uy`

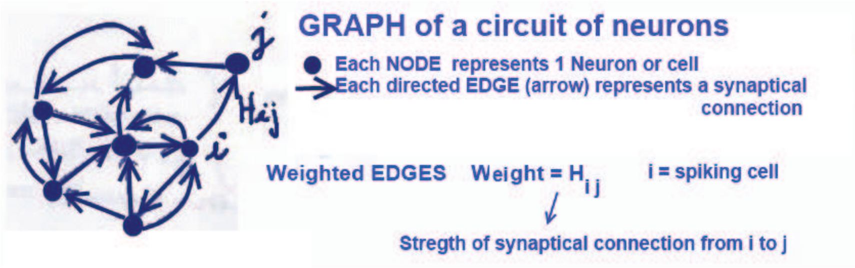
International Symposium on
Neurons, Circuits and Systems
Montevideo, Nov. 30th - Dec.3th., 2014

ORGANIZATION OF THIS TALK

- Object of study: A mathematical system modeling a circuit of "neurons".

(Abstract – General – Simplified)

- Questions to research: Quantitative and qualitative dynamics of the mathematical system.
- Methodology of research: Logical deductive proofs (Rigorous proofs).
- Obtained results: Theorems 1 and 2 (their statements and their proofs).
- Mathematical proofs: Unfortunately not included in this talk, but they are the most enjoyable parts of the work.
- Conclusions: How to interpret the statements of Theorems 1 and 2 and their corollaries. They are necessarily true in the (simplified - general - abstract) mathematical model. But ¿do they necessarily hold for real biological neural networks?



GRAPH of a circuit of neurons

● Each NODE represents 1 Neuron or cell
→ Each directed EDGE (arrow) represents a synaptical connection

Weighted EDGES Weight = H_{ij} i = spiking cell

↓

Stregth of synaptical connection from i to j

Definition

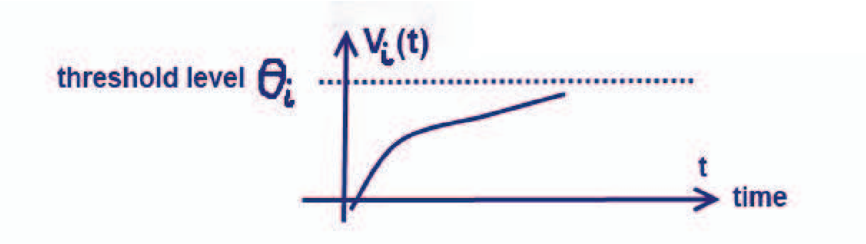
The cell i (node i) is EXCITATORY if $H_{ij} \geq 0 \quad \forall\ j \neq i$.
The cell i (node i) is INHIBITORY if $H_{ij} \leq 0 \quad \forall\ j \neq i$.

Interspike Regime

(One-dimensional neuron)
Governed by a 1-dimensional differential equation:

$$\frac{dV_i}{dt} = F_i(V_i),\ \text{where } F_i > 0,\ \text{while } V_i(t) \leq \theta_i$$

The value θ_i is the *THRESHOLD LEVEL*.



Interspike Regime

(One-dimensional neuron)

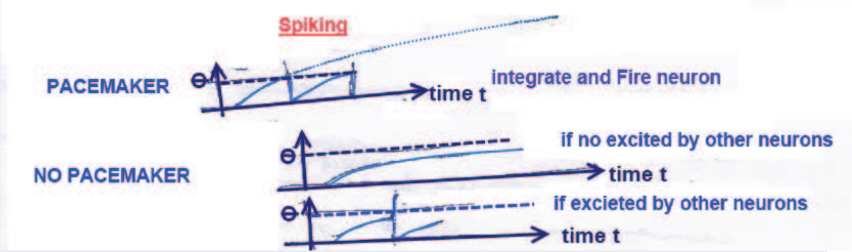
Governed by a 1-dimensional differential equation:

$$\frac{dV_i}{dt} = F_i(V_i), \text{ where } F_i > 0, \text{ while } V_i(t) \leq \theta_i$$

The value θ_i is the *THRESHOLD LEVEL*.

