

TEORÍA DEL DAÑO CONCENTRADO: APLICACIÓN A ESTRUCTURAS DE CONCRETO ARMADO

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Profesor Sergio P.B. Proença, Universidad de São Paulo

- Conceptos fundamentales de Mecánica de la Fractura
- Conceptos fundamentales de la Teoría del Daño Continuo
- TDC: Modelado de elementos esbeltos de concreto armado sometidos a solicitaciones monotónicas
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- Aplicaciones prácticas
- Conclusiones

Engineering Structures

International Journal of Solids and Structures

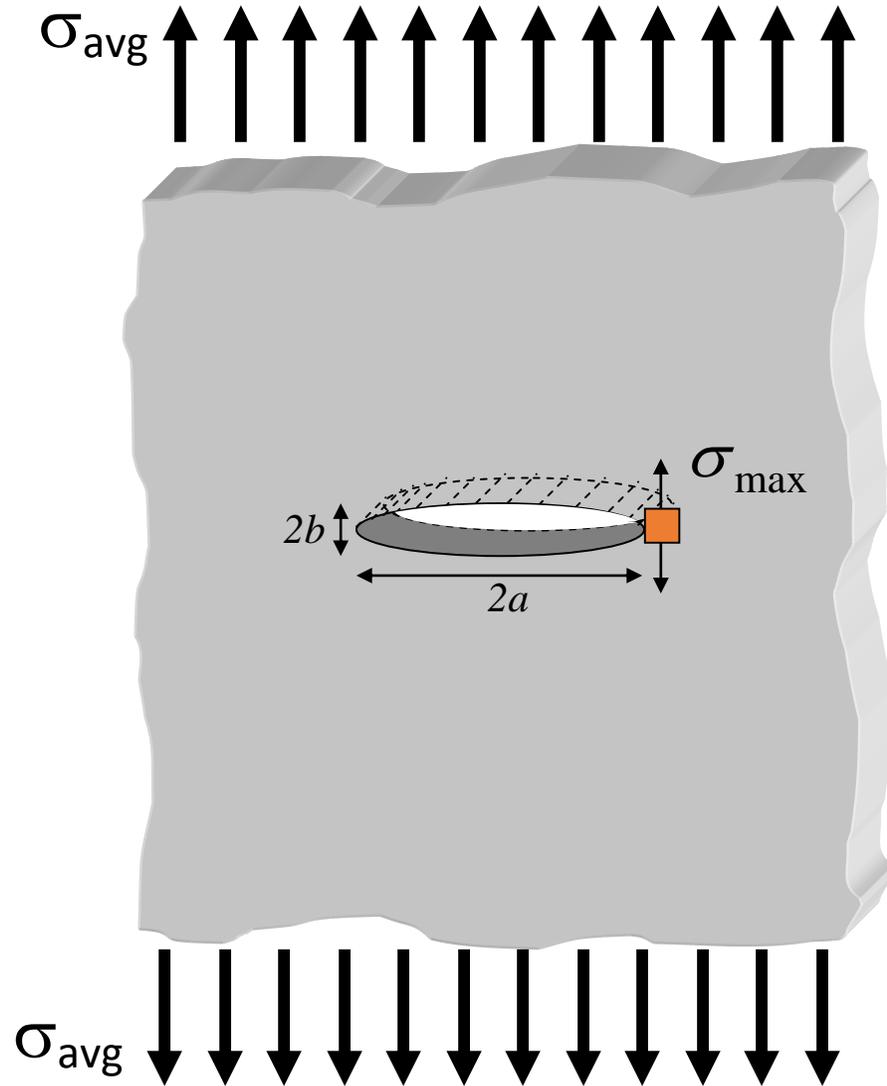
Computers & Structures

Journal of Structural Engineering ASCE

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Factor de concentración de esfuerzos
Agujero elíptico en una placa infinita

