Dynamic Control in the New Electric Grid

Fernando Paganini
Universidad ORT Uruguay

In collaboration with:

- Enrique Mallada, Johns Hopkins University, USA
- Andrés Ferragut, Federico Bliman
  (Universidad ORT Uruguay)

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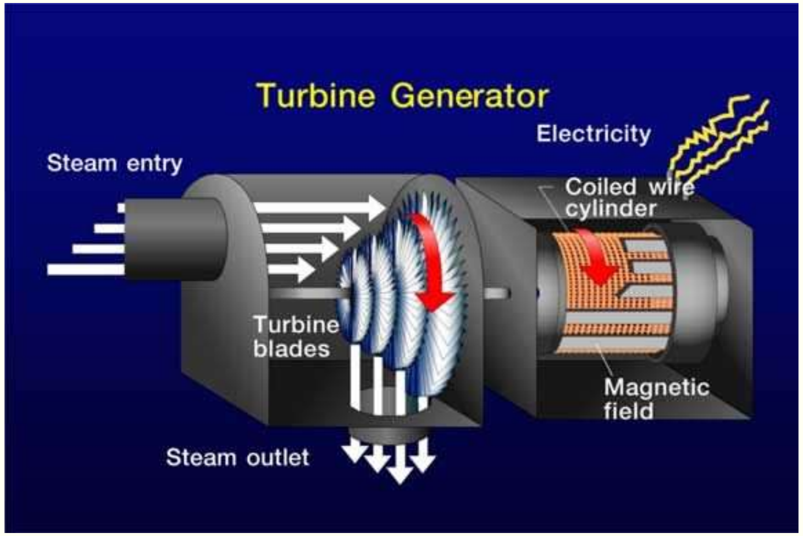
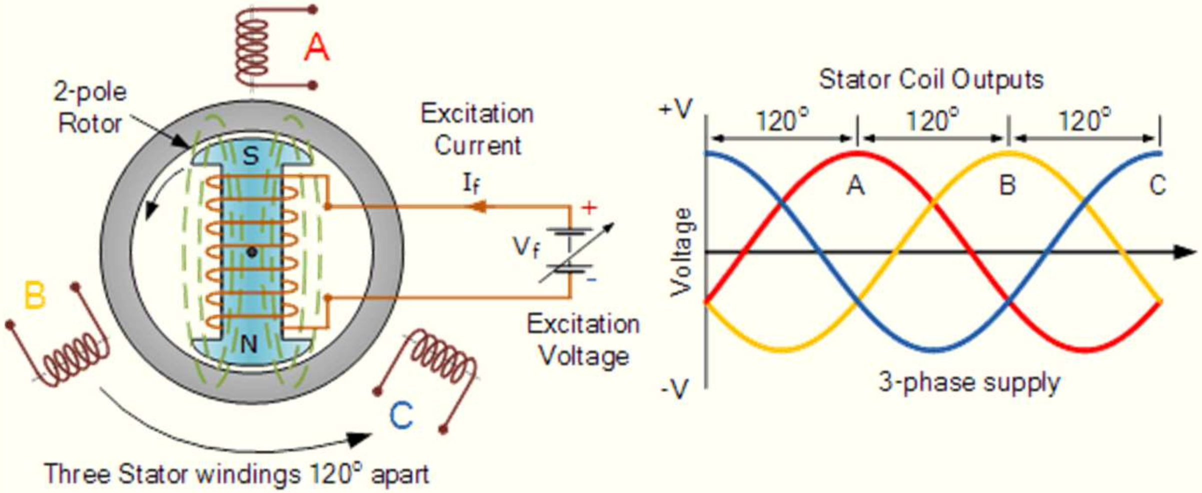
Outline

1. Background on the AC power grid and its dynamic control.
2. Introducing new energy sources: opportunities and challenges.
3. Analysis with multivariable control tools.
4. Active control of load in Smart Grids.
5. Conclusions.

Alternating Current (AC) generation



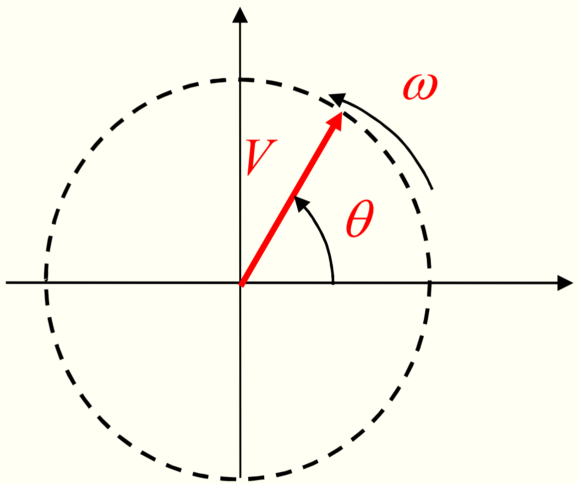
Converts rotational motion to sinusoidal voltage



Phasor representation

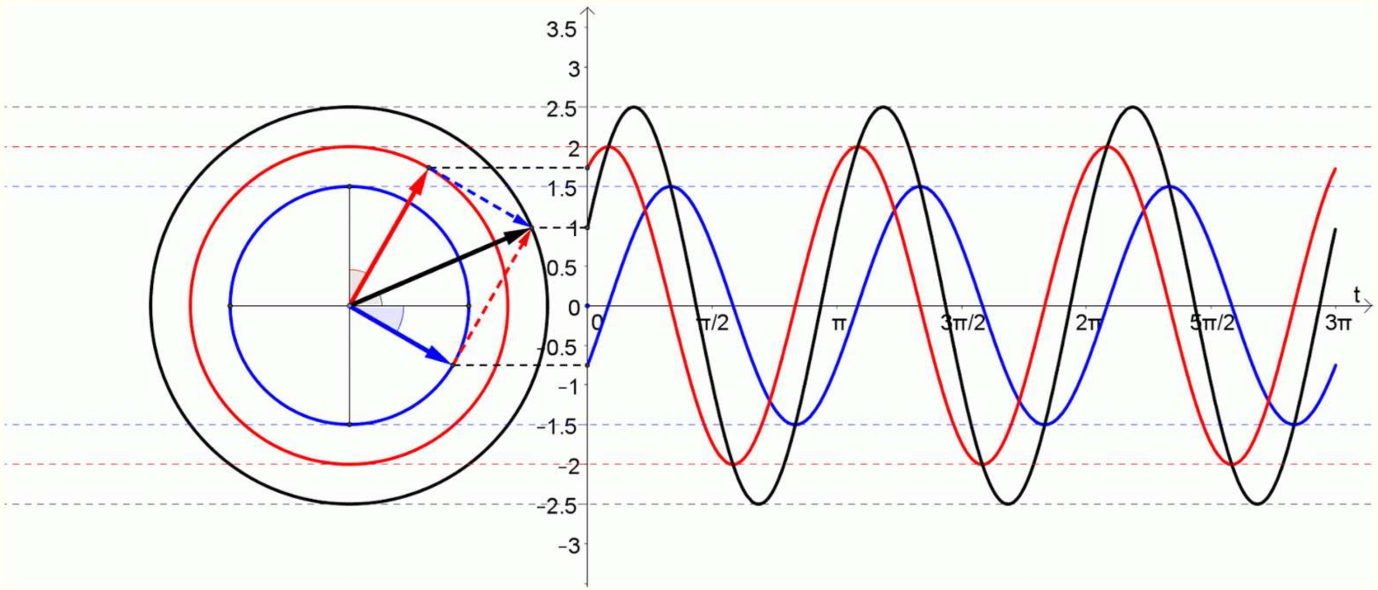
A sinusoid $V\text{sen}(\omega t+\theta)$ is caracterized by:

- Frequency $\omega = 2\pi f$. e.g., $f = 60$ Hz.
- Amplitude V
- Phase θ



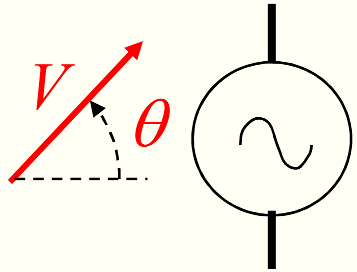
"Phasor": complex number $Ve^{j\theta}$

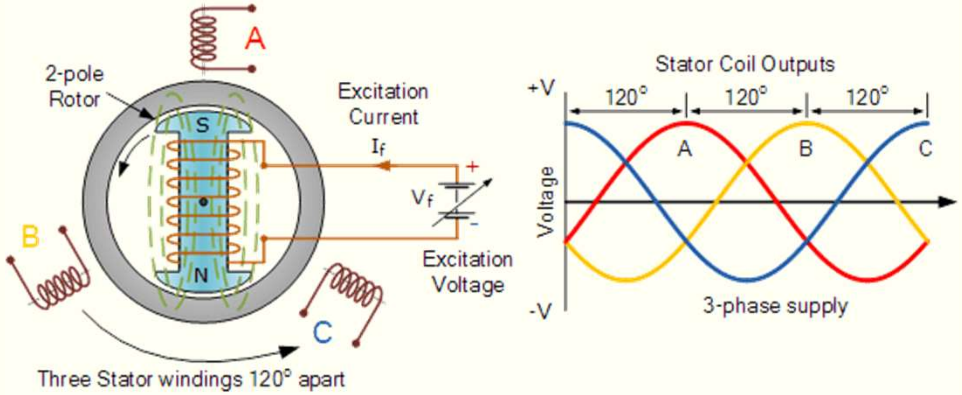
Rotate at speed ω, project to generate sinusoid.



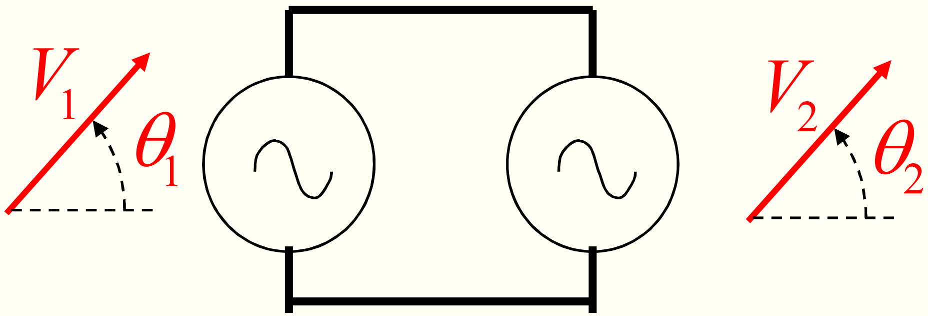
Interconnecting generators

One generator:
(focus on one of 3 phases)





Parallel connection



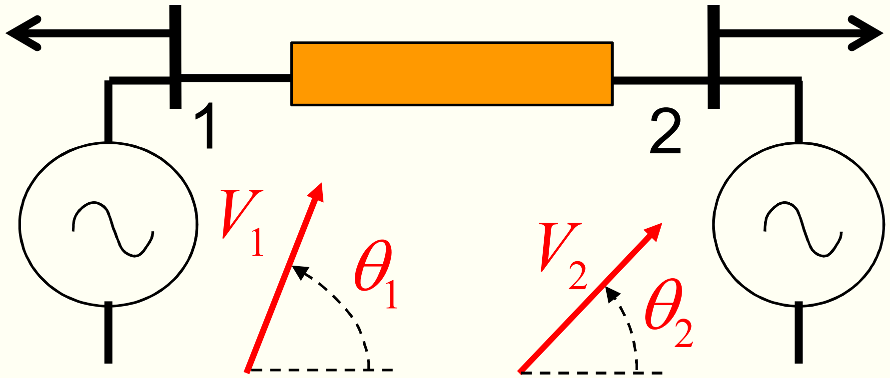
Requirements:

- Same frequency ω
- Same amplitude $V_1 = V_2$
- Same phase $\theta_1 = \theta_2$.

Connection through transmission line

Power load D_1

Power generation G_1



Power load D_2

Power generation G_2

Power balance,
neglect line losses:

$G_1 = D_1 + P_{12}$;

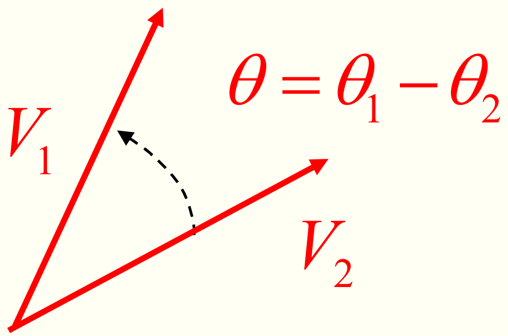
$G_2 + P_{12} = D_2$.

Equilibrium conditions:

- Necessarily, same frequency ω.
- Typically, very similar amplitude: $V_1 \approx V_2$
- Phase may be different: $\theta_1 \neq \theta_2$.

Power transported from node 1 to node 2:

$$P_{12} = \underbrace{b_{12} V_1 V_2}_{K_{ij}} \ \text{sen}(\theta_1 - \theta_2).$$

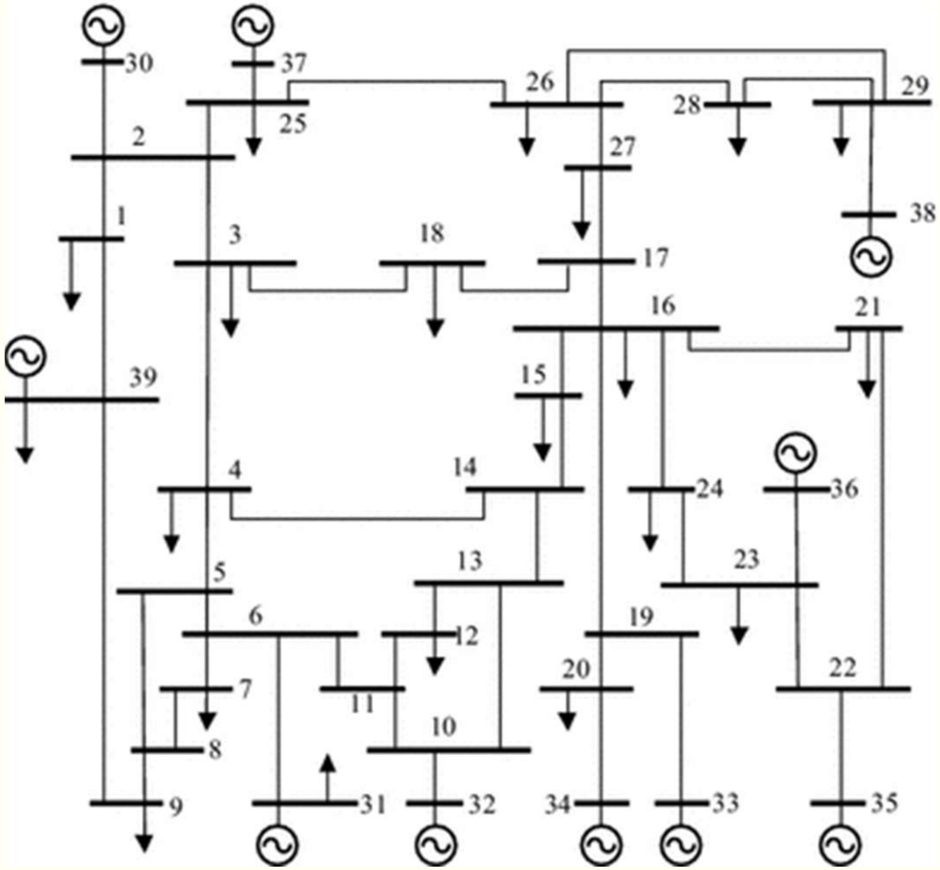


Power system

Equilibrium conditions:

- Same frequency ω
- Approx. same amplitude: V_0
- Different phases: $\theta_i,\ i = 1, \ldots, N$

Phases θ_i must be arranged so that the power flows $P_{ij} = K_{ij}$ sen$(\theta_i - \theta_j)$ satisfy energy balance at each node.



IEEE 39-bus model of New England

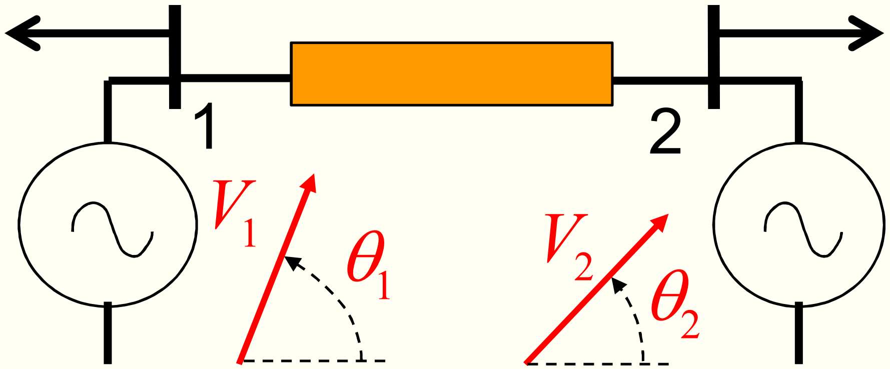
Finding these phases: "power flow" equations.

Non-trivial to solve, even if the total generation covers the total load.