Spiking Regime

(Pacemaker and No Pacemaker neuron)



Interspike Regime

(d-dimensional neuron i)
Its state is described at each instant t by the d real variables, which are the component of a d-dimensional vector $\overline{x}_i(t)$.
Governed by a d-dimensional system of differential equations:

$$\frac{d\overline{x}_i}{dt} = F_i(\overline{x}_i),\ \text{where } F_i : \mathbb{R}^d \mapsto \mathbb{R}^d,\ \text{while } V_i(\overline{x}(t)) \leq \theta_i,$$

such that

$$\frac{dV_i(\hbar x_i(t))}{dt} = \nabla F_i \cdot \frac{d\overline{x}_i}{dt} > 0.$$

The value θ_i is the *THRESHOLD LEVEL*.

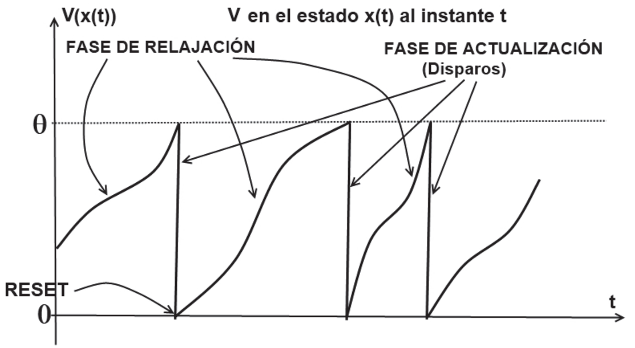
Interspike Regime

(d-dimensional neuron)

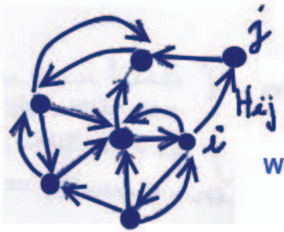
$$\frac{d\overline{x}_i}{dt} = F_i(\overline{x}_i), \text{ where } F_i : \mathbb{R}^d \mapsto \mathbb{R}^d, \text{ while } V_i(\overline{x}(t)) \leq \theta_i,$$

Spiking Regime

(If neither excited nor inhibited by other neurons)



REMARK: The spiking is not necessarily periodic



GRAPH of a circuit of neurons

● Each NODE represents 1 Neuron or cell

→ Each directed EDGE (arrow) represents a synaptical connection

Weighted EDGES Weight = H_{ij} i = spiking cell

↓

Stregth of synaptical connection from i to j

The cell i (node i) is excitatory if $H_{ij} \geq 0 \quad \forall\, j \neq i$.
The cell i (node i) is inhibitory if $H_{ij} \leq 0 \quad \forall\, j \neq i$.

Synaptical Rule

When neuron i spikes (at instant t_0), the potential V_j of neuron $j \neq i$ suffers and "instantaneous" jump:

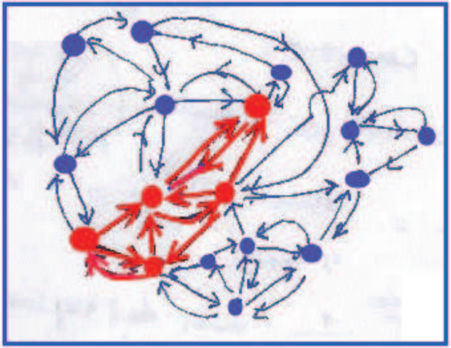
$$V_j(t_0) = V_j(t_0^-) + H_{i,j} \text{ if this number is } < \theta_j, \text{ or}$$

$$V_j(t_0) = 0 \text{ otherwise, and if so, also } j \text{ spikes at instant } t_0.$$

This math. model of the synaptical actions is SIMPLIFIED so:

- Instantaneous jump in the postsynaptical potential V_j.
- The REFRACTORY PHENOMENON holds for the spiking cells.