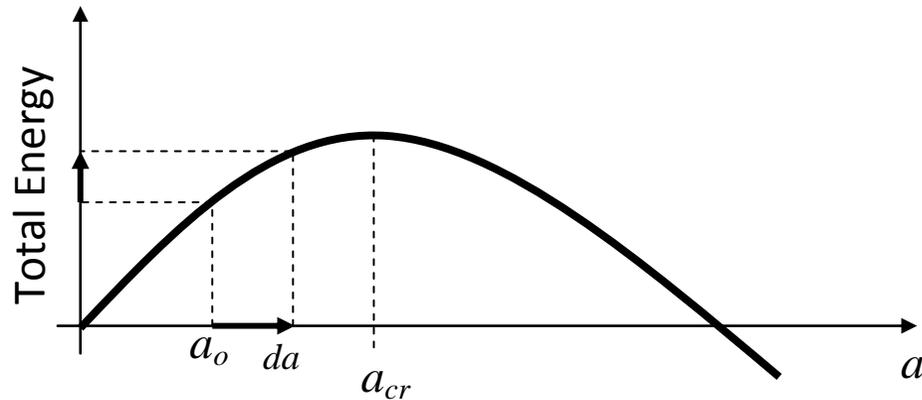


$$\sigma_{max} = SCF \sigma_{avg}$$

$$SCF = 1 + 2 \frac{a}{b}$$

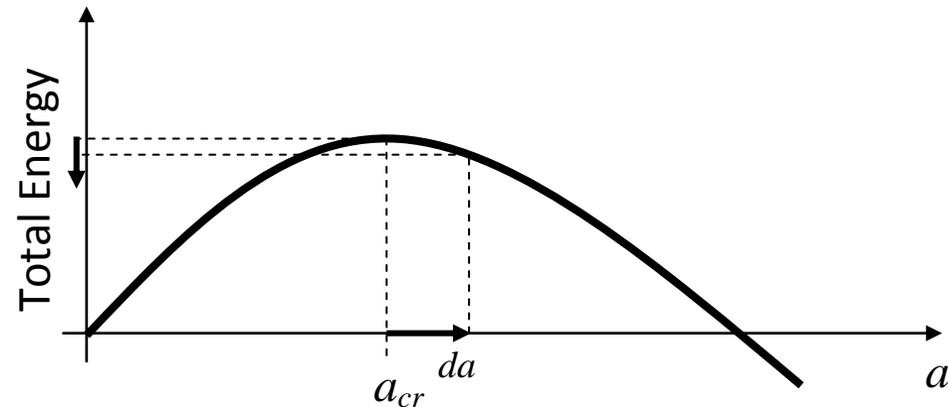
Balance de energía en una placa infinita con agujero elíptico degenerado

$$ET = (U - T_{ext}) + ES = \left(U_0 - \frac{\pi\sigma_{avg}^2}{E} a^2 \right) + 4\Gamma_{sup} a$$



Criterio de fractura:

$$\frac{dET}{da} = 0$$



Criterio de Griffith:

$$ET = (U - T_{ext}) + ES \quad \frac{dET}{da} = 0$$

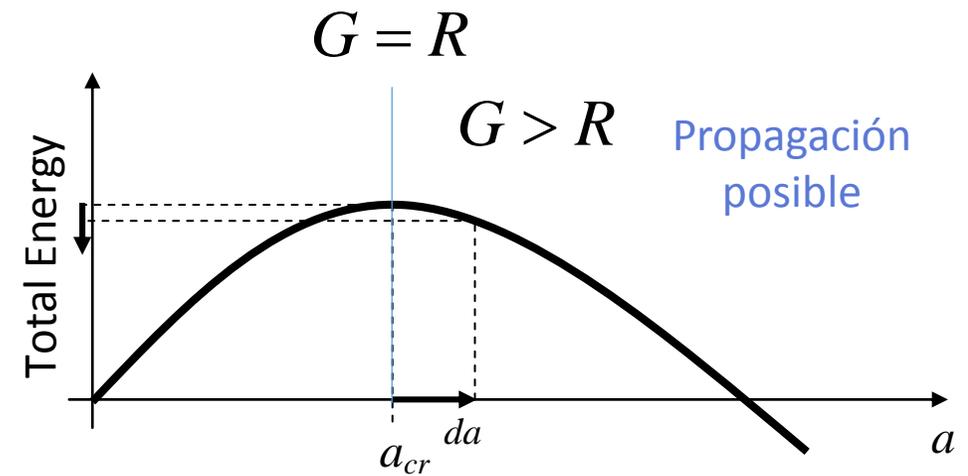
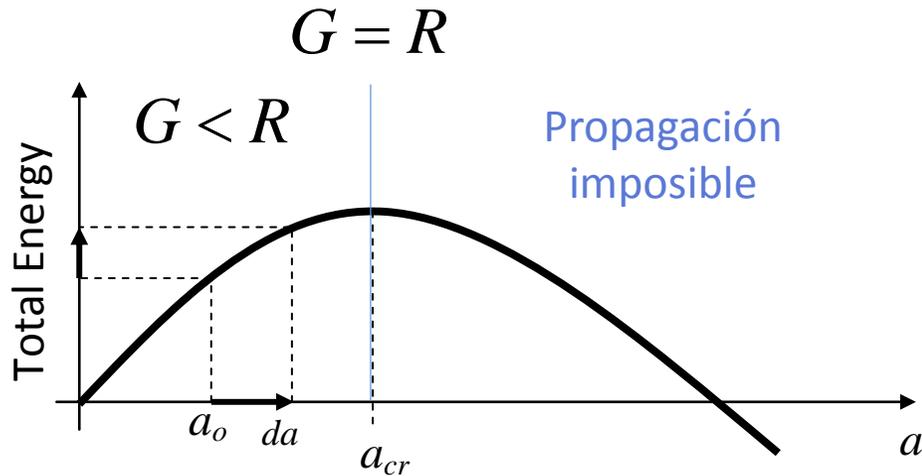
Resistencia al agrietamiento