What happens at imbalance

Power load D_1

Power generation G_1

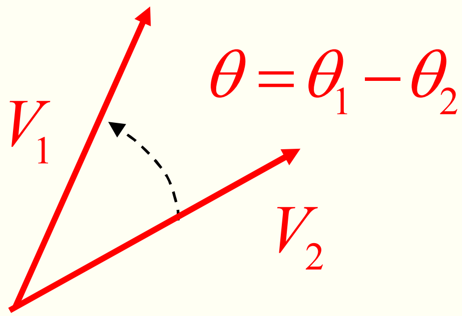


Increased load
$D_2 + \Delta D_2$

G_2

System moves out of equilibrium:

- Generator at node 2 slows down: $\omega_2 \downarrow$
- This "opens" the angle $\theta_1 - \theta_2$.
- $P_{12} = K_{12}$ sen$(\theta_1 - \theta_2)$ $\uparrow$, helps balance node 2.
- But this also affects balance at node 1.

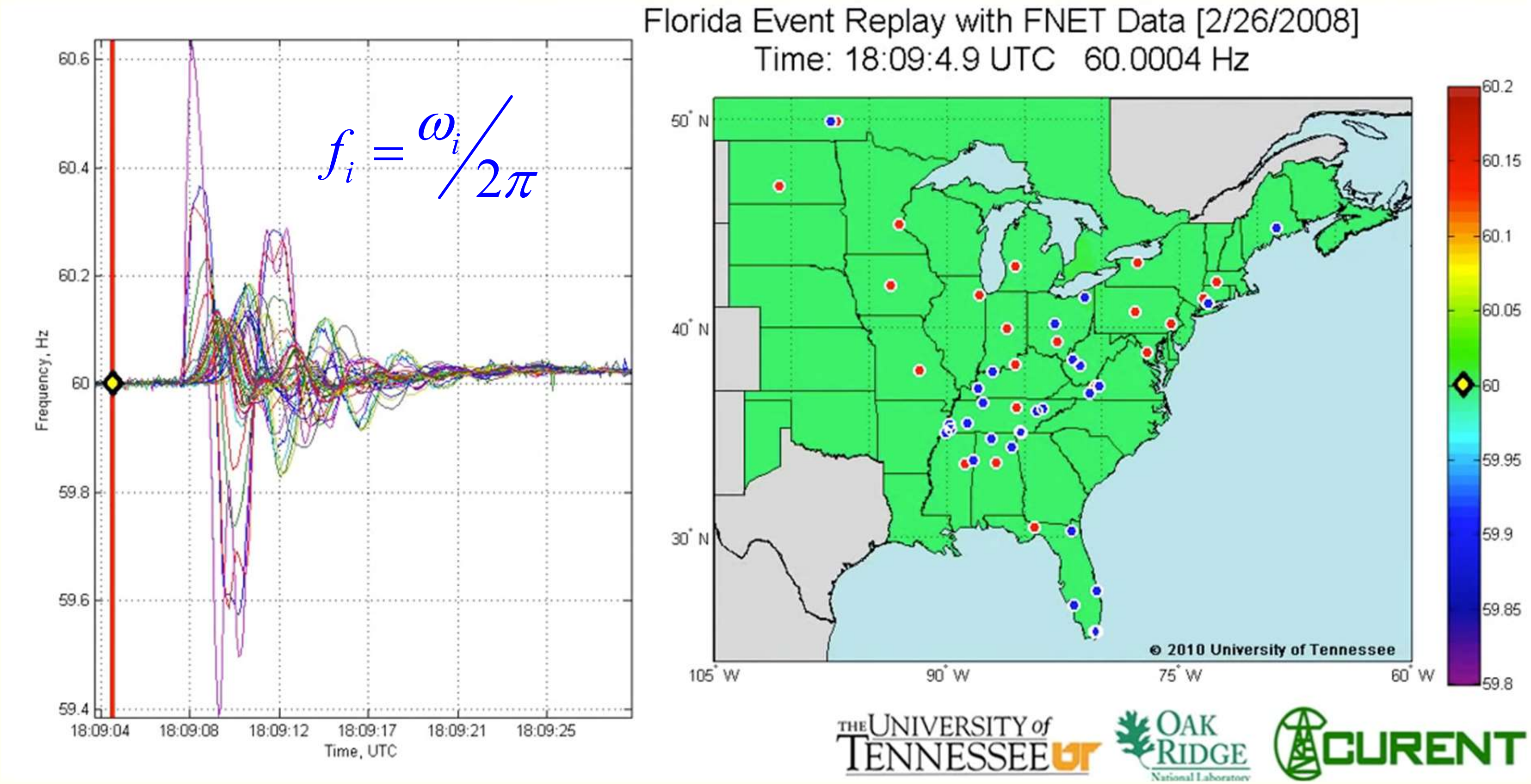


Result: disturbance that affects the entire network.

During this transient, frequency ω_i differs from node to node.

To re-balance we need to control G_1 or G_2.

Real world example

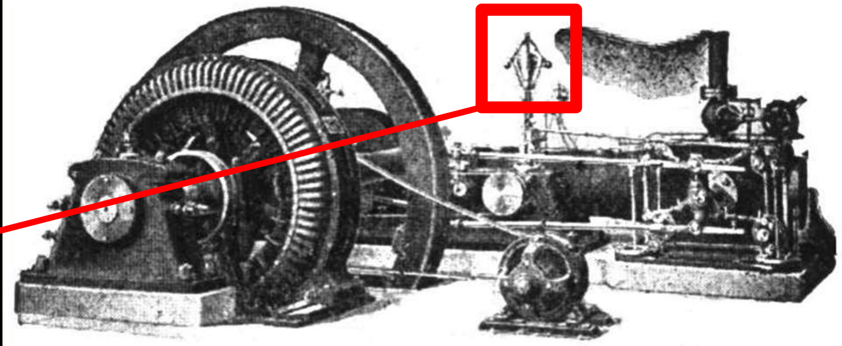


- Loss of load in Florida, affects in seconds the entire continent.
- Oscillations of frequecy observed in both directions.
- Equilibrium is reestablished, with a slightly different frequency.
- How was this achieved?

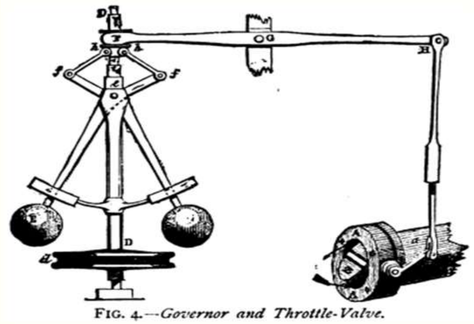
Power balancing actions

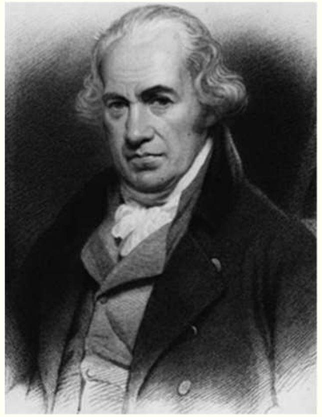
- Passive effect: some loads vary with frequency. $\Delta D = d \cdot \Delta\omega$
- Active "droop control" of generation: $\Delta G = -R^{-1}\Delta\omega$

Old technology!
Flyball converts speed to position of turbine valve.

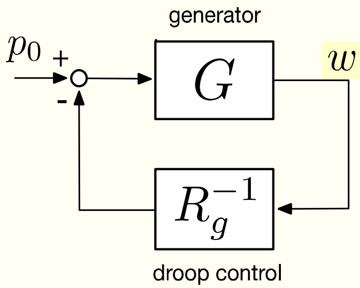


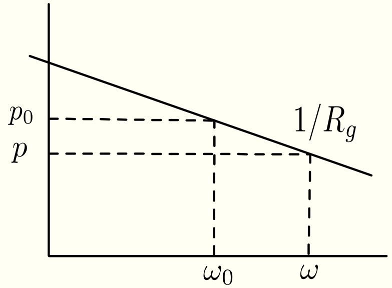
1900s Murray Alternator with Belt-driven Exciter Generator





Watt's Centrifugal Governor 1788



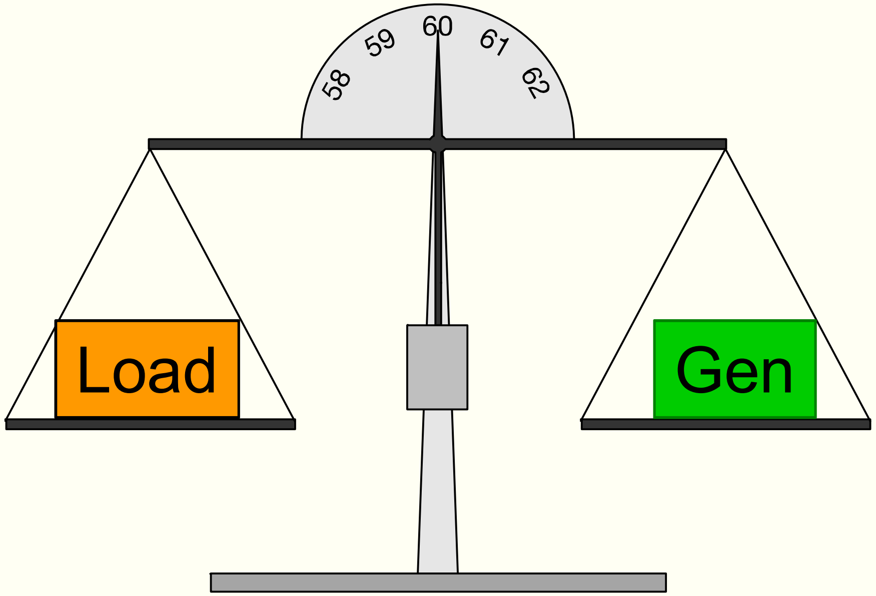


- Effect on power is not immediate, responds in ~ seconds.
- In the meantime: rotating inertia plays a stabilizing role.

Analogy

- Equal-arm balance, pendulum attached to the axis.
- Initially: system in equilibrium at nominal frequency.

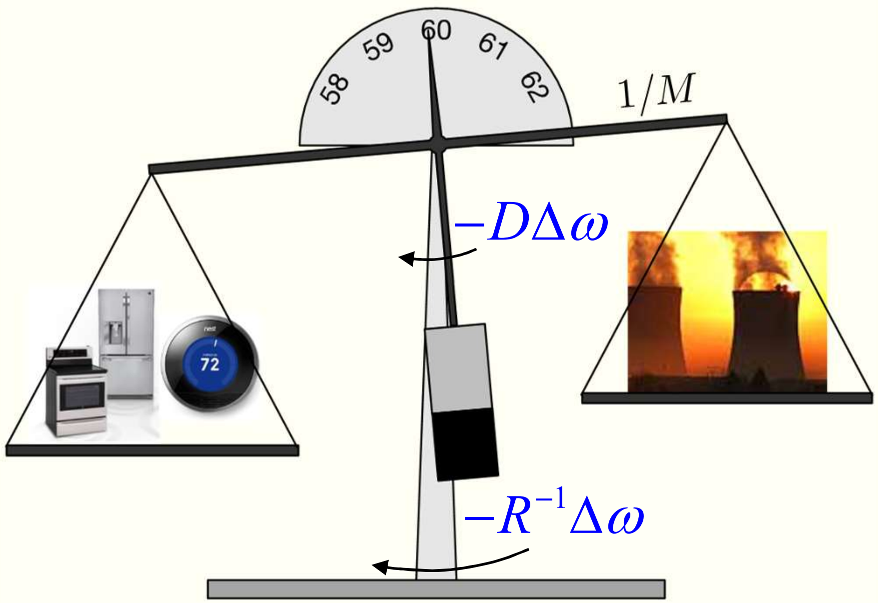
$$f = \omega / 2\pi$$



Imbalance alters frequency

Re-balanced with:

- Passive response $-D\Delta\omega$: torque proportional to deviation.
- Active "Droop Control" $-R^{-1}\Delta\omega$: provides additional torque.
- Intertia $1/M$ plays a role in transient.



Standards

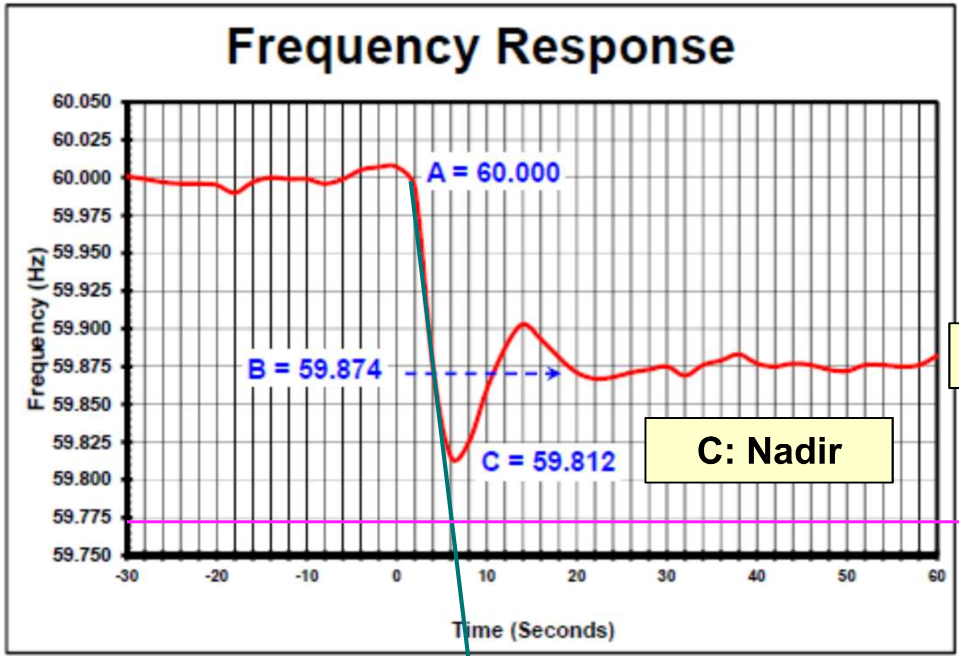


Figure 1. Frequency Response Characteristic

B. Steady state

C: Nadir

UFLS (Under Freq. Load Shedding) Threshold

Slope = RoCoF
(Rate of Change of Freq)

- Characterize response, define actions.

But response varies from node to node! More later…

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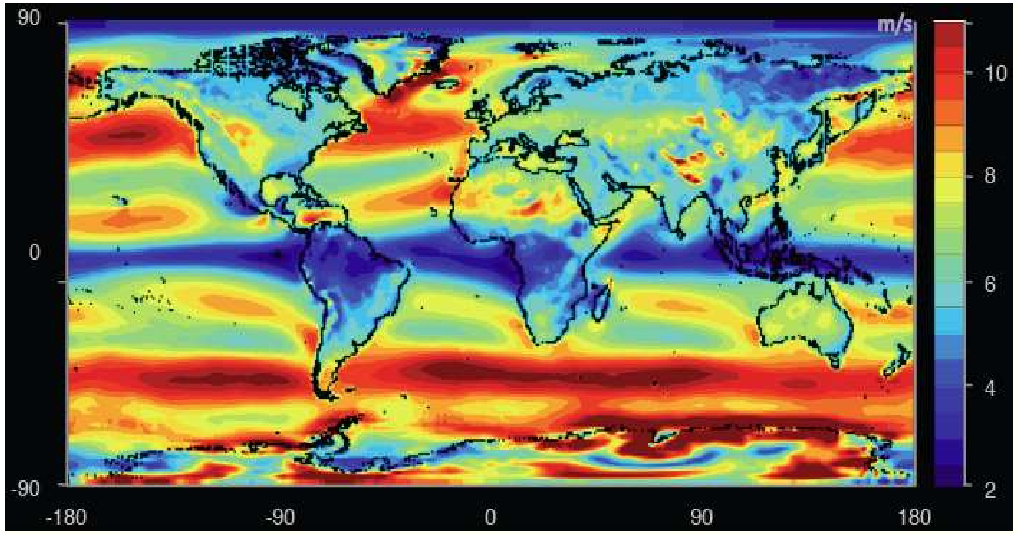
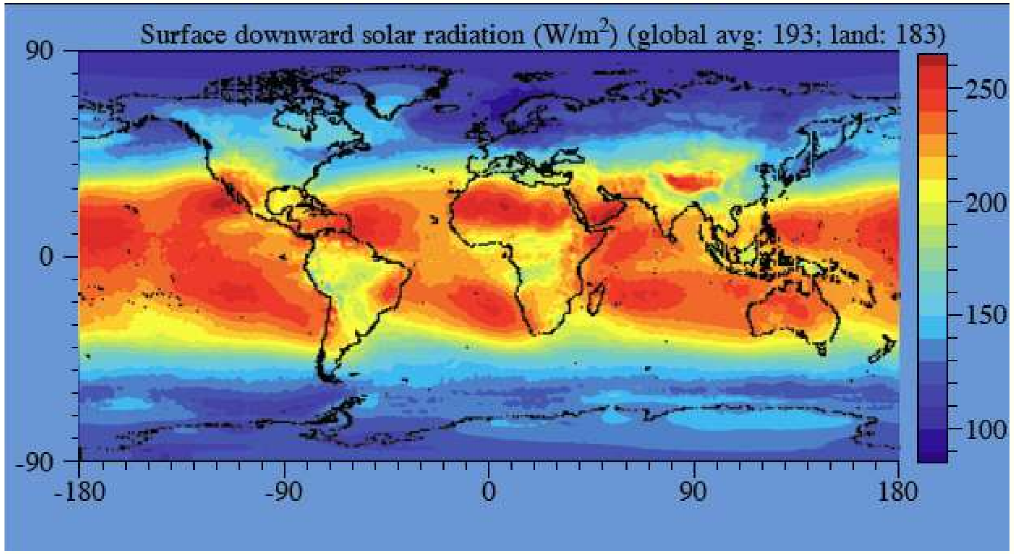
Renewable Energy Potential