Graph of a circuit containing the graph of a subcircuit (in red) which is called a SUBGRAPH



GRAPH (CIRCUIT)

SUBGRAPH (SUBCIRCUIT)

Parameters' Space

Which are the "parameters" of the mathematical system modeling the circuit of neurons? Their values of some of them are NOT numbers but FUNCTIONS or other non numerical MATHEMATICAL STRUCTURES.

For the relaxation and spiking regime of the neurons:

$$(m, F_1, F_2, F_3, \ldots, F_m, \theta_1, \theta_2, \theta_3, \ldots, \theta_m), \text{ where}$$

- m is the number of neurons in the circuit
- F_i is the (vectorial) function at the second member of the system of differential equations $d\overline{x}_i/dt = F_i(\overline{x}_i)$ governing the relaxation regime of the neuron i.
- θ_i is a real number: the threshold level of neuron i.

For the synaptical connections:

$$(G,\ H_{1,2}, H_{1,3}, \ldots, H_{m-1,m}), \text{ where}$$

- G is a Graph Structure: the graph of the circuit, with m nodes $i \in \{1, 2, \ldots, m\}$ and directed and weighted edges $(i, j) : \ i \neq j$
- The weights $H_{i,j}$ of the edges of the graph G: they are (positive or negative or zero) real numbers.

Questions of Research: Dynamics of the Network (circuit or graph)

Which are the qualitative and quantitative mathematical properties that one may obtain, BY LOGICAL DEDUCTION, from the general mathematical model defined above?

EXAMPLES:

- Sequence of spiking instants
- ISI (interspike intervals) of a cell or of a subcircuit.
- Attractors and their basins - periodic orbits - limit cycles
- Synchronization of the spikes of several cells (periodic or non periodic synchronization)
- Waiting times until synchronization
- Recurrence

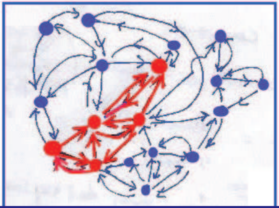
Definition

Recurrent synchronization of spikes in a subcircuit S
if there exists instants $t_1, t_2, \dots, t_n, t_{n+1}, \dots, \dots$
such that at instant t_n all the cells of the subcircuit S spike simultaneously.

In Game Theory the phenomenon of synchronization is called "Grand Coalition".

Remark: Between the simultaneous synchronizations at instants t_n and t_{n+1}, some neurons of the subcircuit may spike.

Sufficient mathematical conditions for recurrent synchronization of the spikes in a subcircuit.



GRAPH (CIRCUIT)

SUBGRAPH (SUBCIRCUIT)

Theorem 1

If

- S *is complete and excitatory:* $H_{i,j} > 0 \ \ \forall i \neq j$ *in* S,
- *at least one cell in* S *is pacemaker,*
- *the number* m *of cells in* S *is large enough in relation to the mininum excitatory weight:*

$$\sqrt{m} \ \geq \ \frac{\max_{j \in S} \theta_j}{\min_{i \neq j \text{ in } S} H_{i,j}},$$

then

all the cells of the subcircuit S *recurrently synchronize spikes while* S *does not receive inhibitions from the cells outside* S.

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all the cells of the subcircuit S recurrently synchronize spikes while S does not receive inhibitions from the cells outside S.

Remark This theorem holds:

- For any initial state of the cells
- Disregarding which are the functions F_i, and if they are similar or mutually very different, and which are the dimensions of vectorial states of the cells.
- No matter if the cells are mutually very different
- No matter if the interactions are mutually very different
- Disregarding how short or long are the refractory periods (but the refractory phenomenon must exist).