$$G = R$$

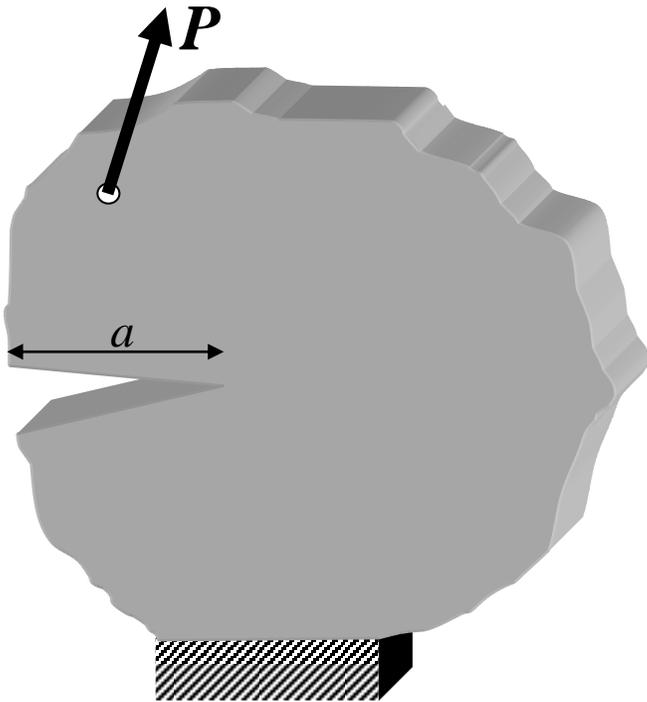
$$G = -\frac{d}{da}(U - T_{ext})$$

$$R = \frac{d}{da}(ES)$$

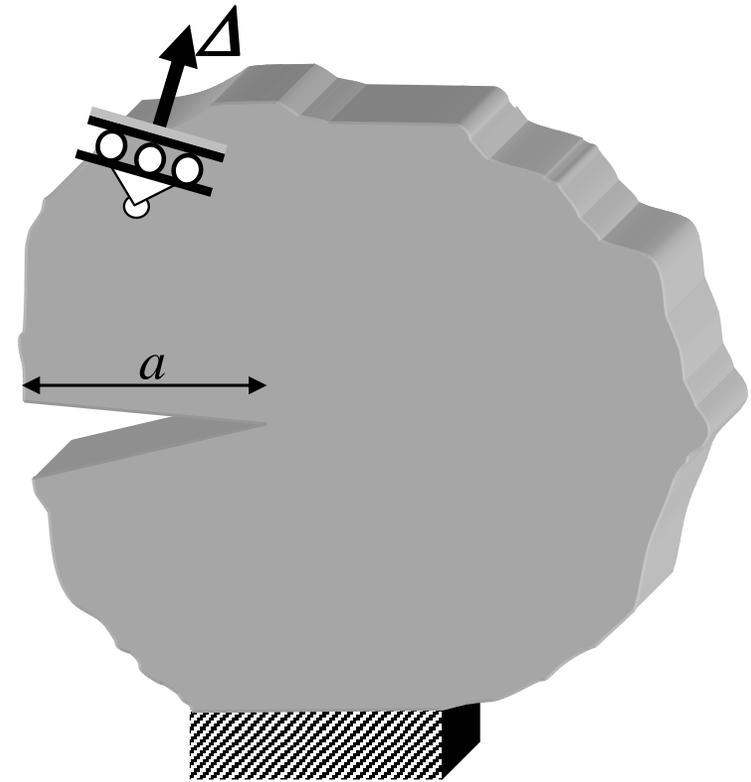
Tasa de restitución de energía o fuerza extensora de la fisura



Cálculo de G en función de la rigidez $K(a)$ o la flexibilidad $F(a)$:



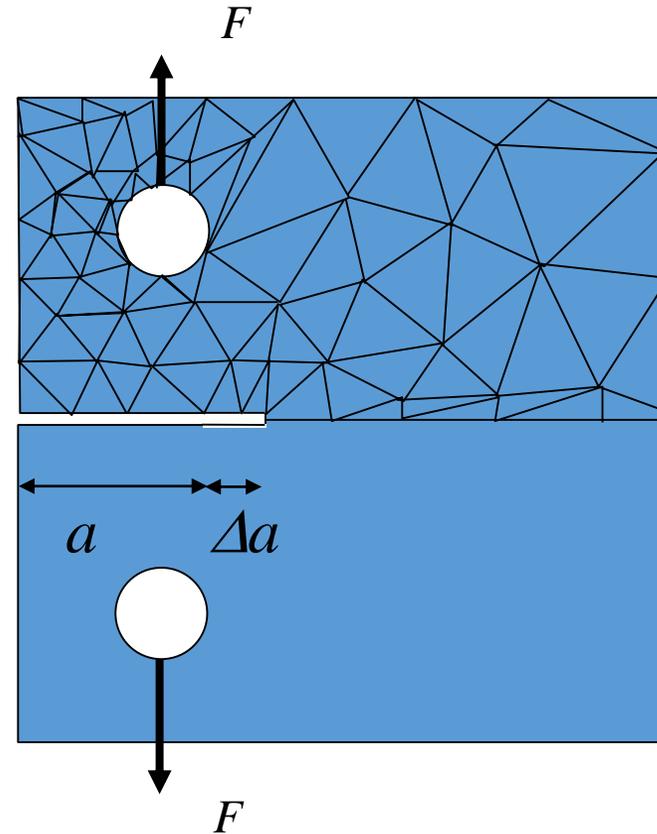
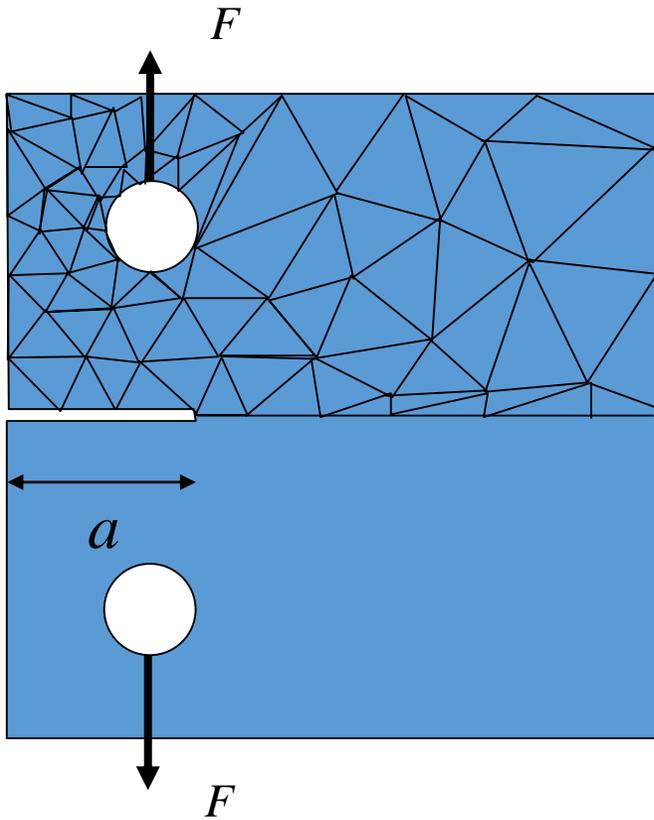
$$G = -\frac{d}{da}(U + PE) =$$
$$= \frac{1}{2} \frac{dF(a)}{da} P^2 = -\frac{1}{2} \frac{dK(a)}{da} \Delta^2$$