Wind power over land
70 – 170 TW



Solar power over land
340 TW

Worldwide

energy demand:
16 TW

electricity demand:
2.2 TW

wind capacity (2009):
159 GW

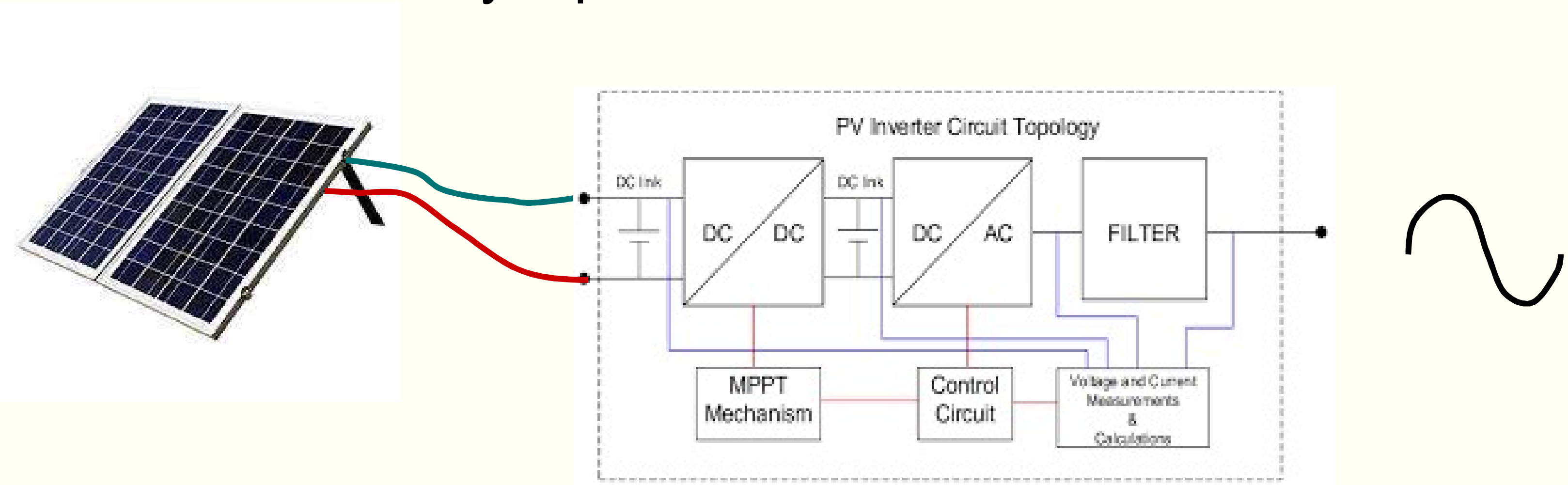
grid-tied PV capacity (2009):
21 GW

Source: Renewable Energy Global Status Report, 2010
Source: M. Jacobson, 2011

Connecting renewables to AC grid

Solar PhotoVoltaic:

- Solar panel produces DC (constant) voltage
- Converted to AC by a power electronic inverter.



Basic components:

- Electronic switch generates square-type wave of frequency ω.
- Filter to "smooth out" the edges.

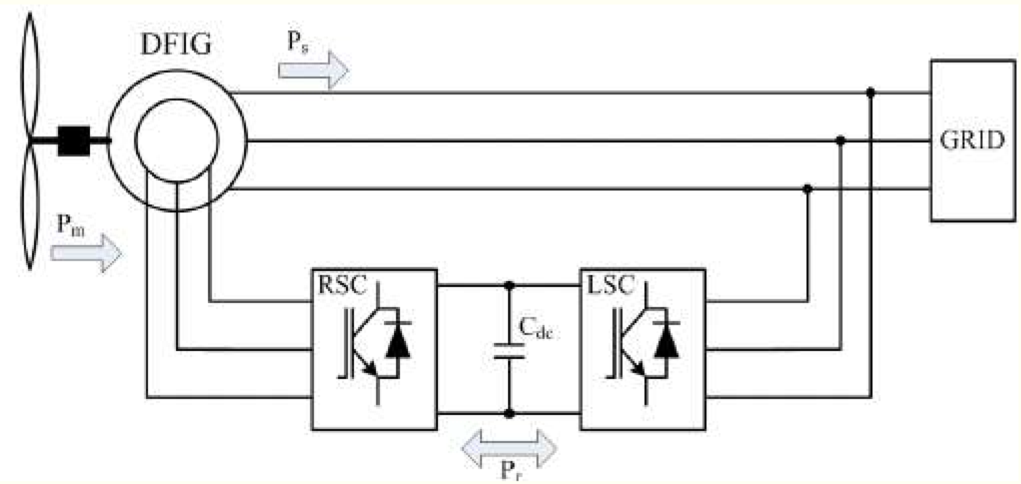
Feature: flexibility to control amplitude, phase.

Connecting renewables to AC grid

Wind Turbine:

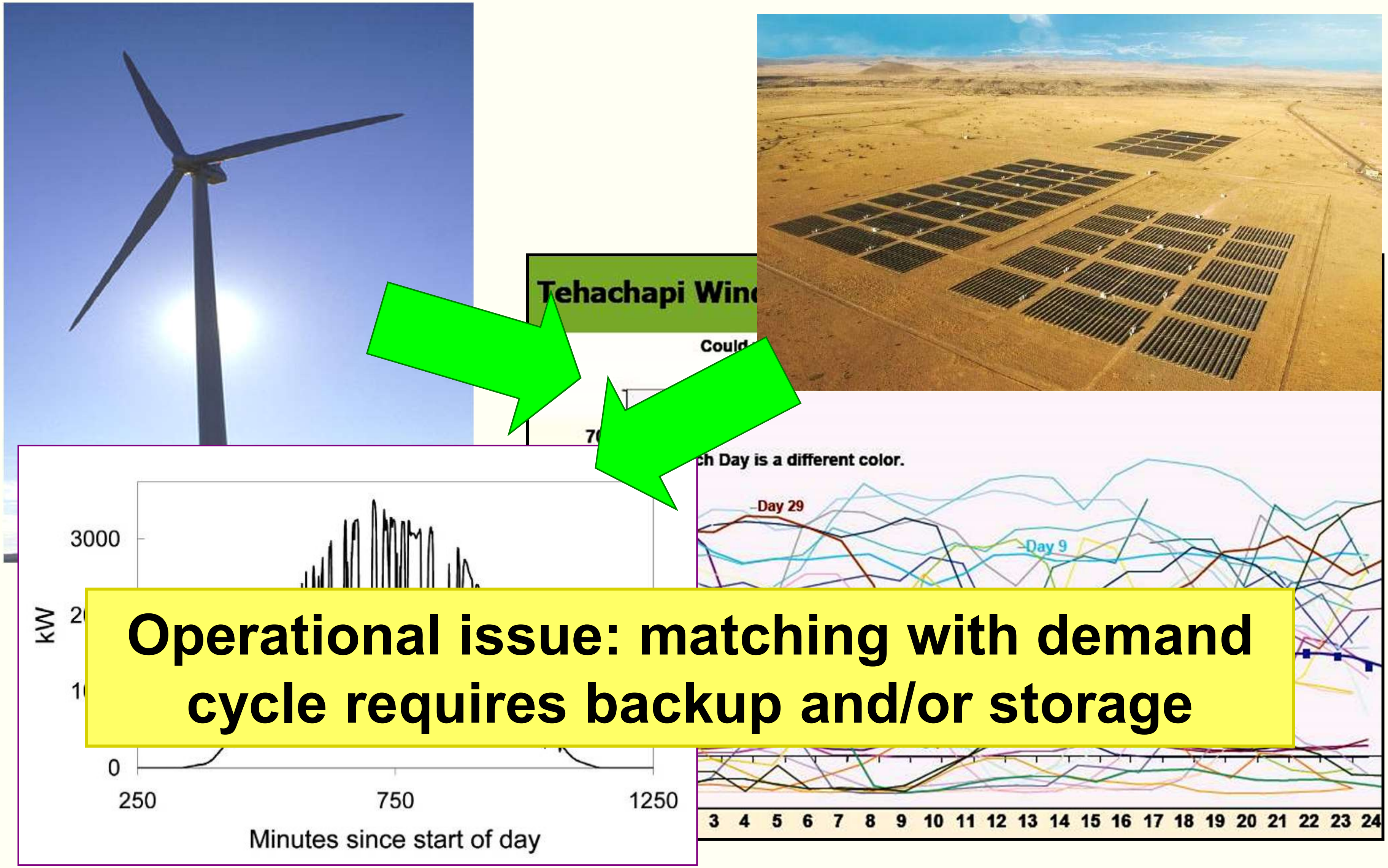
- Rotating blades induce AC voltage.
- Variable wind speed makes frequency conversion necessary.
- Again implemented by power electronics.

Doubly-Fed Induction Generator:



Here as well, electronics provides flexibility to control ω, V, θ.

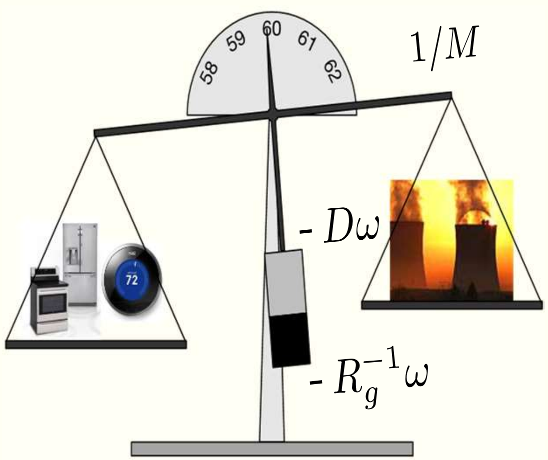
Renewable Challenge I: variable, non-dispatchable sources

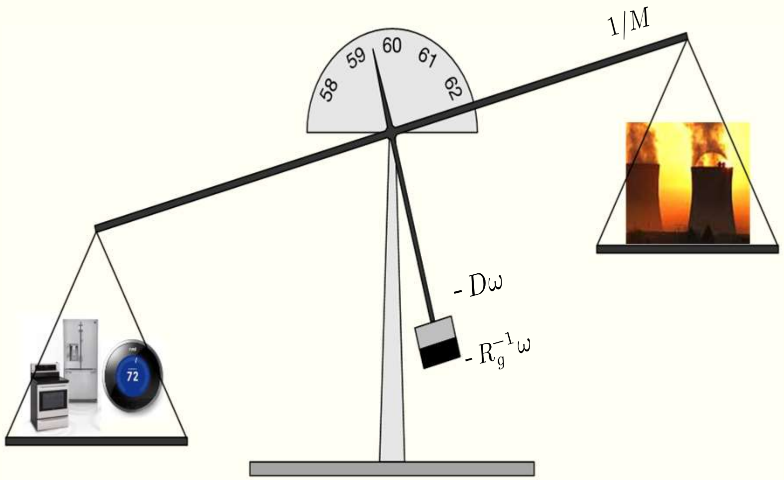


Operational issue: matching with demand cycle requires backup and/or storage

Challenge II: dynamic degradation

- **Loss of rotating inertia:** in renewable generation connected by power electronics.
- **Diminishing frequency dependent loads.**
- **Larger frequency deviations:** Protections may misfire, lead to cascading failure events.





Challenge II: dynamic degradation

Energy Revolution Hiccups

Grid Instability Has Industry Scrambling for Solutions

Sudden fluctuations in Germany's power grid are causing major damage to a number of industrial companies. While many of them have responded by getting their own power generators and regulators to help minimize the risks, they warn that companies might be forced to leave if the government doesn't deal with the issues fast.

By Catalina Schröder

Power struggle: Green energy

Minders of a fragile national power grid say th make it harder to keep the lights on.

December 02, 2013 | By Evan Halper

Germany's Green Ener... Destabilizing Electric Grids

"Energiewende"

JANUARY 23, 2013

RTO Insider

Your Eyes and Ears on the Organized Electric Markets

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FERC: Renewables Must Provide Frequency Response

November 21, 2016

By Rich Heidorn Jr.

In a rulemaking reflecting both reliability concerns and the technological advances of renewable generators, FERC on Thursday proposed revising the *pro forma* Large Generator Interconnection Agreement (LGIA) and Small Generator Interconnection Agreement (SGIA) to require all newly interconnecting facilities to install and enable primary frequency response capability (*RM16-6*).

How to respond to the dynamic challenge

- One option is to emulate traditional behavior:
  – add real inertia (e.g. flywheels), or
  – "Virtual inertia": power electronics controlled to respond like a synchronous generator.

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 28, NO. 2, MAY 2013 1373

Implementing Virtual Inertia in DFIG-Based Wind Power Generation

Mohammadreza Fakhari

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 62, NO. 12, DECEMBER 2017 6209

Optimal Placement of Virtual Inertia in Power Grids

Bala Kameshwar Poolla, Saverio Bolognani, and Florian Dörfler

- A control engineer should ask, however: isn't a heavier system harder to control?

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Dynamic model: synchronous machine

Rotational Newton's law (power $\approx$ torque)

Mechanical power P_i^m

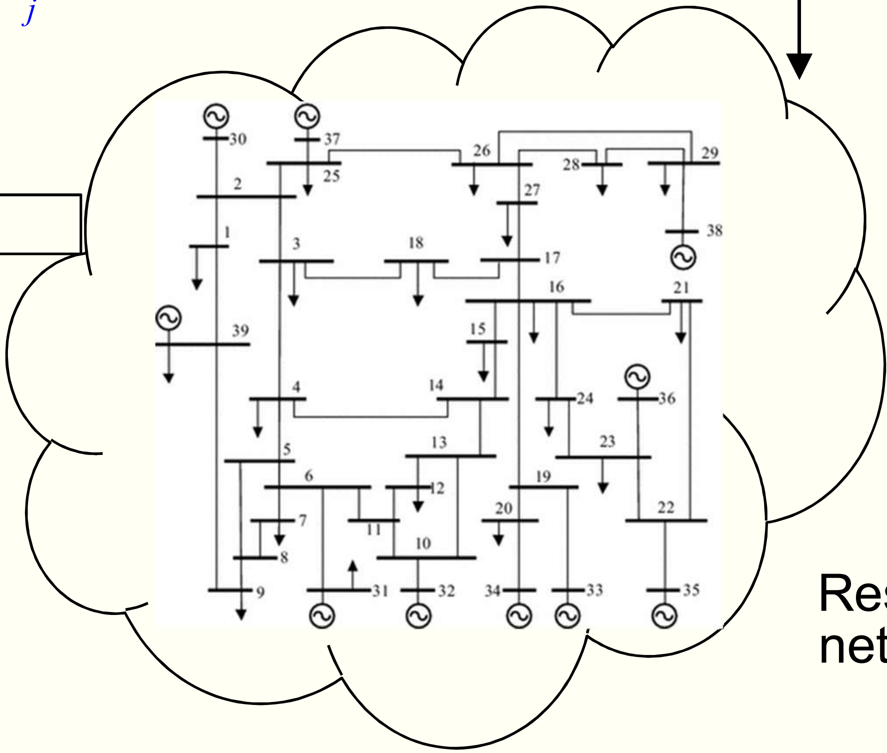
$$m_i \dot{\omega}_i = -d_i \omega_i + P_i^m - P_i^e$$