Theorem 1

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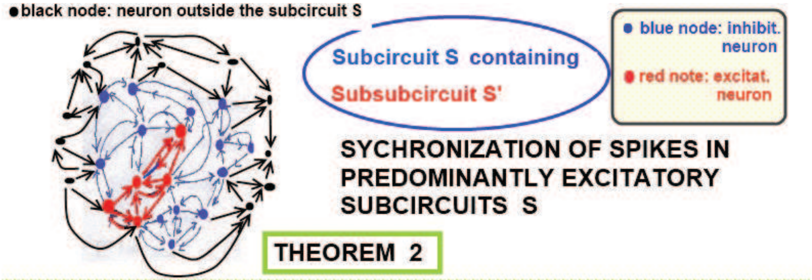
then

all the cells of the subcircuit S *recurrently synchronize spikes while* S *does not receive inhibitions from the cells outside* S.

CHANGING THE CONNECTIONS OF THE SUBCIRCUIT S to be non complete, but still excitatory, provided the number of non null connections is large enough, still produce recurrent synchronization, if certain other mathematical conditions and inequalities hold (work in progress).

Some HISTORY:
1992 Mirollo-Strogatz
1996 Bottani



● black node: neuron outside the subcircuit S

Subcircuit S containing

Subsubcircuit S'

● blue node: inhibit. neuron

● red note: excitat. neuron

SYCHRONIZATION OF SPIKES IN PREDOMINANTLY EXCITATORY SUBCIRCUITS S

THEOREM 2

Theorem 2

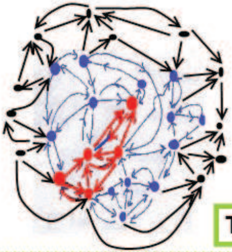
If the subcircuit S contains a sub-subcircuit S' such that

- S' *is complete and excitatory*
- *all the cells in $S \setminus S'$ are inhibitory*
- *at least one neuron of S' es a pacemaker*
- *the number m' of excitatory neurons in S' satisfies the following inequality:*

$$\sqrt{m'} \geq \frac{\left(\max_{j\in S}\theta_j\right) + \left(\max ISI_{S'} / \min_{j\in S} ISI_j\right) \cdot \left(\max_{j\in S\setminus S'} |H_{j,i}|\right)}{\min_{i\neq j, i\in S', j\in S} H_{i,j}},$$

then all the cells of the subcircuit S recurrently synchronize spikes, while they do not receibe inhibitions from the cells outside S.

● black node: neuron outside the subcircuit S



Subcircuit S containing

Subsubcircuit S'

● blue node: inhibit. neuron

● red note: excitat. neuron

SYCHRONIZATION OF SPIKES IN PREDOMINANTLY EXCITATORY SUBCIRCUITS S

THEOREM 2

CONCLUSIONS

1. To avoid recurrent synchr. of spikes in the subcircuit S' composed by excitatory cells connect the nodes of S' with edges coming from inhibitory cells, BUT:
2. If the inhibitory cells connected to S' are themselves excited by the cells of S', then
   - they do not avoid the recurrent syncrhonization of S'.
   - Worst, the inhibitory cells also synchronize spikes with the excitatory cells of S'.
   - So, due to the refractory phenomenon, the inhibitory cells spiking simultaneously with those of S' do not inhibit them.
3. Other cells outside S (in black in the figure), that are inhibited by S but not excited by S', may turn off (do not spike).

25

Sobre ciertas redes de unidades dinámicas acopladas por impulsos.

Presentación en el IV Coloquio Uruguayo de Matemática, Montevideo.

Eleonora Catsigeras

2013

SOBRE CIERTAS REDES DE UNIDADES DINÁMICAS ACOPLADAS POR IMPULSOS

ELEONORA CATSIGERAS

Presentación en
IV Coloquio de Matemática,
del 18 al 20 de diciembre, 2013

Se presentará un modelo matemático abstracto de sistema dinámico determinístico (que proviene de modelos simplificados de redes neuronales biológicas) en que m subsistemas dinámicos (llamados "unidades", "celdas" o "neuronas"), que no son necesariamente mutuamente idénticos, evolucionan independientemente excepto en instantes de "disparo", determinados por cada neurona en forma ciega al estado global de la red.