Cálculo de G por elementos finitos:

$$\Pi = \frac{1}{2}[\mathbf{u}]^T [\mathbf{K}][\mathbf{u}] - [\mathbf{u}]^T [\mathbf{F}]$$

$$\mathcal{G} = -\left(\frac{\partial \Pi}{\partial a}\right)_{load} = -\frac{1}{2}[\mathbf{u}]^T \frac{\partial [\mathbf{K}]}{\partial a} [\mathbf{u}]$$



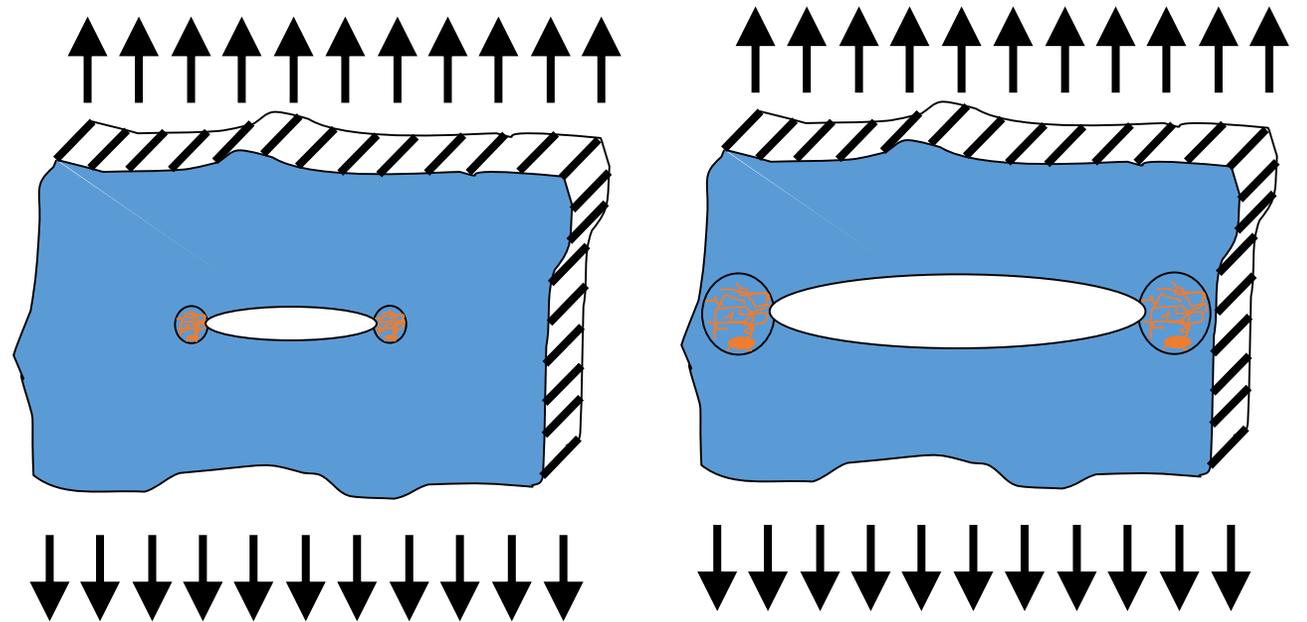
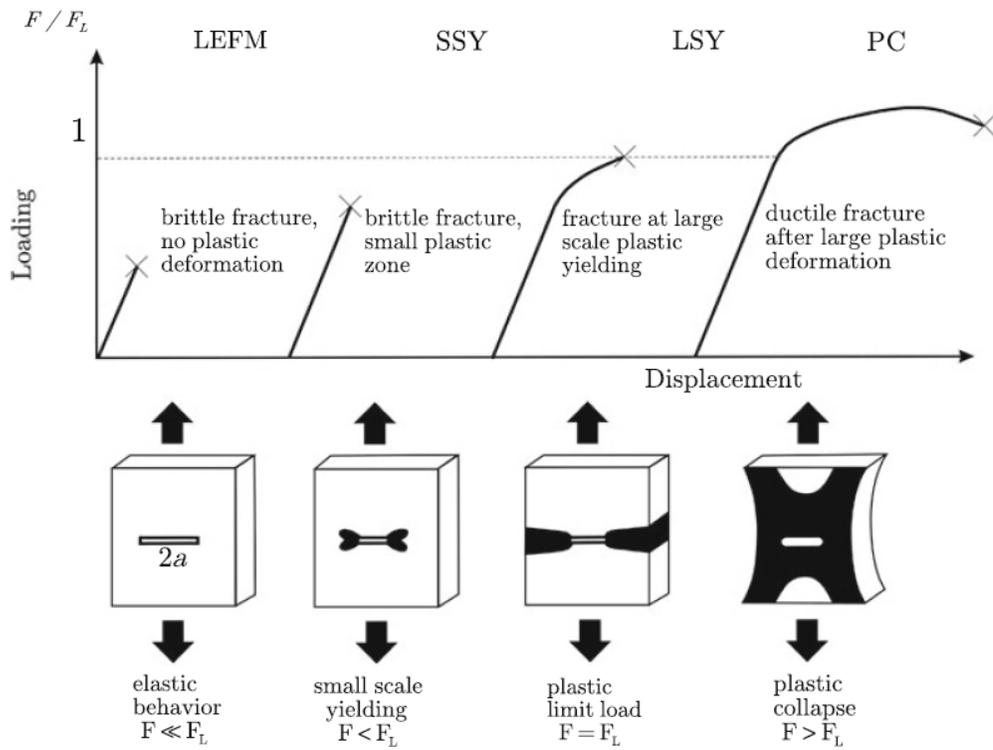
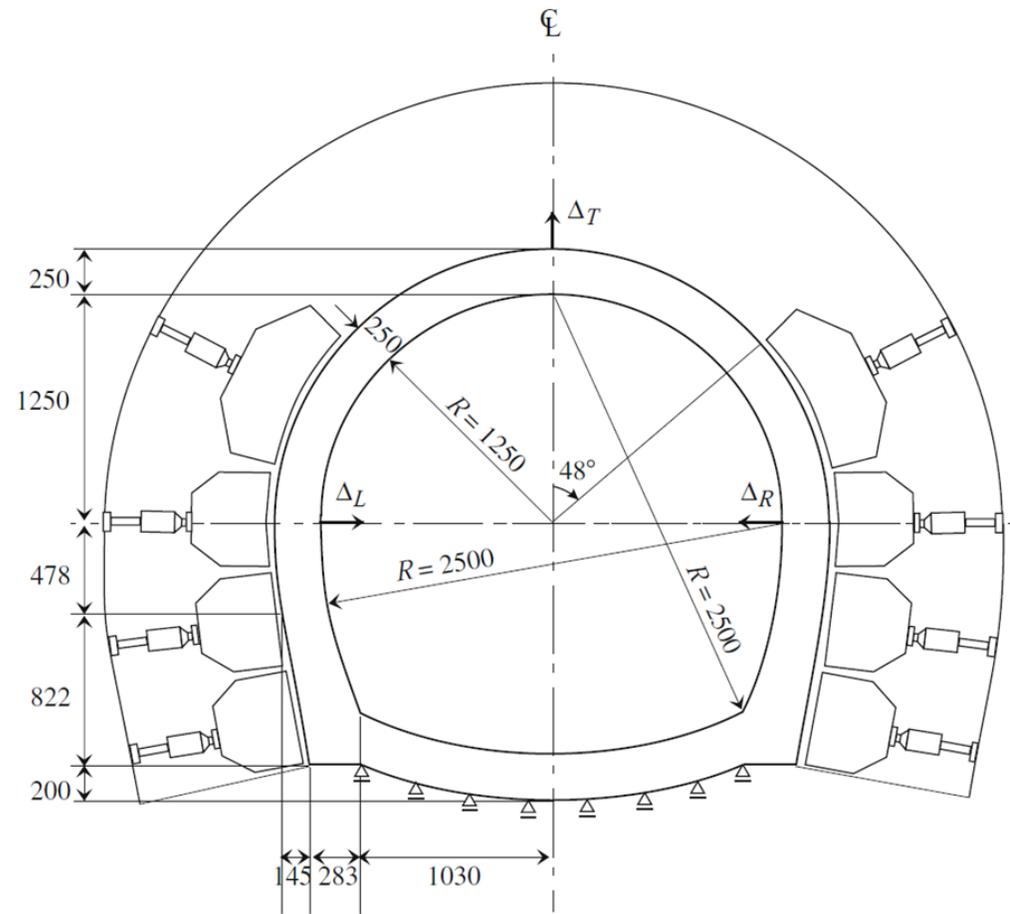