Angular frequency ω_i $\rightarrow$ $\int$ $\rightarrow$ Angle θ_i

Electrical power sent to load and rest of network

$$P_i^e = D_i + \sum_j K_{ij} \sin(\theta_i - \theta_j)$$

Angles $\theta_j,\ j \neq i$



Rest of the network

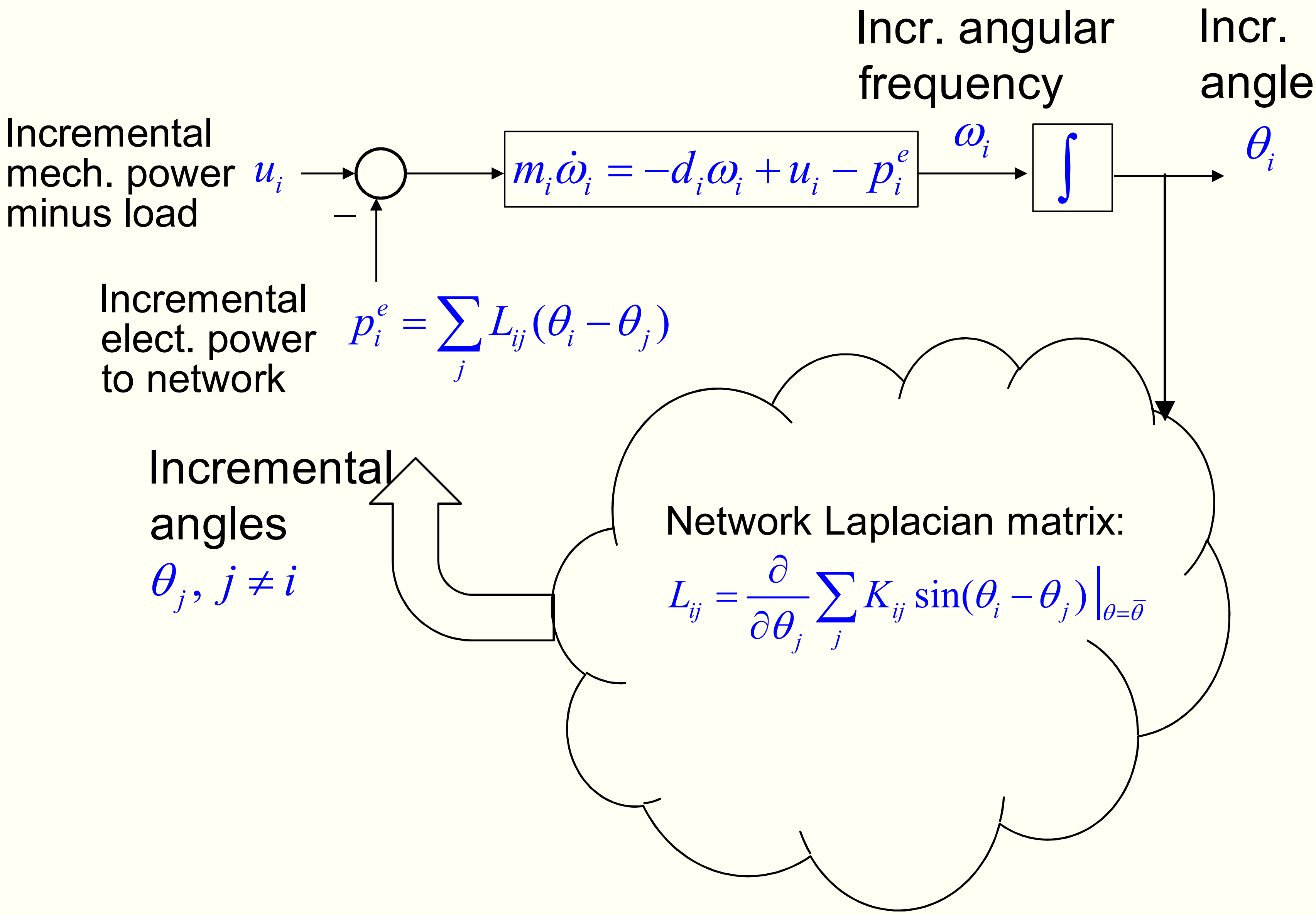
Equilibrium point:

$\omega_i = \overline{\omega};$ $\qquad \theta_i = \overline{\omega} t + \overline{\theta}_i;$

$\overline{P}_i^m - \overline{P}_i^e - d\overline{\omega} = 0;$

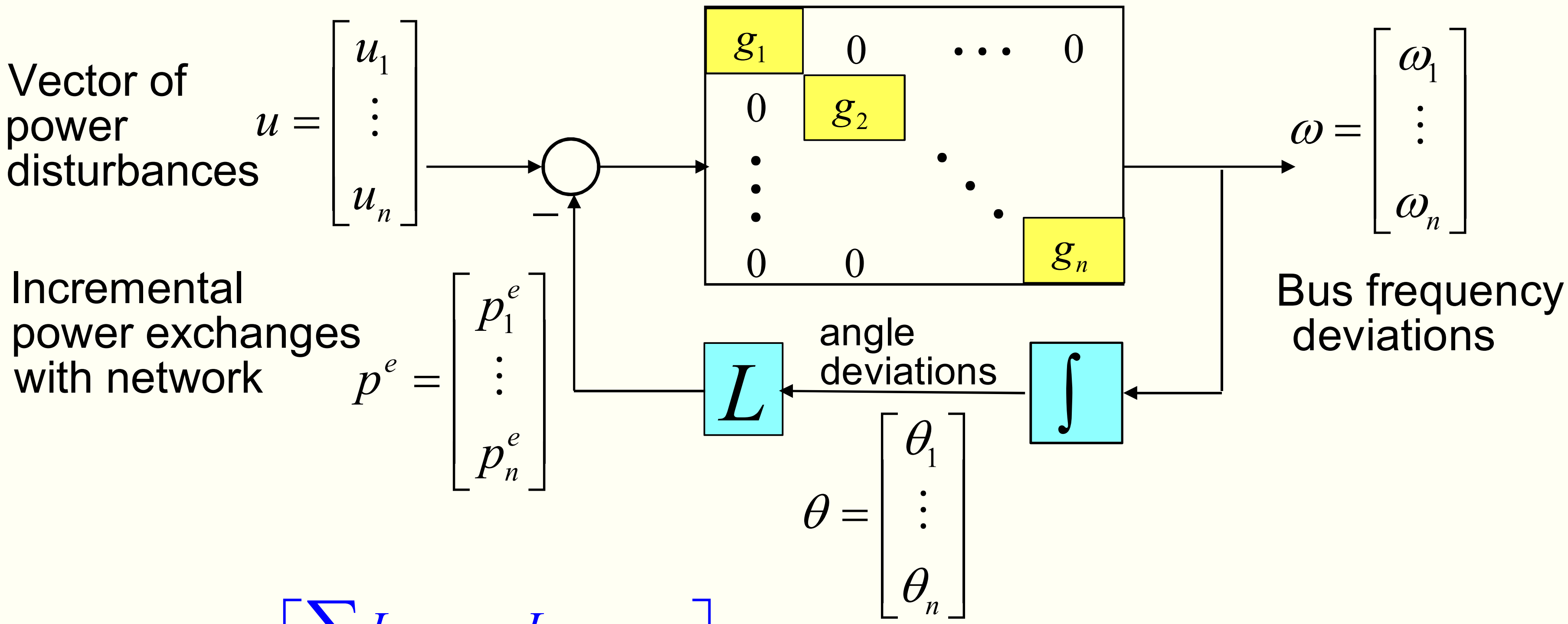
$\overline{P}_i^e = \overline{D}_i + \sum_j K_{ij} \sin(\overline{\theta}_i - \overline{\theta}_j)$

Linearize around the equilibrium point



Linearized model in vector form

"Swing equation" model for machine g_i: $\boxed{m_i \dot{\omega}_i = -d_i \omega_i + u_i - p_i^e}$



Matrix $L = \begin{bmatrix} \sum_{j \neq 1} L_{1j} & -L_{12} & \cdots \\ -L_{12} & \sum_{j \neq 2} L_{2j} & \ddots \\ \vdots & \ddots & \ddots \end{bmatrix}$ is weighted graph Laplacian.

Symmetric, positive semidefinite.

Linearized model in Laplace Transforms

Swing equation model

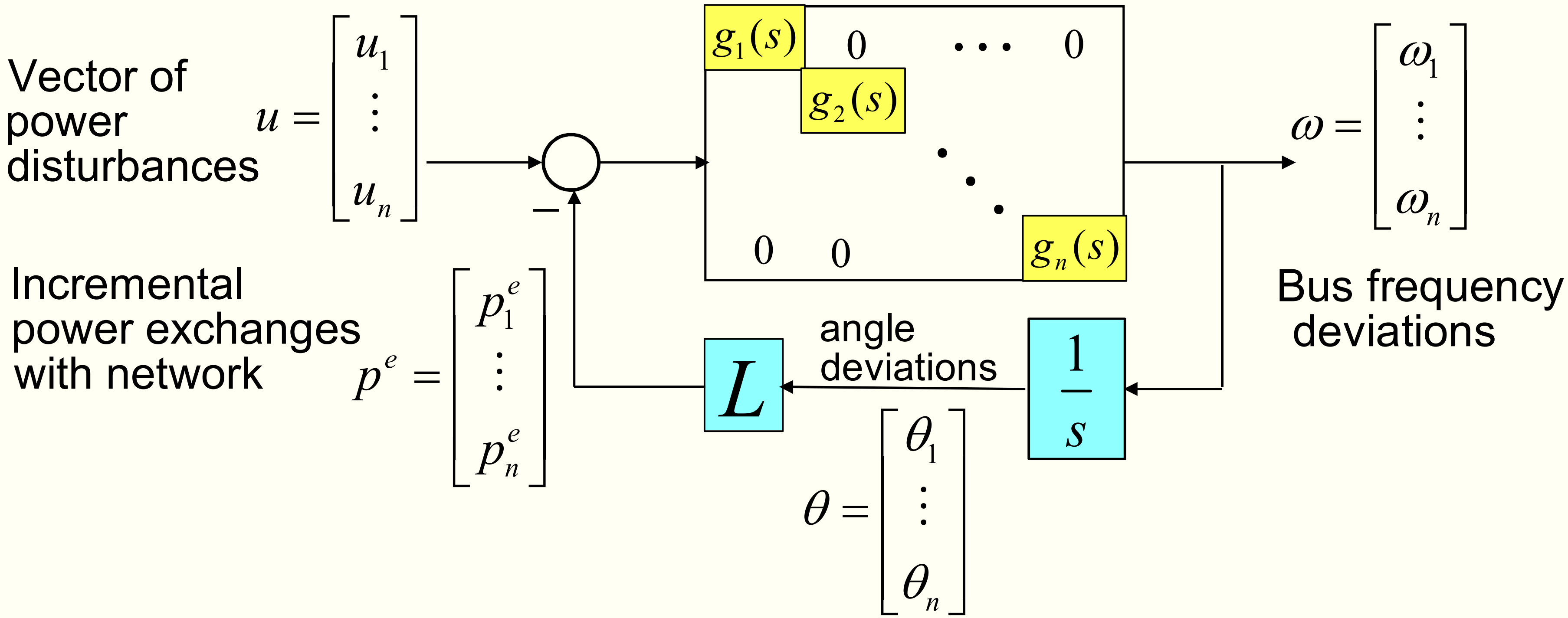
$$m_i \dot{\omega}_i = -d_i \omega_i + u_i - p_i^e$$

$$\boxed{g_i(s) = \frac{1}{m_i s + d_i}}$$

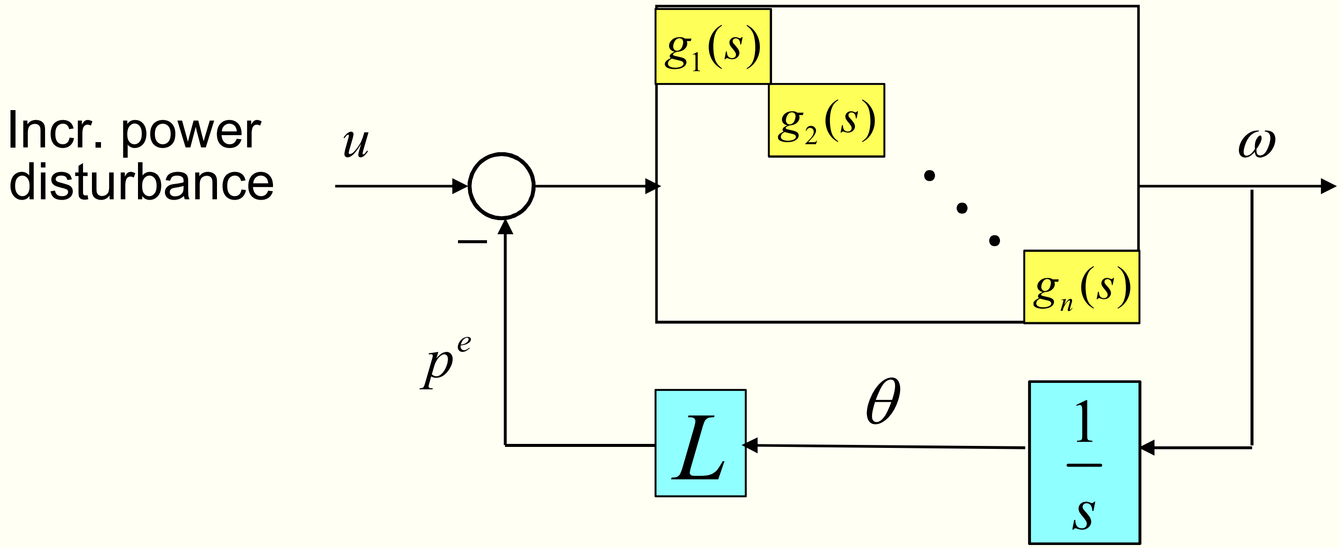
With 1st order turbine droop control

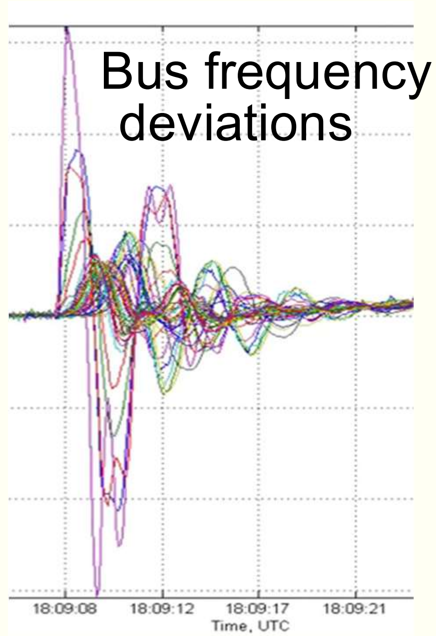
$$\begin{cases} m_i \dot{\omega}_i = -d_i \omega_i + q_i + u_i - p_i^e \\ \tau_i \dot{q}_i = -r_i^{-1} \omega_i - q_i \end{cases}$$

$$\boxed{g_i(s) = \frac{\tau_i s + 1}{m_i \tau_i s^2 + (m_i + \tau_i d_i) s + d_i + r_i^{-1}}}$$



How do we measure frequency response?



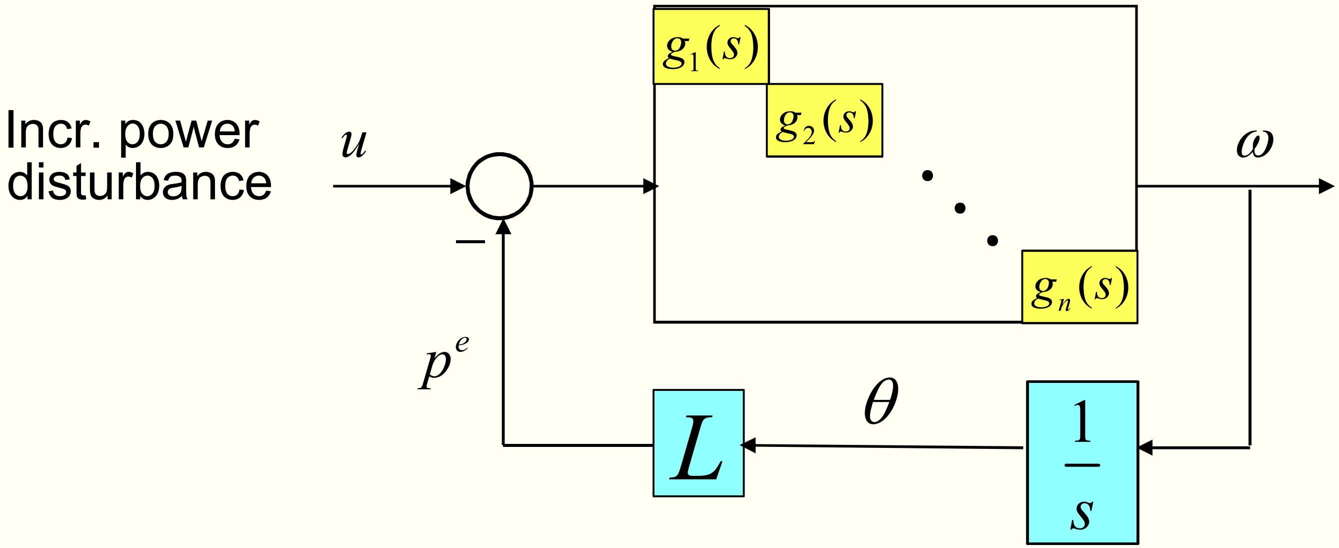


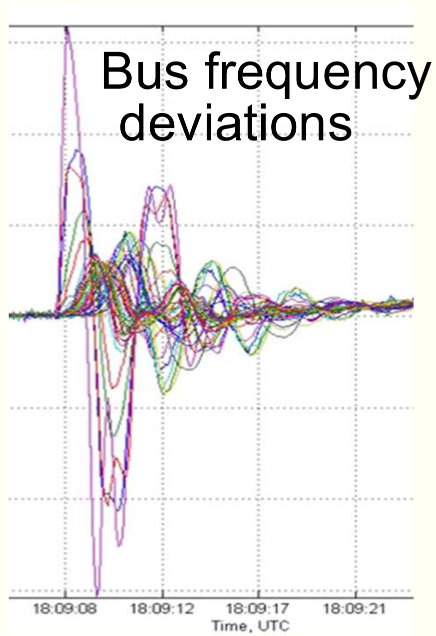
Robust Control approach

[Tegling-Bamieh-Gayme ’15, Simpson-Porco et al ’17,…]

- Global view of synchronization performance.
- Use signal norm of vector $\omega(t)$, or other coherency measure $\left(\text{e.g., } \theta_i(t)-\theta_j(t)\right)$, for specific classes of disturbances (noise, sinusoids,...)
- Alternatively, an operator norm of the mapping $u(t) \mapsto \omega(t)$.
- Different criteria ($\mathcal{H}_2$, $\mathcal{H}_\infty$,...), different interpretations.

