Stable and Unstable Sets of C^0 Perturbations of Expansive Homeomorphisms of Surfaces

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Abstract

Let M be a compact metric space and $g : M \to M$ be an homeomorphism C^0-close to an expansive map of M. In general, it is not true that g is also expansive, but it still has some properties resembling the expansivity. In fact, if we identify pairs of points whose g-orbits stay nearby, both for the future and the past, we obtain an equivalence relation $\sim$. The quotient space $M/\sim$ is a compact, metric space and g induces an expansive homeomorphism $\tilde{g}$ on that quotient. If M is a surface, we show that for any $\bar{x} \in M/\sim$ the local stable (unstable) set of $\bar{x}$ contains non trivial compact, connected and locally connected subsets (i.e. continuous arcs).

1 Introduction

Let M be a metric space and $f : M \longrightarrow M$ an expansive homeomorphism with an expansivity constant $\alpha > 0$, that is:

$$\text{if for all } n \in \mathbb{Z} \ \ dist(f^n(x), f^n(y)) < \alpha \ \text{ then } x = y.$$

In [8] Lewowicz obtained a topological classification of expansive homeomorphisms of surfaces. More precisely, he proved that S^2 does not support such an homeomorphism, and that expansive homeomorphisms are conjugate to Anosov diffeomorphisms if $M = T^2$ and to pseudo-Anosov maps if M has genus strictly bigger than one.
The dynamics of an expansive homeomorphism has a close relationship with the topology of the manifold supporting it. This relationship is strongly used in the above cited paper and can be quickly understood thinking that an expansive map distinguishes each point of the ambient space from the dynamical viewpoint.

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