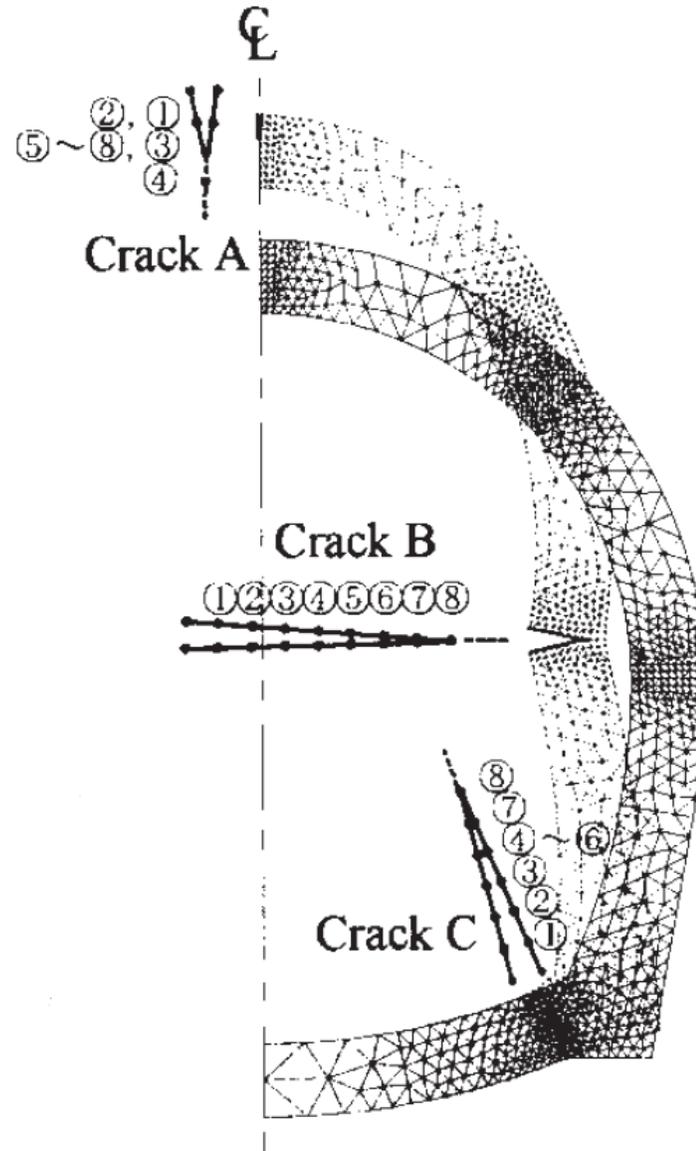
Fig. 3.29 Stages of plastic deformation in a body with crack