How do we measure frequency response?





Robust Control approach

Positives: analytical results capture role of parameters, e.g. inertia.

Limitations. Restrictive assumptions:

- Homogeneous machines, swing model:

$$g_1(s) = g(s) = \frac{1}{ms+d} \quad i = 1, \ldots N.$$

- Reconcile with power engineering metrics?

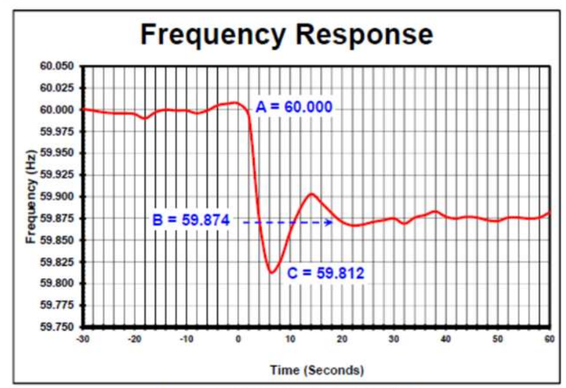


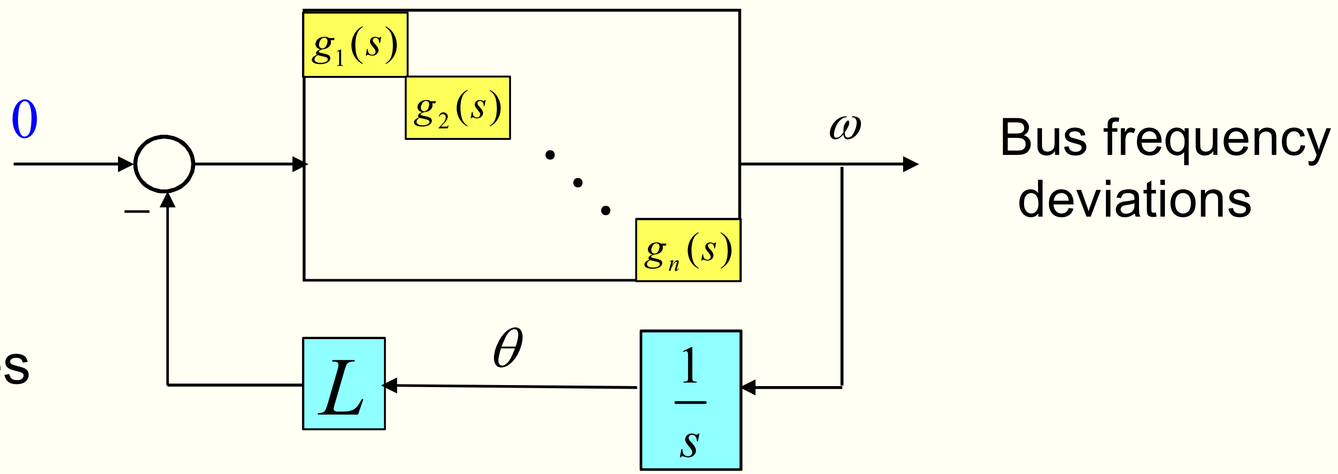
Figure 1. Frequency Response Characteristic

Bridging the Theory-Practice gap

[F. Paganini & E. Mallada, *IEEE Transactions on Automatic Control*, 2020, published online in early access]

$$u = \underbrace{\begin{bmatrix} u_{01} \\ \vdots \\ u_{0n} \end{bmatrix}}_{u_0}, \quad t \geq 0$$

Step disturbances



- General machine model $g_i(s)$.
- Heterogeneous scale, can solve analytically under a proportionality assumption.

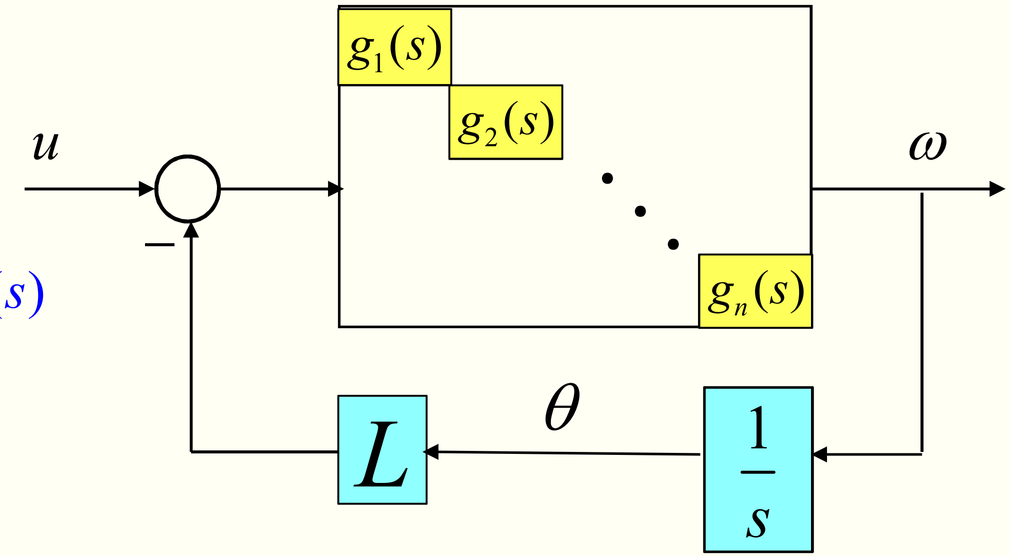
Decompose response:

- System-wide component, to which we apply power engineering metrics.
- Vector of deviations, to which we apply control theory metrics.

Proportionality assumption

$$g_i(s) = \frac{1}{f_i} g_0(s)$$

- f_i : machine rating, relative to a representative machine $g_0(s)$
- Larger units respond less!



Special cases:

- in swing equation model:

$$m_i = m\, f_i; \quad d_i = d\, f_i.$$

$$g_0(s) = \frac{1}{m\ s + d}$$

i.e., m_i proportional to d_i

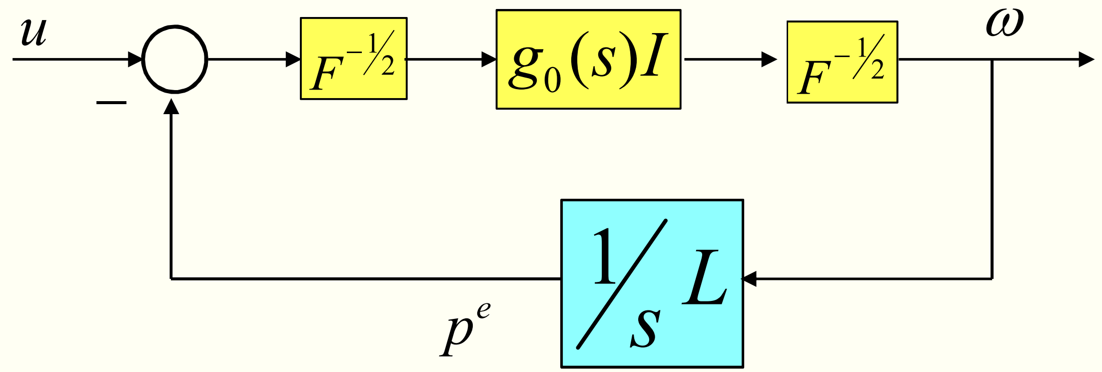
- in model with turbine droop control, also:

$$r_i^{-1} = r^{-1} f_i; \qquad \tau_i = \tau.$$

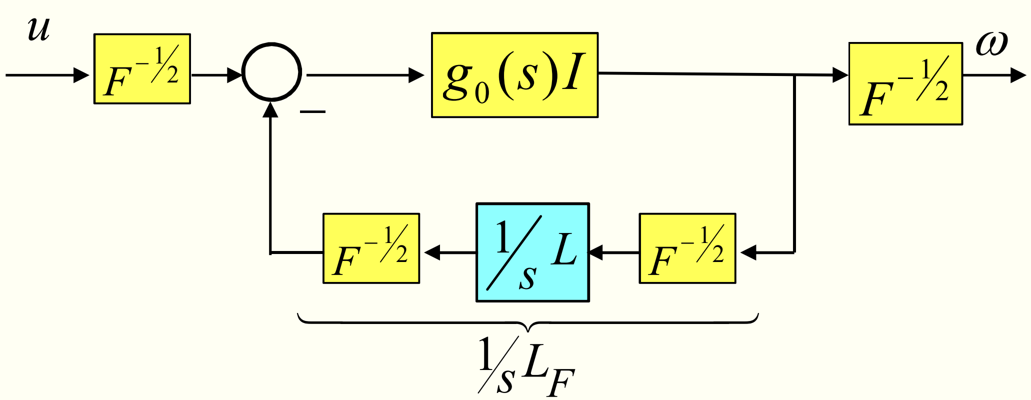
$$g_0(s) = \frac{\tau s + 1}{m\tau s^2 + (m + \tau d)s + d + r^{-1}}$$

- Not exactly satisfied in practice, but order of magnitude is correct.
- Far more realistic than homogeneity.
- Will later validate approach with real world data.

Proportionality → Diagonalization



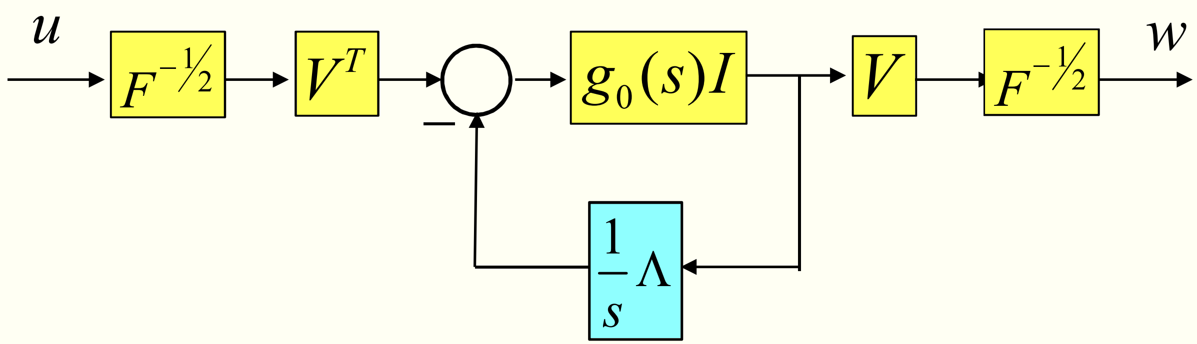
$$F = \begin{bmatrix} f_1 & 0 & \cdots & 0 \\ 0 & f_2 & 0 & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & f_n \end{bmatrix}$$



Scaled Laplacian $L_F = F^{-1/2} L F^{-1/2}$

Pos. semidefinite, diagonalize:

$$L_F = V \underbrace{\begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & \lambda_1 & 0 & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_{n-1} \end{bmatrix}}_{\Lambda} V^T$$



Step response decomposition

$$u(t)=u_0,\ t\geq 0 \longrightarrow \boxed{V^T F^{-1/2}} \longrightarrow \begin{bmatrix} h_0(s) & 0 & \cdots & 0 \\ 0 & h_1(s) & 0 & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_{n-1}(s) \end{bmatrix} \longrightarrow \boxed{F^{-1/2} V} \xrightarrow{\ \omega(t)\ }$$

$$\boxed{\begin{aligned} h_i(s) &= \frac{s\ g_0(s)}{s+\lambda_k g_0(s)}, \\ 0 = \lambda_0 &< \lambda_1 \leq \cdots \leq \lambda_{n-1} \end{aligned}}$$

Compute in Laplace:

$$\omega(s) := \underbrace{\left(\sum_i f_i\right)^{-1} \sum_i u_{0i}\tilde{h}_0(s)}_{\bar{\omega}(s)} \underbrace{\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}}_{\mathbf{1}} + \underbrace{F^{-1/2} V_\perp \tilde{H}(s) V_\perp^T F^{-1/2} u_0}_{\tilde{\omega}(s)}; \qquad \tilde{h}_i(s) = h_i(s)/s$$

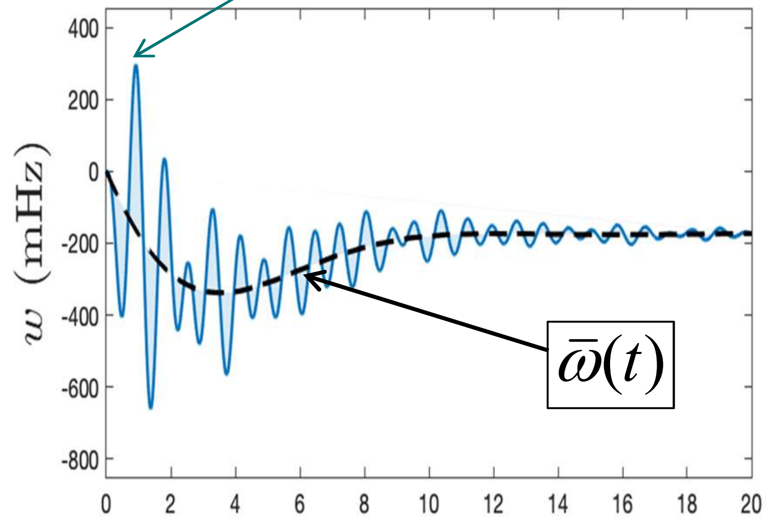
Time domain response:

$$\boxed{\omega(t) = \bar{\omega}(t)\ \mathbf{1} + \tilde{\omega}(t)}$$

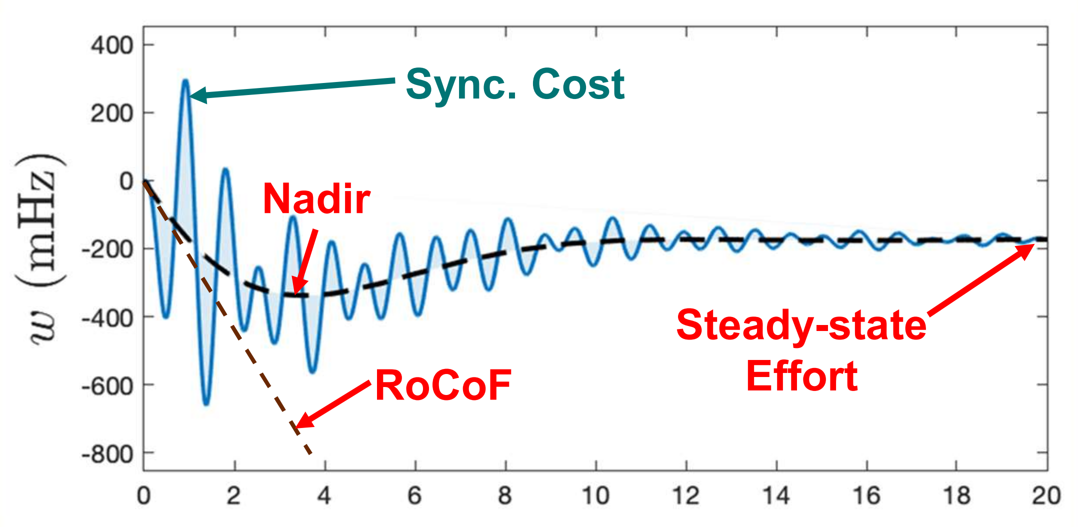
- Scalar $\bar{\omega}(t)$ is a system frequency, applies equally to all nodes.
- Vector $\tilde{\omega}(t)$ of individual node deviations from synchrony. Transient term.

One component

$$\omega_i(t) = \bar{\omega}(t) + \tilde{\omega}_i(t)$$



Step response decomposition



$$\boxed{\omega(t) = \bar{\omega}(t)\ \mathbf{1} + \tilde{\omega}(t)}$$

System frequency $\bar{\omega}(t)$.

- Coincides with motion of the center of inertia: $\bar{\omega}(t) = \left(\sum_i m_i\right)^{-1} \sum_i m_i \omega_i(t).$
- Depends on generators and imbalance, not the network L.
- We can apply standard metrics to this object: Nadir, RoCoF,...

