Combinatorial dynamics of strip patterns of quasiperiodic skew products in the cylinder

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We extend the results and techniques from [1] to study the combinatorial dynamics (*forcing*) and entropy of quasiperiodically forced skew-products on the cylinder. For these maps we prove that a cyclic permutation τ forces a cyclic permutation ν as interval patterns if and only if τ forces ν as cylinder patterns. This result gives as a corollary the Sharkovskiĭ Theorem for quasiperiodically forced skew-products on the cylinder proved in [1].

Next, the notion of s-horseshoe is defined for quasiperiodically forced skew-products on the cylinder and it is proved, as in the interval case, that if a quasiperiodically forced skew-product on the cylinder has an s-horseshoe then its topological entropy is larger than or equals to $\log(s)$.

Finally, if a quasiperiodically forced skew-product on the cylinder has a periodic orbit with pattern τ, then $h(F) \geq h(f_\tau)$, where f_τ denotes the *connect-the-dots* interval map over a periodic orbit with pattern τ. This implies that if the period of τ is $2^n q$ with $n \geq 0$ and $q \geq 1$ odd, then $h(F) \geq \frac{\log(\lambda_q)}{2^n}$, where $\lambda_1 = 1$ and, for each $q \geq 3$, λ_q is the largest root of the polynomial $x^q - 2x^{q-2} - 1$. Moreover, for every $m = 2^n q$ with $n \geq 0$ and $q \geq 1$ odd, there exists a quasiperiodically forced skew-product on the cylinder F_m with a periodic orbit of period m such that $h(F_m) = \frac{\log(\lambda_q)}{2^n}$. This extends the analogous result for interval maps to quasiperiodically forced skew-products on the cylinder.

Moreover, there is a natural question that arises in this setting: *Does Sharkovskiĭ Theorem holds when restricted to curves instead of general strips?*

We answer this question in the negative by constructing a counterexample: We construct a map having a periodic orbit of period 2 of curves (which is, in fact, the upper and lower circles of the cylinder) and without any invariant curve.

In particular this shows that there exist quasiperiodic skew products in the cylinder without invariant curves.

References

[1] Roberta Fabbri, Tobias Jäger, Russel Johnson, and Gerhard Keller. A Sharkovskii-type theorem for minimally forced interval maps. *Topol. Methods Nonlinear Anal.*, 26(1):163–188, 2005.