Función de resistencia al agrietamiento: $R = R(\Delta a)$

$$G = R(\Delta a)$$



Shi et al. 2001





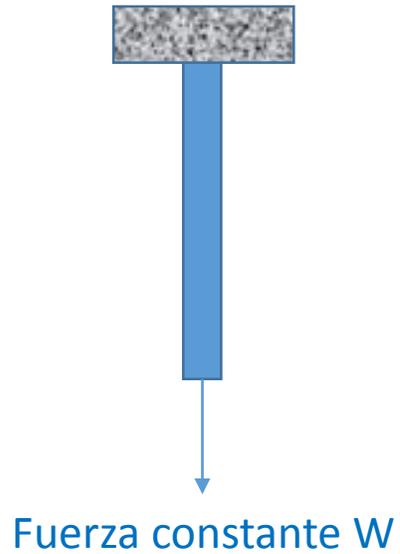
Venezuela 1997



Venezuela 1997

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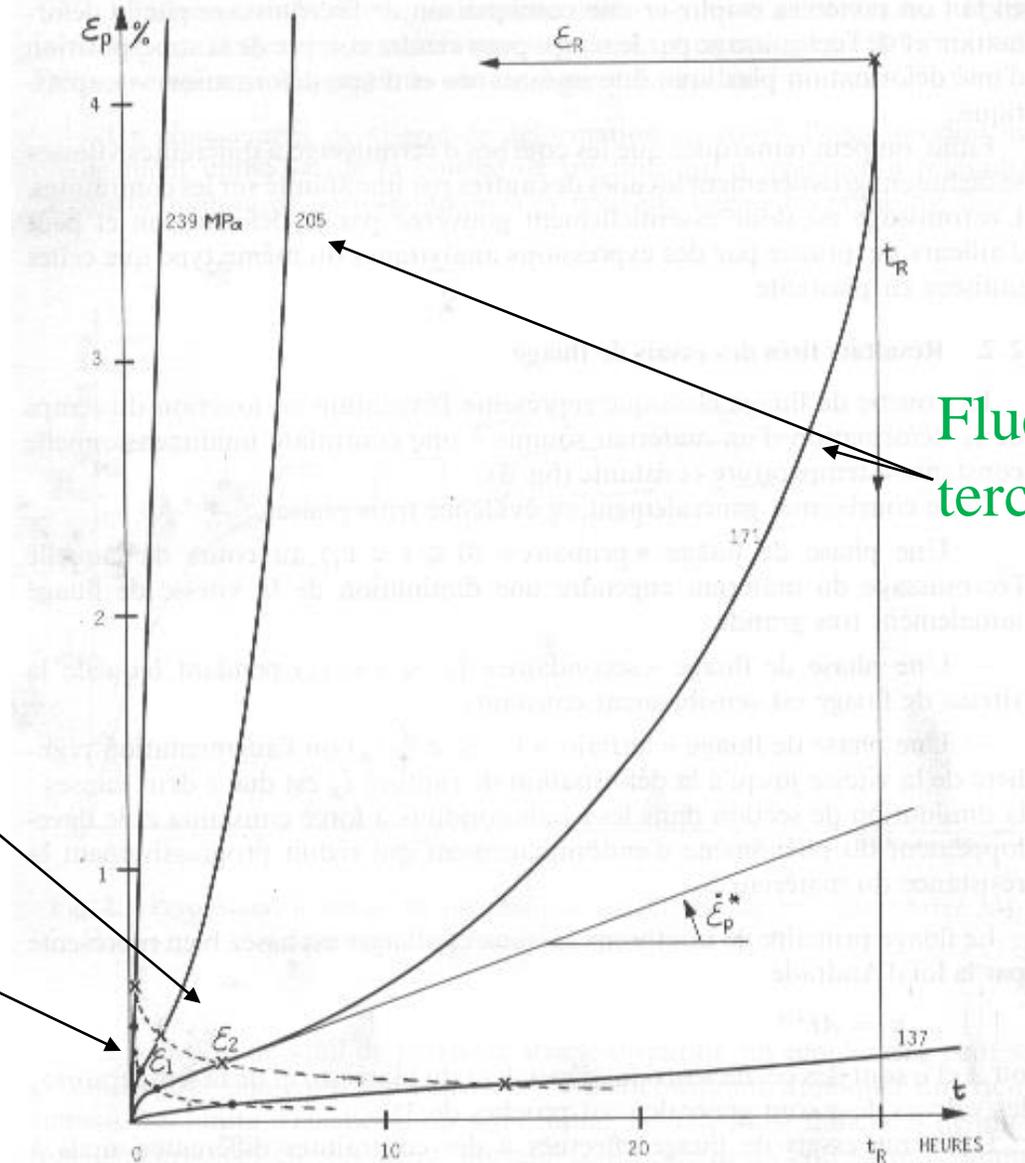
Teoría clásica del daño continuo: modelado de la fluencia terciaria (Kachanov 1958, Rabotnov 1969)