Vector of oscillatory components $\tilde{\omega}(t)$

- Depends on both network and generator model.
- We use its L_2 norm as synchronization cost: $\| \tilde{w} \|_2^2 = \int_0^\infty |\tilde{w}(t)|^2\ dt$

System frequency: swing equation model

System frequency: has a first order response,

$$\overline{\omega}(t) = \left(\sum_i d_i\right)^{-1} \sum_i u_{0i}\left(1 - e^{-d/_m t}\right).$$

No overshoot:

Nadir $=$ steady-state value.

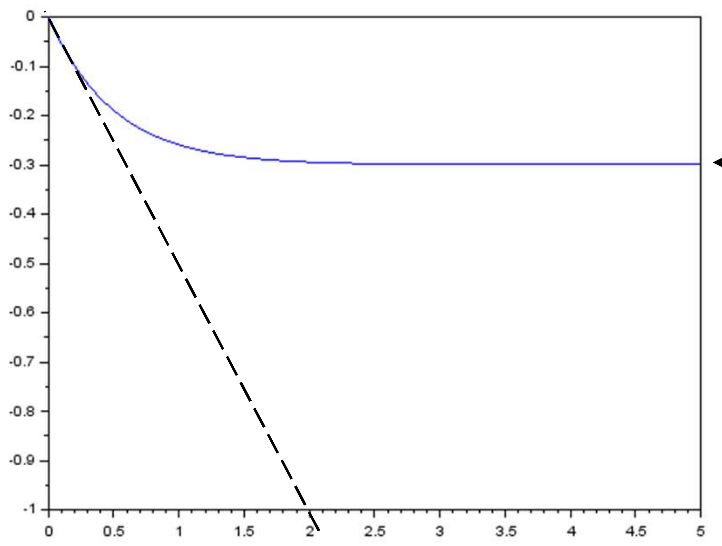
Independent of inertia

Maximal RoCoF:

initial response.

Inertia appears directly

$$||\dot{\bar{w}}||_\infty = \frac{|\sum_i u_i|}{\sum_i f_i}\frac{1}{m}$$



$$||\bar{w}||_\infty = \frac{|\sum_i u_i|}{\sum_i f_i}\frac{1}{d}$$

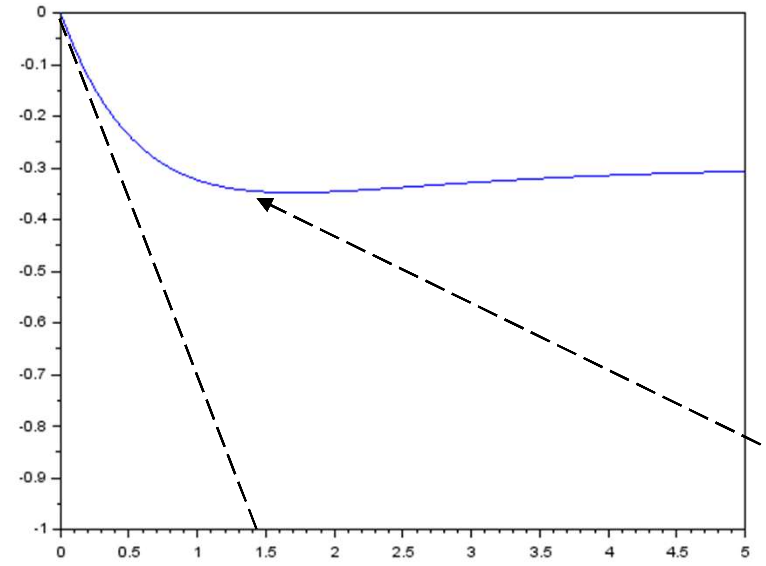
Model including turbine droop control

System frequency. 2nd order response. e.g. underdamped case:

$$\overline{\omega}(t) = \frac{\sum_i u_{0i}}{\sum_i d_i + r_i^{-1}} \left[1 - e^{-\eta t} \left(\cos(\omega_d t) - \frac{(\gamma - \eta)}{\omega_d} \sin(\omega_d t) \right) \right]$$

Maximal RoCoF:
can show that,
like in swing case,

$$||\dot{\bar{w}}||_\infty = \frac{|\sum_i u_i|}{\sum_i f_i} \frac{1}{m}$$



Steady state

$$\frac{\sum_i u_{0i}}{\sum_i d_i + r_i^{-1}}$$

Nadir at overshoot.
Decreases (mildly)
with inertia.

$$||\bar{w}||_\infty = \frac{|\sum_i u_{0i}|}{\sum_i f_i} \frac{1}{d + r^{-1}} \left(1 + \sqrt{\frac{\tau r^{-1}}{m}} e^{-\frac{\eta}{\omega_d}\left(\phi + \frac{\pi}{2}\right)} \right)$$

Synchronization cost

In swing dynamics.

Depends on inertia, but limits indicate influence is mild.

Low inertia case:

$$\|\tilde{w}\|_2^2 \xrightarrow{m\to 0} \sum_{k,l=1}^{n-1} \frac{\gamma_{kl} z_{0k} z_{0l}}{d(\lambda_k + \lambda_l)}.$$

High inertia case:

$$\|\tilde{w}\|_2^2 \xrightarrow{m\to\infty} \sum_{k=1}^{n-1} \frac{\gamma_{kk} z_{0k}^2}{2d\lambda_k}$$

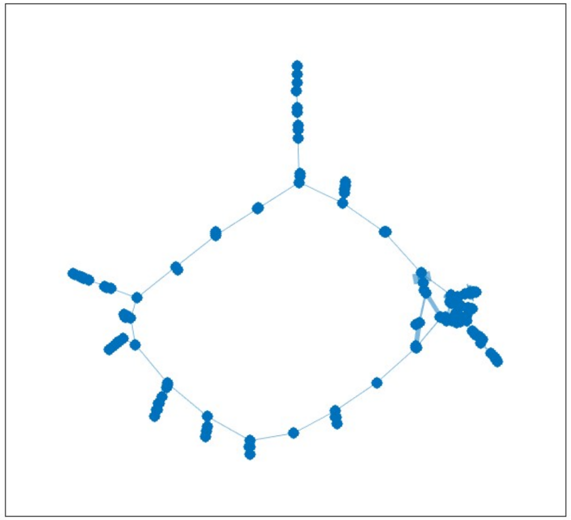
Model with turbine droop control

High inertia ~ swing model: $\|\tilde{w}\|_2^2 \xrightarrow{m\to\infty} \sum_{k=1}^{n-1} \frac{z_{0k}^2 \gamma_{kk}}{2\lambda_k d} \cdot \frac{d}{r^{-1} + d}.$

Expressions for low inertia $(m \to 0)$ are more involved, but the limit is again finite. We will compare numerically.

Simulation Study: Icelandic Grid

- Real network, sparse topology
- Heterogeneous ratings.
- Parameters not proportional.

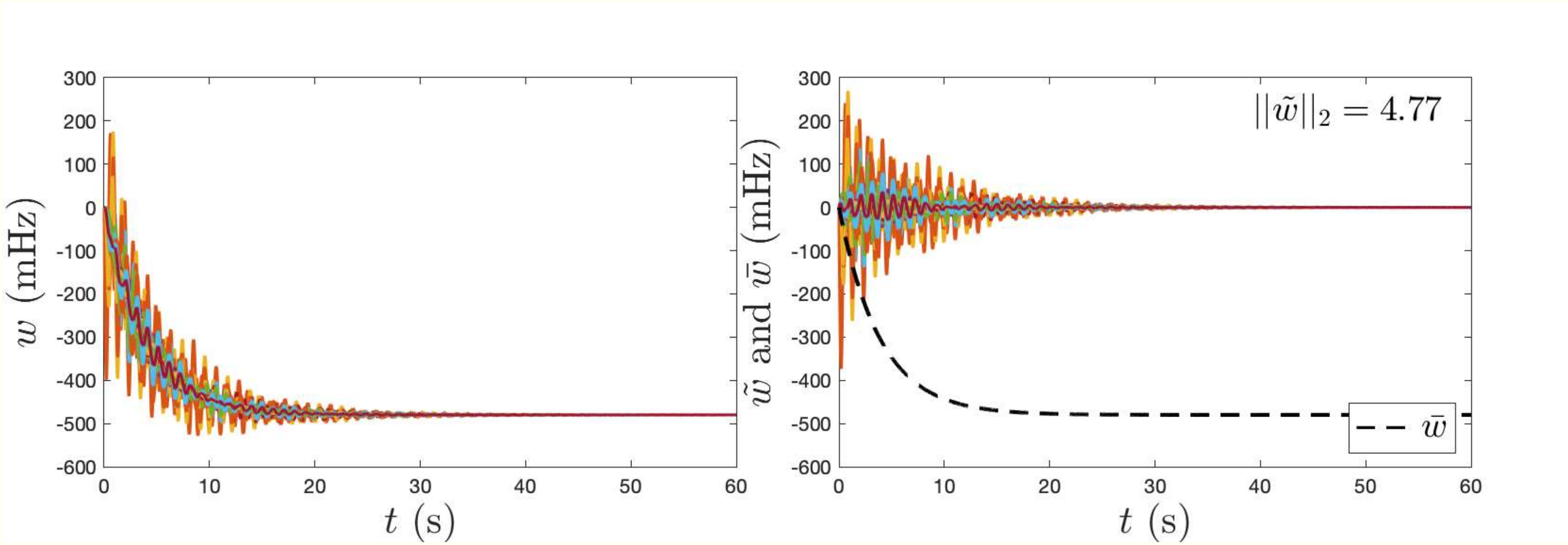


Synthetic data with proportionality:

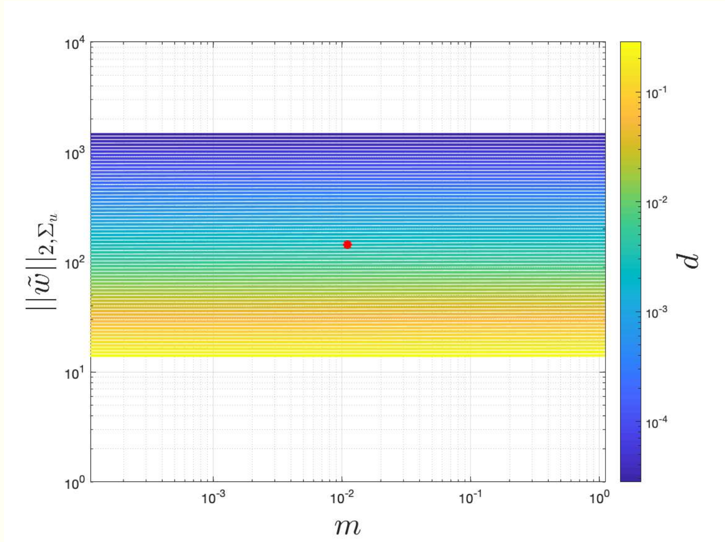
- Real network, graph, admittances $\rightarrow$ Laplacian L
- Real values of inertia m_i. Define $m = \frac{1}{n}\sum_{i=1}^{n} m_i$, rating $f_i = {m_i}/{m}$
- Synthethic d, r^{-1} so the proportional system has the same total damping, total droop control. Average value for τ.

Swing dynamics

Step response and its decomposition, disturbance in bus 2

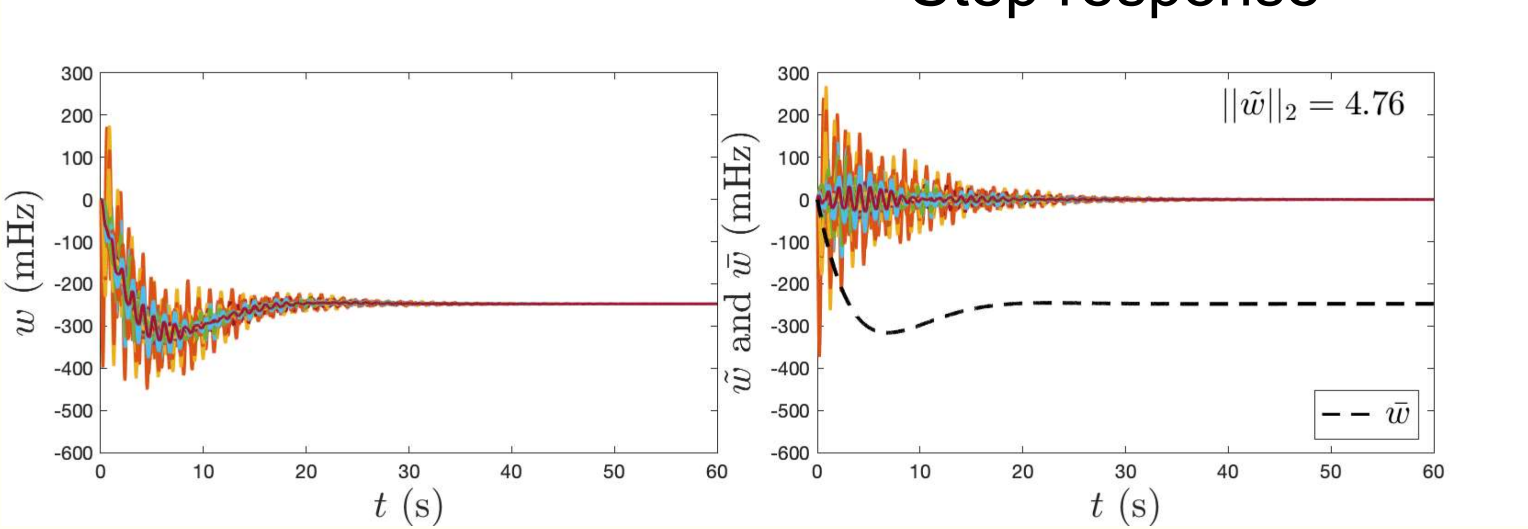


Synchronization cost
as a function of:
inertia m, damping d.
Red dot: nominal values.