En sus instantes de disparo cada neurona envía una señal instantánea de acople excitatoria (colaborativa) o inhibitoria (no colaborativa, o competitiva o antagonista) a algunas de las otras. Se asume como hipótesis el principio de Dale (establecido en el sistema nervioso de los animales). Esto es, cada neurona es o bien excitatoria o bien inhibitoria. Es decir no es colaborativa con algunas y competitiva con otras.

En primer lugar se establecerán condiciones suficientes (ejemplos: que el grafo de la red sea completo y sean todos los acoples colaborativos, o que tenga un subgrafo completo colaborativo con suficiente cantidad de neuronas en relación a otros parámetros) para que la red sincronice totalmente. Esto es, periódicamente, sin necesidad de un reloj externo marcapasos o de un sistema del tipo "master-slaves", todas las neuronas disparan simultáneamente, con un período determinado por la red (que en general no coincide con el período espontáneo de ninguna neurona por separado si estuviera aislada de la red). Se acotará superiormente el tiempo de espera hasta la sincronización completa de la red, en función de los parámetros.

En segundo lugar, en el caso de redes en que existan además neuronas inhibitorias (competitivas con las demás) se definirá "riesgo de muerte" de una neurona cualquiera. Esto es un coeficiente que mide qué tan posible es que el estado de esa neurona se mantenga bajo el umbral de disparo para todo tiempo futuro a partir de un cierto instante. Se verá que una red que sincroniza, aunque tenga algunas neuronas inhibitorias, minimiza el riesgo de muerte de todas sus neuronas.

En último lugar se definirá la cantidad total de información de la red (a tiempo finito). Se verá que la cantidad de información es nula si todas las neuronas son idénticas y colaborativas, pero que puede ser positiva si las neuronas son muy diferentes entre sí en relación a otros parámetros de la red, aunque sean todas colaborativas y la red sincronice ("principio de la necesaria diversidad").

Como conclusión, se observará que la cantidad de información crece si se agregan neuronas competitivas, pero la sincronización puede ser destruida si el número de neuronas competitivas es demasiado grande, y el riesgo de muerte de todas (incluso de las competitivas) aumenta si la sincronización se destruye.

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INSTITUTO DE MATEMÁTICA Y ESTADÍSTICA "PROF. ING. RAFAEL LAGUARDIA" (IMERL), FACULTAD DE INGENIERÍA, UNIVERSIDAD DE LA REPÚBLICA, URUGUAY
E-mail address: `eleonora@fing.edu.uy`

Redes Cooperativas Acopladas por Impulsos

Eleonora Catsigeras

IMERL - Fac. Ingeniería
Universidad de la República - Uruguay

`eleonora@fing.edu.uy`

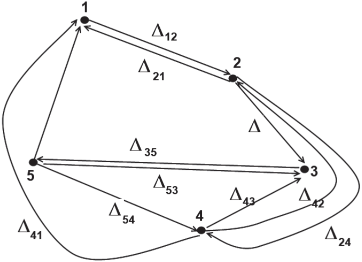
Presentación en el
IV Coloquio Uruguayo de Matemática
Montevideo, del 18 al 20 de diciembre de 2013

Objeto de estudio

Dinámica determinista de un red N
de $m \geq 2$ células (o neuronas: sub-sistemas dinámicos)
$i \in \{1, 2, \ldots, m\}$
acopladas por impulsos instantáneos.

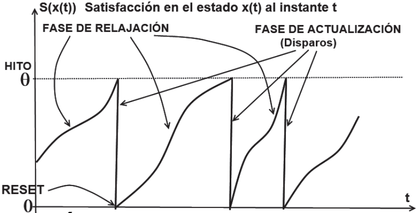
• CÉLULA O NEURONA i: subsistema dinámico autónomo
(*Dinámica libre de* i)

• ACOPLAMIENTOS $\Delta_{i,j}$ $\forall$ $(i, j) : i \neq j$



Dinámica libre de i:

Estado x_i de i en función del tiempo t
si i estuviera desacoplada de la red N. DOS FASES:



- **FASE DE RELAJACIÓN**
  - $\dot{x}_i = f_i(x_i)$ ecuación diferencial n-dim; flujo solución $x_i(t) = \Phi_i(x_i(0), t)$; $x_i \in X_i$
  - $S_i : X_i \mapsto [0, \theta_i]$ **función de satisfacción** ,
  - θ_i **goal ó hito ó threshold level**

$\frac{d}{dt}(S_i(x_i(t)) =< \bigtriangledown S_i, f_i \geq v_i > 0 \;\; \forall \; x_i \in S_i^{-1}[0, \theta_i)$

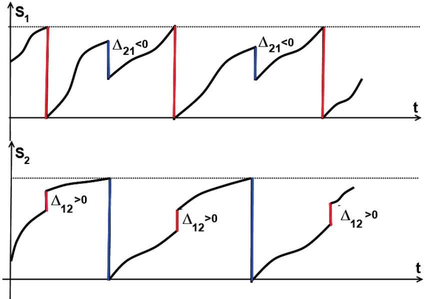
- **FASE DE ACTUALIZACIÓN - UPDATE RULE:**
  - $S_i(x_i(t^-)) = \theta_i \quad \Rightarrow \quad x_i(t) \in S_i^{-1}(0)$
  - **Disparo o spike** Es la discontinuidad en el estado $x_i(t)$ que se produce cuando la variable de satisfacción S_i alcanza el hito θ_i: S_i se resetea a cero instantáneamente.

ACOPLAMIENTOS EN LA RED:

$$I(t) := \{1 \leq i \leq m \colon i \text{ dispara en el instante } t\}$$

REGLA DE INTERACCIONES INSTANTÁNEAS:

$$S(x_j(t)) := S(x_j(t^-)) + \sum_{i \in I(t),\ i \neq j} \Delta_{ij} \text{ si es } < \theta_j, \quad := 0 \text{ si es } \geq \theta_j .$$



Célula i es **Cooperativa:** $\Delta_{i,j} \geq 0 \ \forall j \neq i$, $\Delta_{i,j} > 0$ para algún $j \neq i$.
Célula i es **Antagonista:** $\Delta_{i,j} \leq 0 \ \forall j \neq i$, $\Delta_{i,j} < 0$ para algún $j \neq i$.
Principio de Dale (hipótesis) Cada célula o bien es cooperativa o bien es antagonista.
Red Cooperativa: Todas las células son cooperativas

Definiciones

Espacio (funcional) de parámetros de la red

$$\text{Param}(N) = \left\{ \{(\phi_i, S_i, \theta_i)\}_{1\leq i\leq m},\ \{\Delta_{i,j}\}_{1\leq i,j\leq m,\ i\neq j\} \right\}$$

Topología C_0 en el espacio de parámetros.
Fenómeno robusto o persistente $\forall\ N$ que exhibe el fenómeno, existe un entorno V tal que toda red N' en ese entorno también exhibe el fenómeno.

Muerte de célula i en instante t_0
Para todo $t > t_0$ la célula i no dispara.
t_0 es el mínimo $t_0 \geq 0$ para el que ocurre lo anterior.

SINCRONIZACIÓN DE DISPAROS: existe $\{t_n\}_{n\geq 0}$
$0 < t_0 < t_1 < \ldots < t_n \to +\infty$ tal que $I(t_n) := \{1, \ldots, m\}$.
SINCRONIZACIÓN DE DISPAROS PERIÓDICA: existe $\{t_n^*\}_{n\geq 0}$ tal que $I(t_n^*) \neq \emptyset$ ($I(t_n^*)$ se llama n-ésimo **cluster**),
$I(t) = \emptyset$ si $t_n < t < t_{n+1}$, y existe $p \geq 1$ tal que

$$I(t_{hp}^*) = \{1, \ldots, m\}\ \forall\ h \in \mathbb{N}^+, \qquad I(t_n^*) = I(t_{np}^*)\ \forall\ n \in \mathbb{N}^*.$$

Características y ejemplos:
No hay reloj externo ni neuronas masters y otras slaves
Ejemplo de luciérnagas. Otros ejemplos en biología, neurociencias y ecología.
Ejemplos en redes en economía y otras ciencias sociales.

Resultados de sincronización

Teorema

1a) *Si la red es cooperativa con grafo completo, y si el número* m *de células es suficientemente grande, i.e.*

$$\sqrt{m} \geq \max \left\{ \sqrt{3}, \frac{\max_j \theta_j}{\min_{i \neq j} \Delta_{i,j}} + 1 \right\}$$

entonces la red **sincroniza disparos.** *El transitorio* T *entre un disparo simultáneo de todas las neuronas de la red y el siguiente, está acotado superiormente por* $T \leq \max_{1 \leq i \leq m} \dfrac{\theta_i}{v_i}$.

1b) *Si además la dinámica libre de cada neurona es tal que* $dS_i/dt = g_i(S_i)$, *entonces la sincronización de disparos es* **periódica con período** $p \geq 1$*: cada* p *disparos en la red, todas las neuronas disparan juntas. Hay* p **clusters diferentes** *entre un disparo de la red completa y el siguiente.*

1c) *Si además las neuronas son mutuamente similares, i.e.*

$$\frac{(\min_i \theta_i) \cdot (\min_i \min_{x_i \in X_i} < \bigtriangledown S_i, g_i >)}{(\max_i \theta_i) \cdot (\max_i \max_{x_i \in X_i} < \bigtriangledown S_i, g_i >)} > 1 - \frac{\min_{i \neq j} \Delta_{ij}}{\max_i \theta_i},$$

entonces $p = 1$*. Es decir, todas las neuronas disparan juntas cada vez que una de ellas dispara. Hay un solo cluster formado por todas las neuronas de la red.*

Demostración de sincronización

Hipótesis: $\sqrt{m} \geq \max\left\{\sqrt{3}, \dfrac{\max_j \theta_j}{\min_{i \neq j} \Delta_{i,j}} + 1\right\}$

Sean $t_1^* < t_2^* < \ldots < t_n^* < \ldots$ los instantes en que por lo menos una neurona de la red dispara.

Sea $K :=$ parte entera $\left(\dfrac{\max_j \theta_j}{\min_{i \neq j} \Delta_{i,j}}\right) + 1.$ (Obs: $K^2 < m$)

Afirmación A) A más tardar en el instante t_K ya dispararon todas las neuronas por lo menos una vez.

Afirmación B) Si en el instante t_n disparan por lo menos K neuronas simultáneamente, entonces disparan todas simultánemente.

Afirmaciones A) y B) $\Rightarrow$ existe un instante t_0 en que todas las neuronas disparan simultáneamente. De lo contrario la cantidad de neuronas sería menor que K^2 lo cual contradice la hipótesis. ☐

Riesgo y factor de protección (Definiciones)

INTERFERENCIAS NEGATIVAS EXTERNAS A LA RED

Riesgo de muerte intrínseco de i, relativo a las otras neuronas de la red

$$R_i = \frac{\theta_i / v_i}{\max_{1 \leq j \leq m}(\theta_j / v_j)} \in (0, 1]$$

Riesgo de muerte neto en red de i en el h-ésimo interspike interval $ISI_i^{(h)}$ de la neurona i:

$$R_i'^{(h)} = \frac{\max\{0,\ \theta_i - \sum_{j \in I(t): t \in ISI_i^{(h)}} \Delta_{j,i}\} \,/\, v_i}{\max_{1 \leq j \leq m}(\theta_j / v_j)} \in (0, 1].$$

Factor de protección de la red a la neurona i en el h-ésimo interspike interval de i:

$$P_i^{(h)} = \min\Big\{1,\ \frac{\sum_{j \in I(t): t \in ISI_i^{(h)}} \Delta_{j,i}}{\theta_i}\Big\}.$$

Protección negativa si la red es antagonista.
Protección ≥ 0 si la red es cooperativa. Es 1 si $\sum_j \Delta_{j,i} \geq \theta_i$