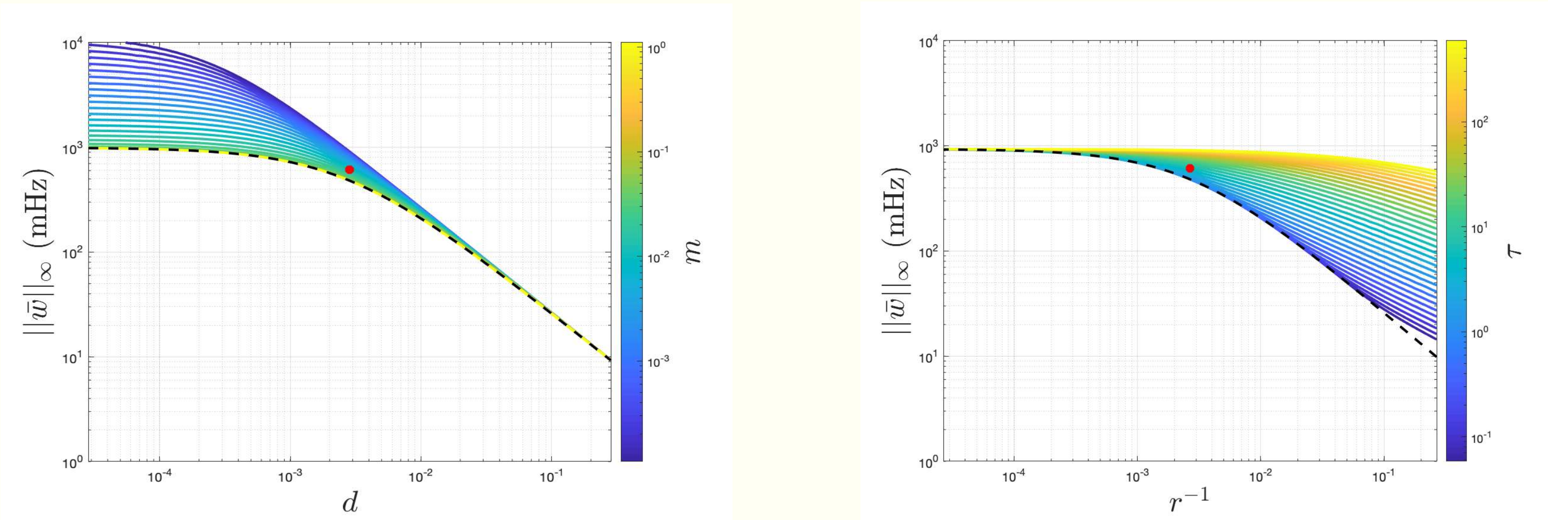


Turbine dynamics

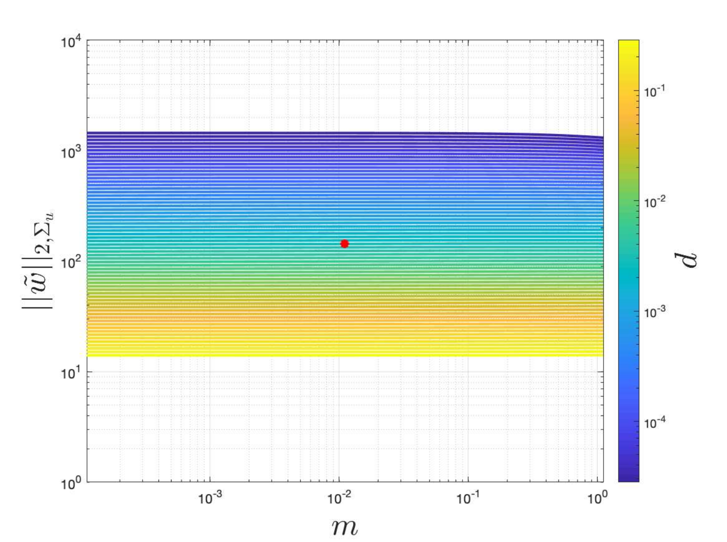
Step response



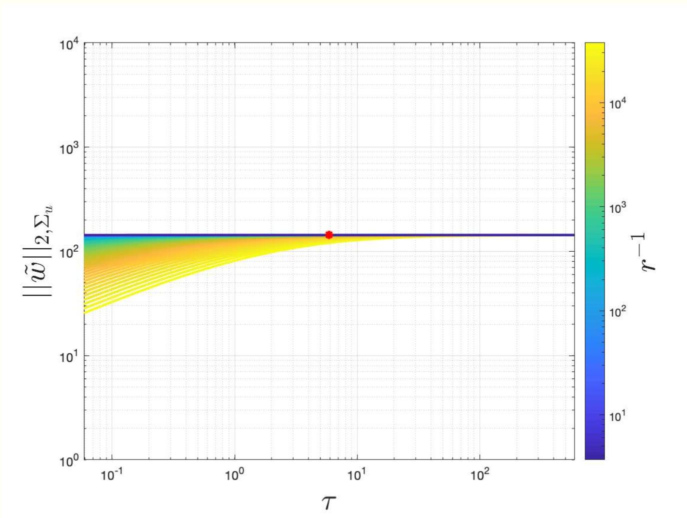
Nadir as a function of parameters

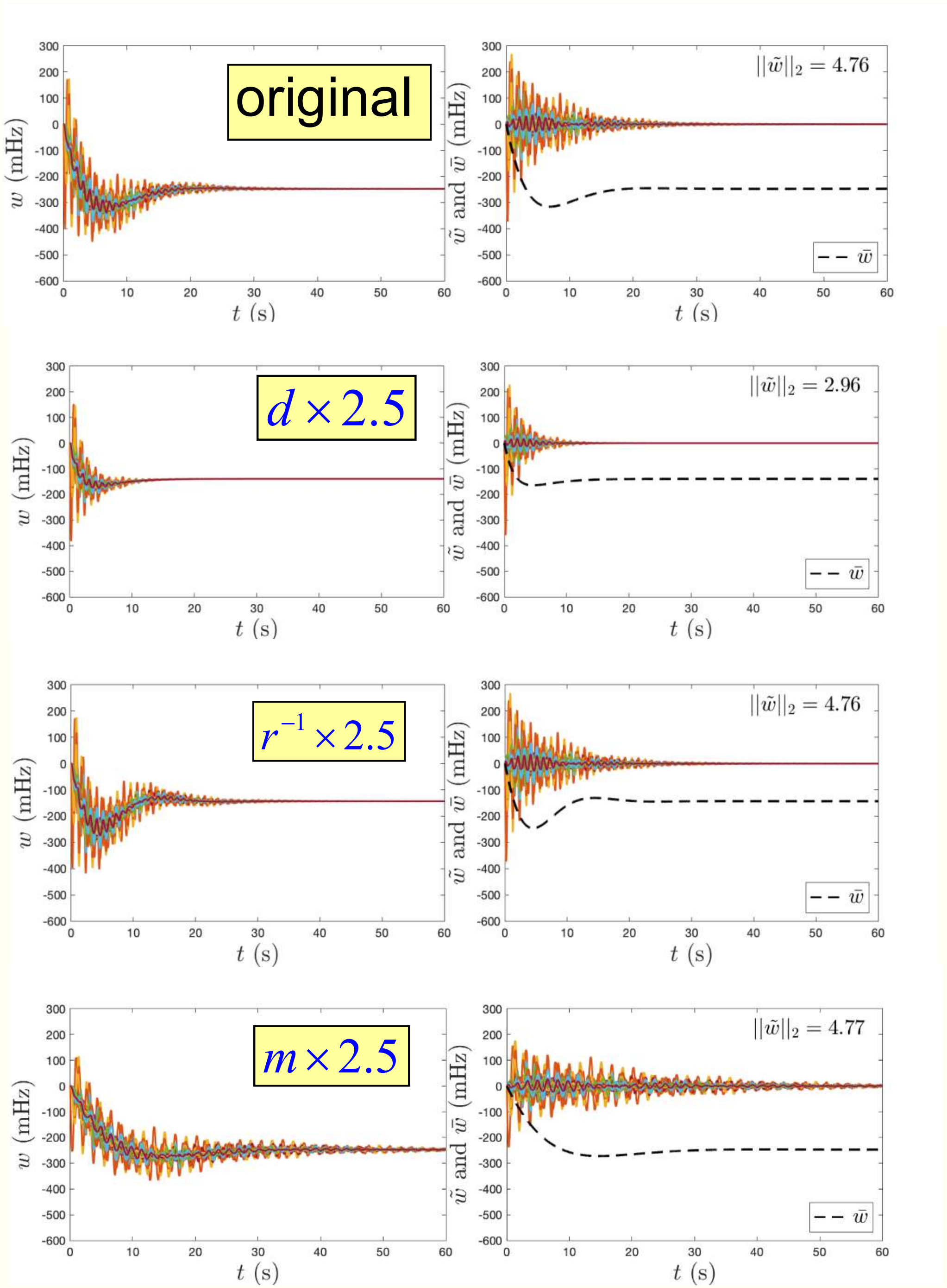


Turbine dynamics



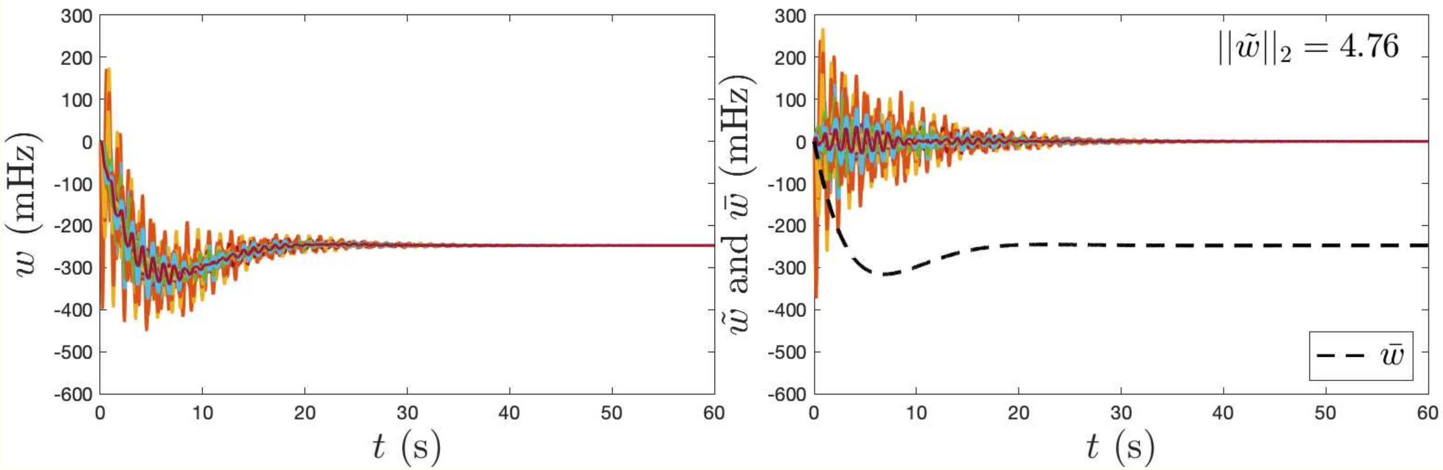
Synchronization cost



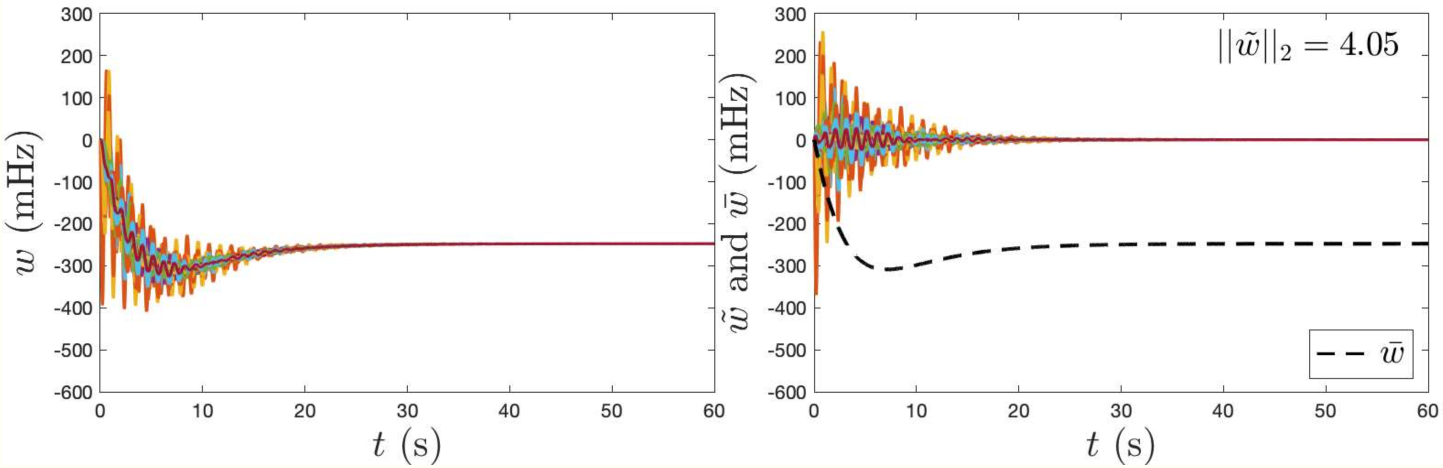


Validation with real Icelandic grid

Synthetic, proportional parameters



True, non-proportional parameters



Summary of our analysis

- Reconciled power engineering metrics and standards with a global view of performance.
- Models matter! Swing model misses key features, important to include droop control lags.
- Role of inertia less dramatic than in conventional wisdom. Lighter systems are also faster to control.
- Short-term damping d is a more crucial parameter.
- "Cyber-physical" options for a grid of less inertia:
  1. Control the inverters of renewable energy sources (e.g. iDroop, Jiang et al. '19.)
  2. Load-side frequency regulation: demand response may provide regulation service (e.g. Zhao et al '14).

Outline

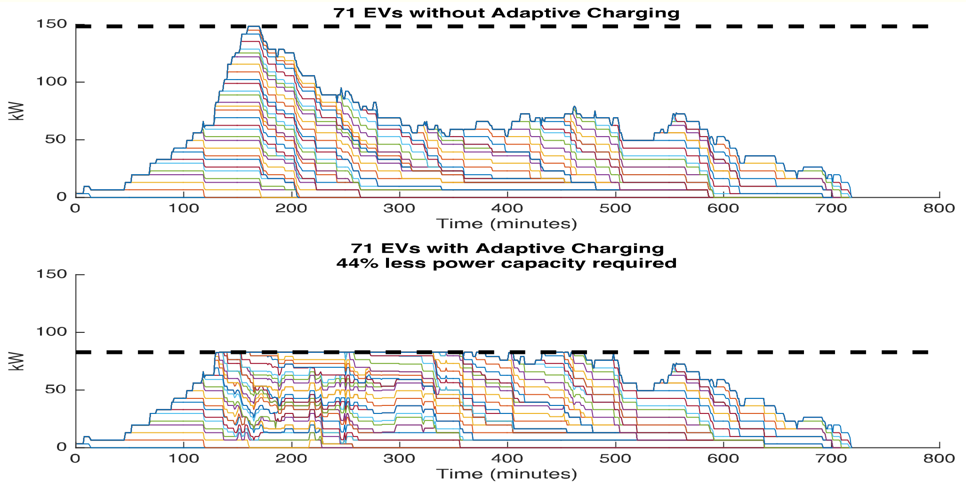
1. Background on the AC power grid and its dynamic control.
2. Introducing new energy sources: opportunities and challenges.
3. Analysis with multivariable control tools.
4. Active control of load in Smart Grids.
5. Conclusions.

Time scales of power balancing

- Frequency regulation classification:
  - Primary FR or "droop control". Decentralized feedback at each machine achieves power balance away from nominal frequency.
  - Secondary FR. Correct back to nominal frequency, through actions coordinated by the System Operator (SO).
- Traditionally, SO generates "Area Control Error" signal.
- Certain generators are dedicated to tracking these signals.
- Alternative: can a smarter control of load provide regulation?

Aggregates of deferrable loads

- Smart Grids enable deferring service for some kinds of loads.
- e.g., peak shaving in an EV charging facility [Low et al., ‘17]



data: g2016mtv2000

- Another use of controlled deferral: tracking a reference signal provided by the SO for frequency regulation.

Related work on load side secondary regulation:

- Model predictive control of deferral [Subramanian et al ‘13].
- Thermostatically controlled loads [Koch et al ’11, Hao- et al ’14].
- Building HVAC systems [Lin-Barooah-Meyn-Middlekoop’15]

Queueing model of deferrable loads

[F. Bliman, F. Paganini, A. Ferragut, *IEEE Trans. on Smart Grid*, 2017]

Loads arrive for service, at rate λ → **service power controlled by aggregator** → served loads depart

Q_k: energy required by $k-$th arrival .

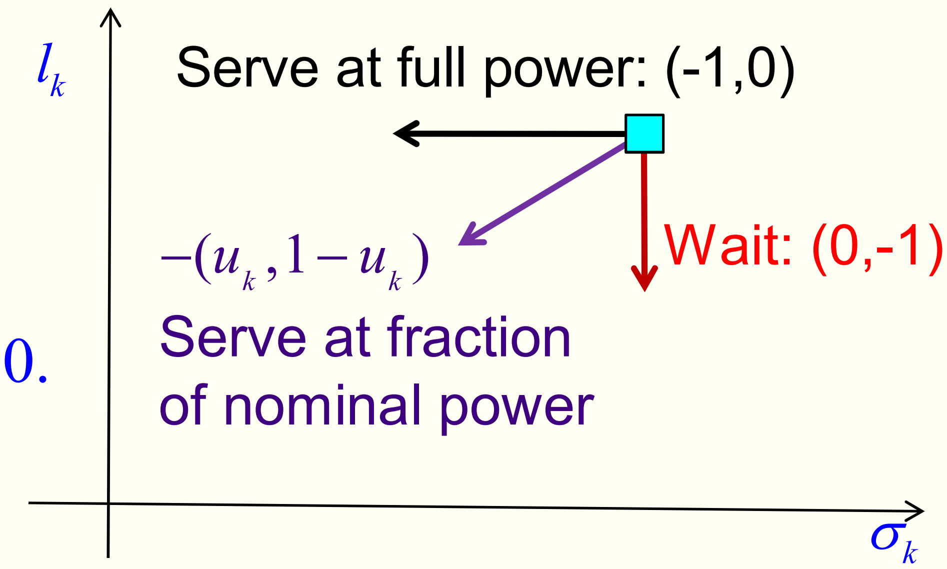
p_0 : common nominal power

$\sigma_k = Q_k / p_0$: service time at nominal power.

l_k : laxity (spare time).

Control deferred service through u_k: fraction of nominal power.

- Departs when reaching $\sigma = 0.$
- Misses deadline at $l = 0.$

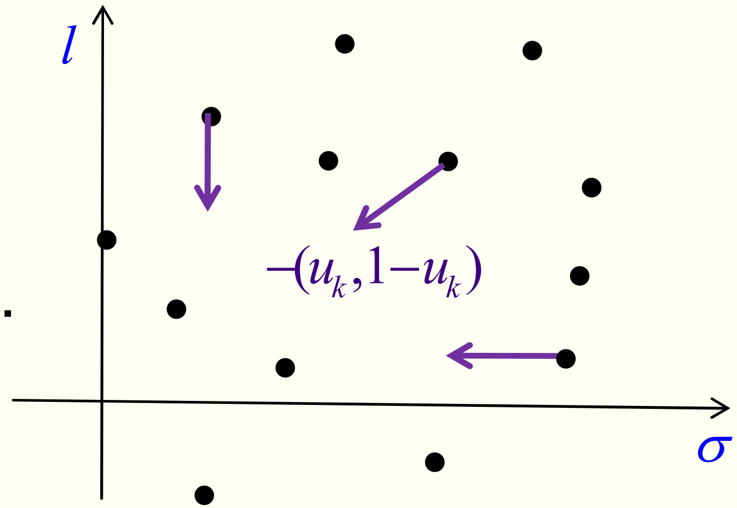


Controlling a large population

Choosing u_k for each load present,

total power consumption $p(t) = p_0 \sum_k u_k$

can track reference $r(t)$ provided by SO.