Proposición:

$$R_i'^{(h)} = (1 - P_i^{(h)}) R_i \quad \text{es nulo cuando el factor de protección es } 1.$$

Resultados sobre riesgo y factor de protección

Teorema

2a) *(En las hipótesis del Teorema 1a (si la red es cooperativa de grafo completo y con suficiente cantidad de neuronas) entonces el factor de protección* $P_i^{(h)}$ *de cada neurona* i *en todo intervalo inter-spike* h*, es positivo. Luego el riesgo neto* R_i' *de muerte de la neurona* i *acoplada a la red, por interferencias negativas externas a la red, es menor estrictamente que el riesgo intrínseco* R_i *de la misma neurona si no estuviera acoplada a la red.*

2b) *Si además todas las neuronas son similares (hipótesis del Teorema 1b), entonces el factor de protección* P_i *de cada neurona es igual al máximo posible 100%, y su riesgo neto de muerte* R_i' *es el mínimo posible 0%.*

Cantidad de información (Definiciones)

$t_0^* < t_1^* \ldots < t_n^* <$ instantes en que por lo menos una neurona dispara.

- $I(t_n) = \{i : i$ dispara en el intante $t_n\} \neq \emptyset$.

Cantidad potencial de conjuntos diferentes en cada disparo: 2^m.

Cantidad potencial de información en cada disparo: $\log_2(2^m) = m$.

- **Spiking code (código de disparo):** Sucesión $\{I(t_n)\}_{n \geq 0}$. Depende de la condición inicial de todas las neuronas en la red.
- **Pattern de longitud finita $k \geq 1$:** Palabra de longitud k en el spiking code.

$$\pi_{n_0,k} = \Big(I(t_{n_0}), I(t_{n_0+1}), \ldots, I(t_{n_0+k-1}) \Big).$$ Depende del estado inicial.

- **Pattern recurrente de longitud $k \geq 1$:** palabra π_k de longitud k tal que existe algún estado inicial y una sucesión $n_j \to +\infty$ que lo realiza:

$$\pi_k = \pi_{n_j,k} \quad \forall\ j \in \mathbb{N}.$$

- $\Pi_k := \{\pi_k\}$ conjunto de todos los patterns recurrentes de longitud k.
- $\#\Pi_k$: cantidad de patterns recurrentes de longitud $k \geq 1$ diferentes que la red exhibe.
- **Cantidad de información** que la red puede procesar en forma recurrente:

$$H = \sup_{k \geq 0} \log \#\Pi_k.$$

Si $H = +\infty$ se define **entropía:** $h = \limsup_{k \to +\infty} \dfrac{\log \#\Pi_k}{k}$.

entropía = velocidad de crecimiento exponencial de la cantidad de información que la red puede procesar en forma recurrente.

Resultados sobre la cantidad de información

Teorema

2 a) *Si la red sincroniza periódicamente todos sus disparos con período* $p \geq 1$ *(por ej. en las hipótesis del teorema 1b), entonces* $H = \log_2 p < \dfrac{\log_2 m}{2}$.

2 b) *Si además las células son mutuamente similares, entonces* $H = \log_2 1 = 0$.

Conclusiones (redes cooperativas)

A) Cuando se maximiza el factor de protección de cada neurona (haciéndola 100% y logrando 0% de riesgo neto de muerte de cada una), se minimiza la cantidad de información H en la red, haciéndola nula.

B) Las redes cooperativas con suficiente cantidad de neuronas sincronizan y dan un factor de protección positivo a cada una de sus neuronas.
Si no son periódicas o si son periódicas con período p mayor que 1, entonces su cantidad de información total H(durante todo el tiempo futuro) es positiva. Si son periódicas, H es finita igual a $\log p$.

C) **Necesaria diversidad de células para tener cierta riqueza dinámica:**
Las redes cooperativas que pueden procesar una cantidad positiva de información están necesariamente compuestas por neuronas diversas, cuyas dinámicas intrínsecas difieren sensiblemente entre sí.