For instance: to track reference $r(t)$,

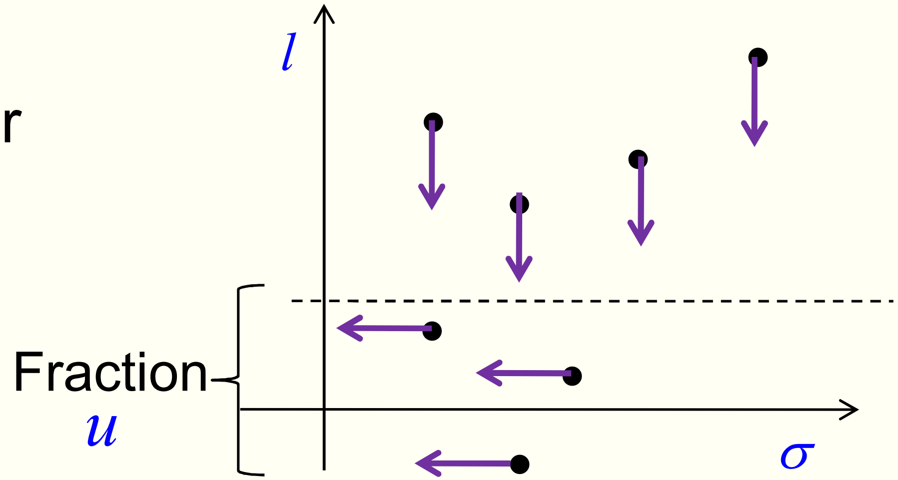
with $n(t)$ loads present, serve a

fraction $u(t) = r(t) \big/ n(t) p_0$ at full power

Least-Laxity-First scheduling:

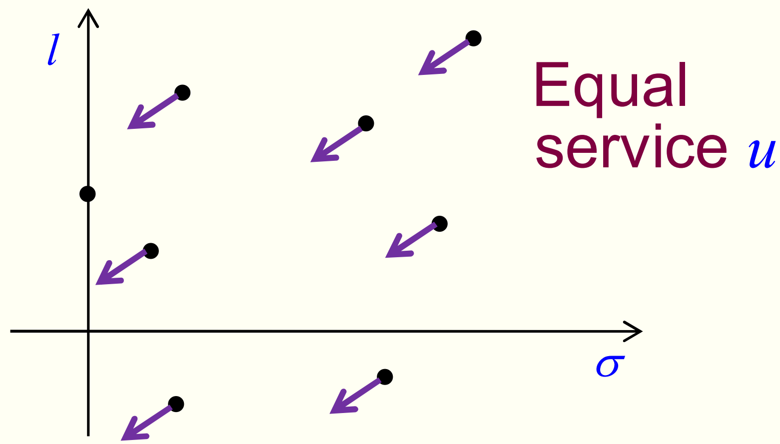
choose loads with smallest laxity.

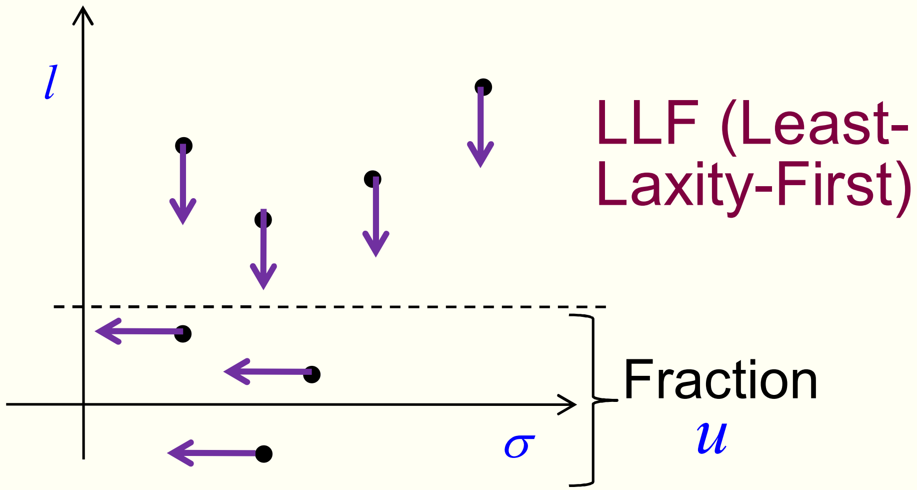
Helps enforce deadlines.



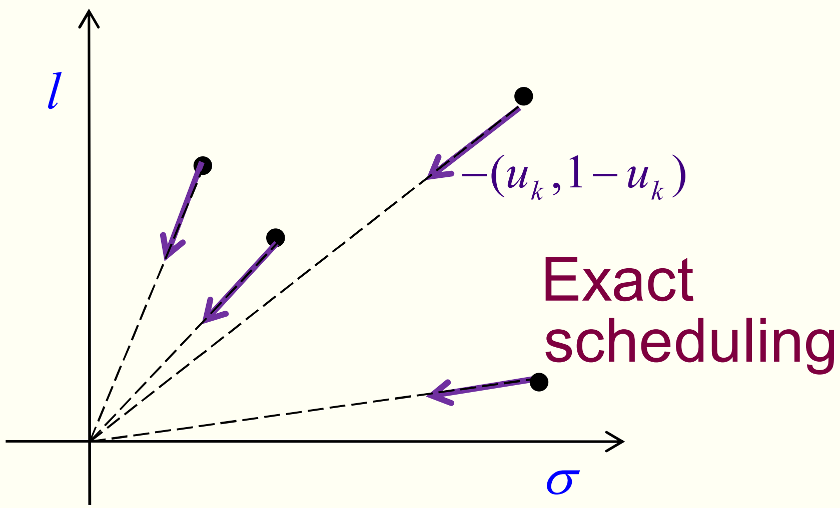
Limitation: requires load micromanaging by aggregator.

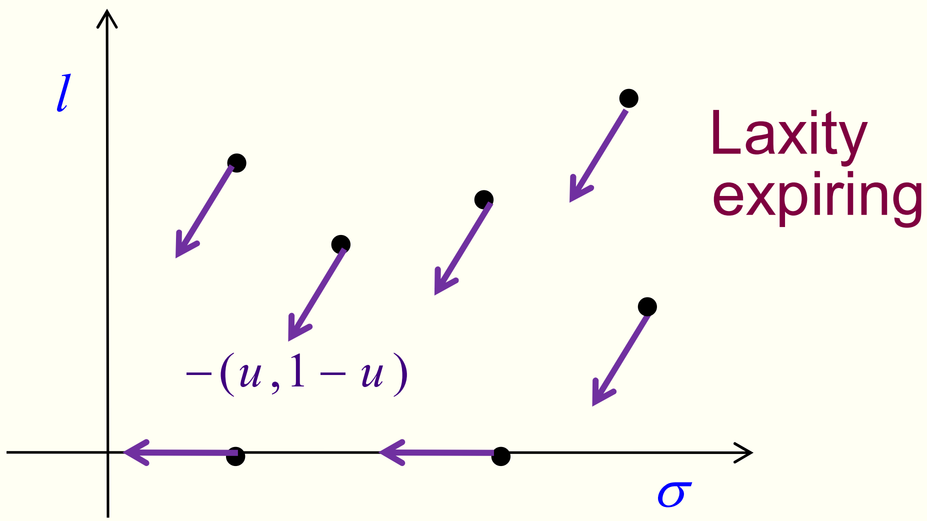
Strategies with soft deadlines





Alternatives for firm deadlines





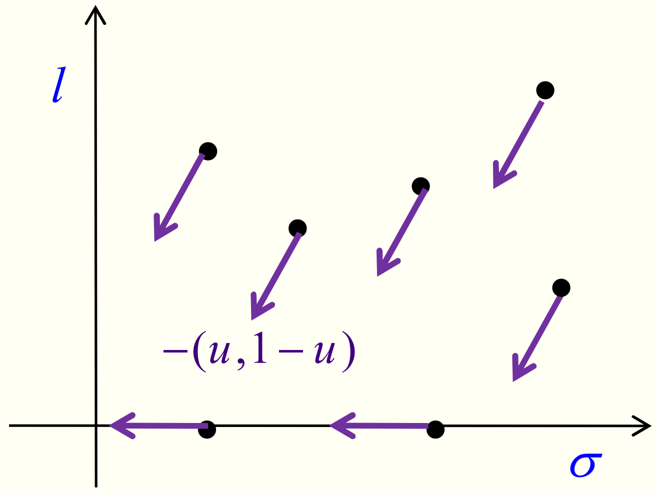
Models for laxity expiring case

State variables are load populations:

$n(t)$: with remaining laxity.

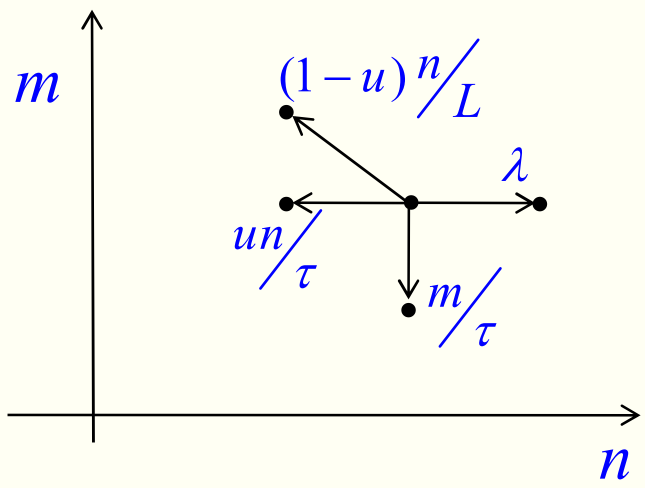
$m(t)$: with expired laxity.

Control $u(t)$ applied to loads with laxity.



Markov chain model:

- Poisson (λ) arrivals.
- $\sigma_k \sim \exp(1/\tau)$, $l_k \sim \exp(1/L)$



Macroscopic fluid flow model:

$$\dot{n}(t) = \lambda - \frac{1}{\tau} n(t)u(t) - \frac{1}{L} n(t)(1-u(t))$$

$$\dot{m}(t) = \frac{1}{L} n(t)(1-u(t)) - \frac{1}{\tau} m(t)$$

$$p(t) = p_0\left[n(t)u(t) + m(t)\right].$$

Macroscopic model with randomness:

diffusion model, with cont. time noise.

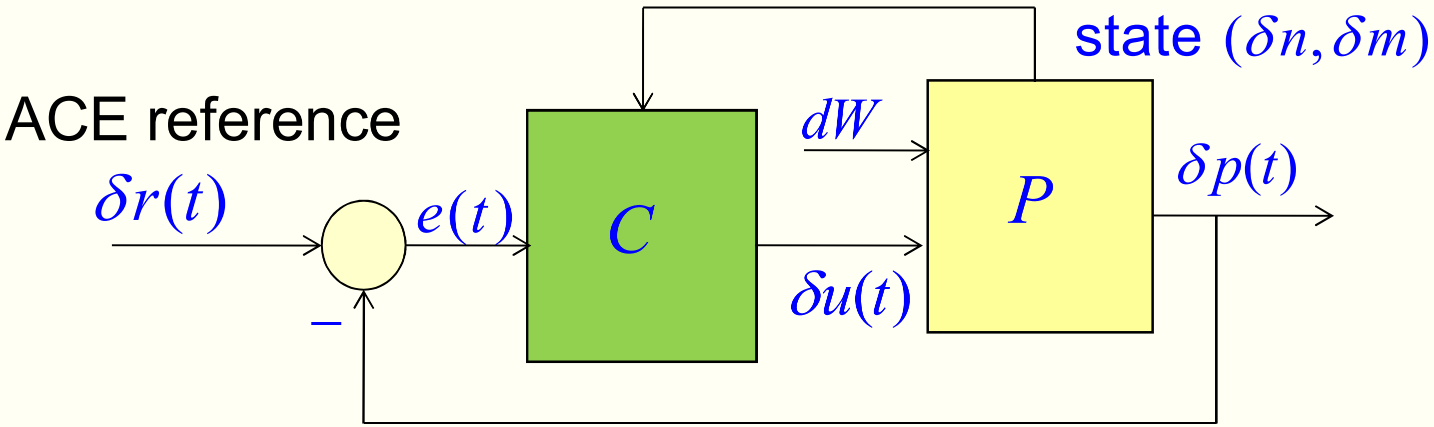
Control using diffusion model

Incremental model, linearized around an equilibrium point:

$$d\overbrace{\begin{bmatrix}\delta n\\ \delta m\end{bmatrix}}^{dx}=\overbrace{\begin{bmatrix}-\nu & 0\\ \frac{1-u^*}{L} & -\frac{1}{\tau}\end{bmatrix}}^{A}\overbrace{\begin{bmatrix}\delta n\\ \delta m\end{bmatrix}}^{x}dt+\overbrace{\begin{bmatrix}\sqrt{\lambda} & -\sqrt{\alpha\lambda} & -\sqrt{(1-\alpha)\lambda} & 0\\ 0 & 0 & \sqrt{(1-\alpha)\lambda} & -\sqrt{(1-\alpha)\lambda}\end{bmatrix}}^{B_1}\overbrace{\begin{bmatrix}dW_1\\ dW_2\\ dW_3\\ dW_4\end{bmatrix}}^{dW}+\overbrace{\begin{bmatrix}-\frac{n^*}{\tau}+\frac{n^*}{L}\\ -\frac{n^*}{L}\end{bmatrix}}^{B_2}\delta u$$

$$\delta p=\overbrace{\begin{bmatrix}p_0 u^* & p_0\end{bmatrix}}^{C}\begin{bmatrix}\delta n\\ \delta m\end{bmatrix}+\overbrace{p_0 n^*}^{D}\,\delta u$$

$\mathcal{H}_2-$optimal control of $u(t)$ so that power tracks a regulation signal:



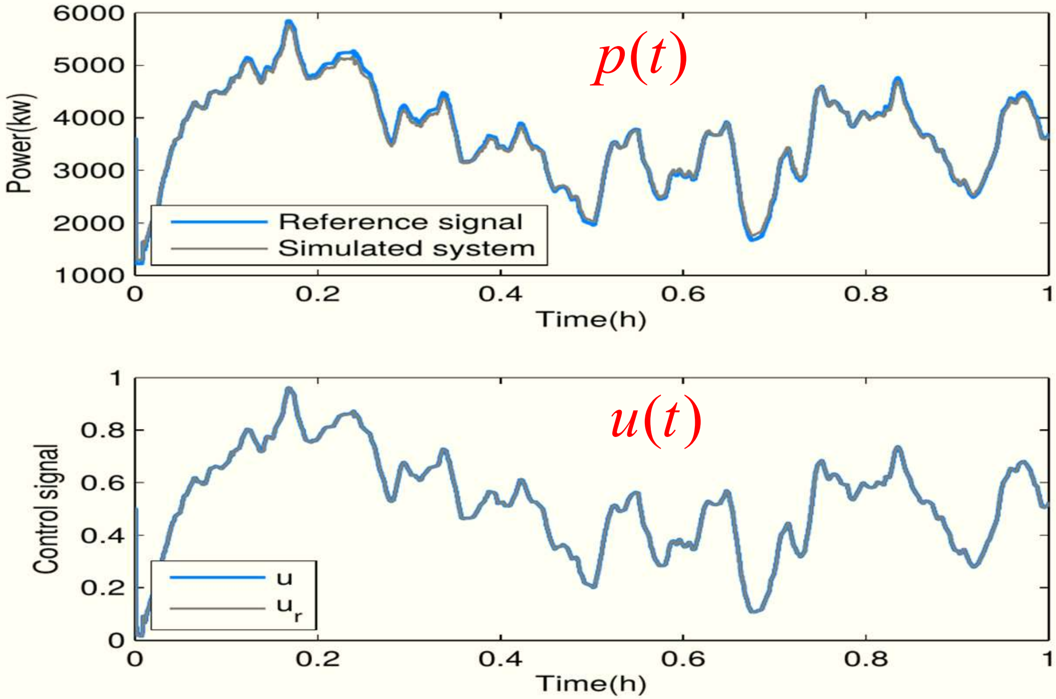
Distributed Implementation

Aggregator entity tracks the states n, m: loads must notify when they arrive, run out of laxity or leave the system.

Aggregator receives $r(t)$ from SO. Computes and broadcasts $u(t)$:

- Loads able to modulate their power (e.g., EVs) apply load $u\ p_0$.
- $ON-OFF$ loads turn on with probability u.

Discrete simulation



Summary of the approach

- Deferrable loads can play a role in frequency regulation.
- Aggregator entity manages the total consumption for a large number of loads.
- Macroscopic fluid/diffusion model from queueing theory, used for H_2 - optimal control design.
- Distributed implementation.
- Other uses of the queuieng model for deferrable loads:
  - Minimal variance load scheduling [Nakahira-Ferragut-Wierman, Performance Evaluation Review, 2018]
  - Proportional fairness for EV charging in overload [Zeballos-Ferragut-Paganini, *IEEE Transactions on Smart Grid* 2019]

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Conclusions

- The power grid has always relied on feedback control to achieve instantaneous power balance.
- The integration of renewable sources poses new challenges: lighter systems, faster control.
- Also, new opportunities: controlling inverters, or using Smart Grids for load-side regulation.
- Mathematical modeling remains essential. Many open research questions to address!

¡Gracias!