Ensayo de fluencia

Fluencia secundaria

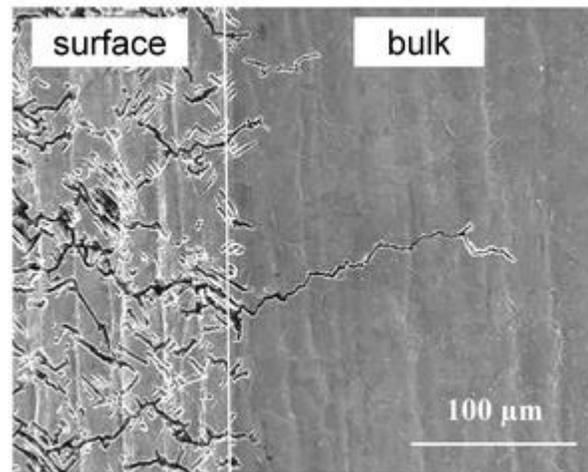
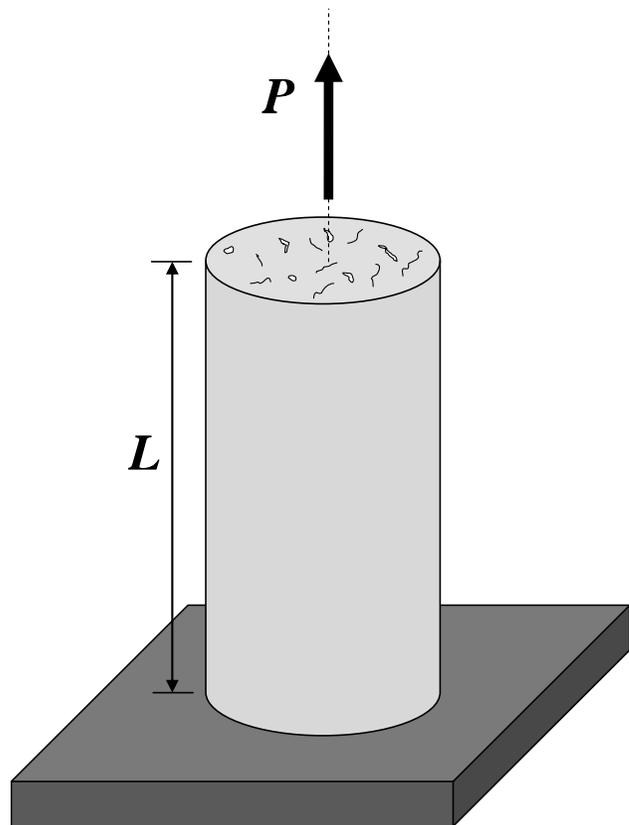
Fluencia primaria



Fluencia terciaria

Fig. 3. Essais de fluage à différentes contraintes. Superalliage IN 100 à 1 000 °C.

Variable de daño:



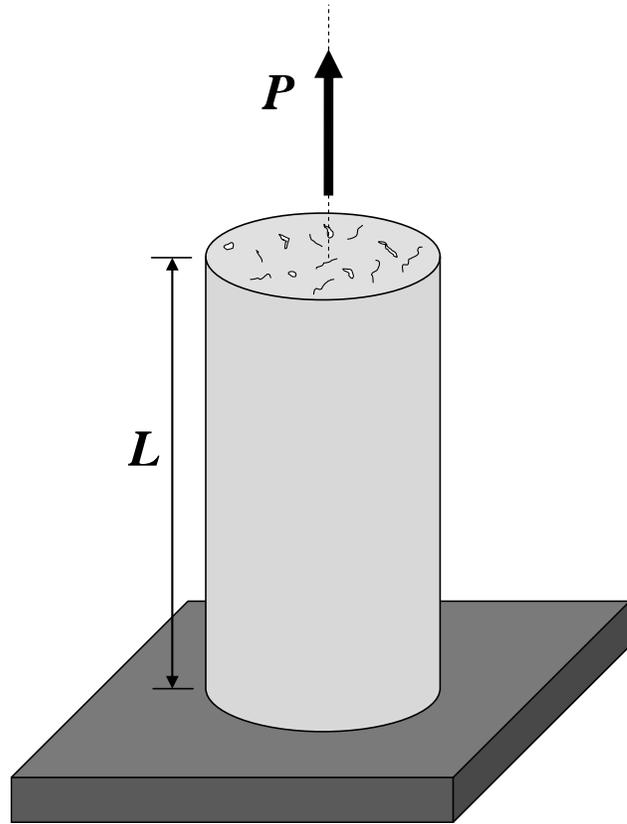
Área de micro defectos

$$\omega = \frac{A_d}{A}$$

$$0 \leq \omega \leq 1$$

Área total

Esfuerzo efectivo de Rabortnov:



$$\bar{\sigma} = \frac{P}{A - A_d}$$

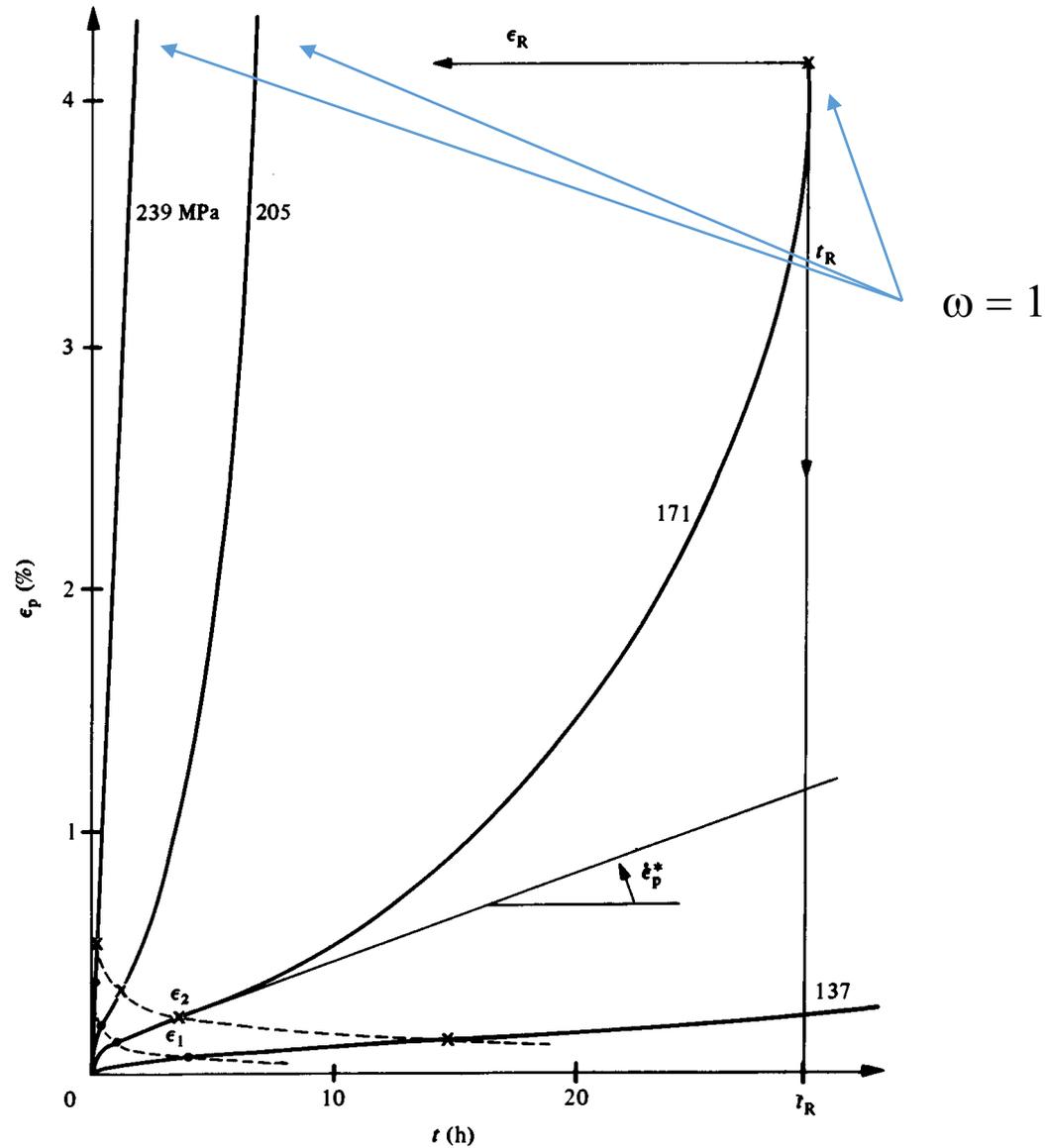
$$\omega = \frac{A_d}{A}$$

$$\bar{\sigma} = \frac{P}{A - A_d} = \frac{P}{A - \omega A}$$

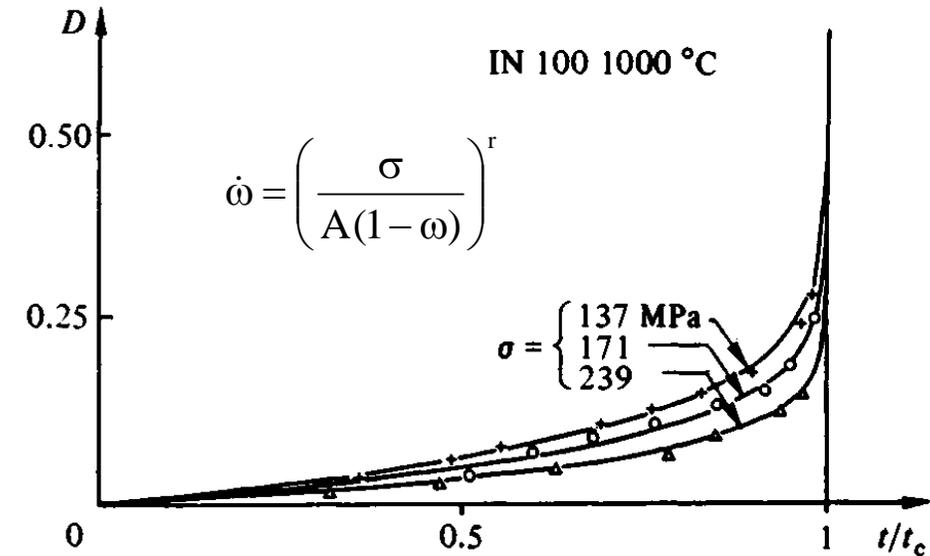
$$\bar{\sigma} = \frac{\sigma}{1 - \omega}$$

Teoría clásica del daño continuo: modelado de la fluencia terciaria (Kachanov 1958, Rabotnov 1969)

Creep tests at different stress levels: IN 100 superalloy at 1000 °C.



Creep damage evolution for the alloys IN 100 and AU2GN.



Modelado del ablandamiento por deformación (Lemaitre 1970)

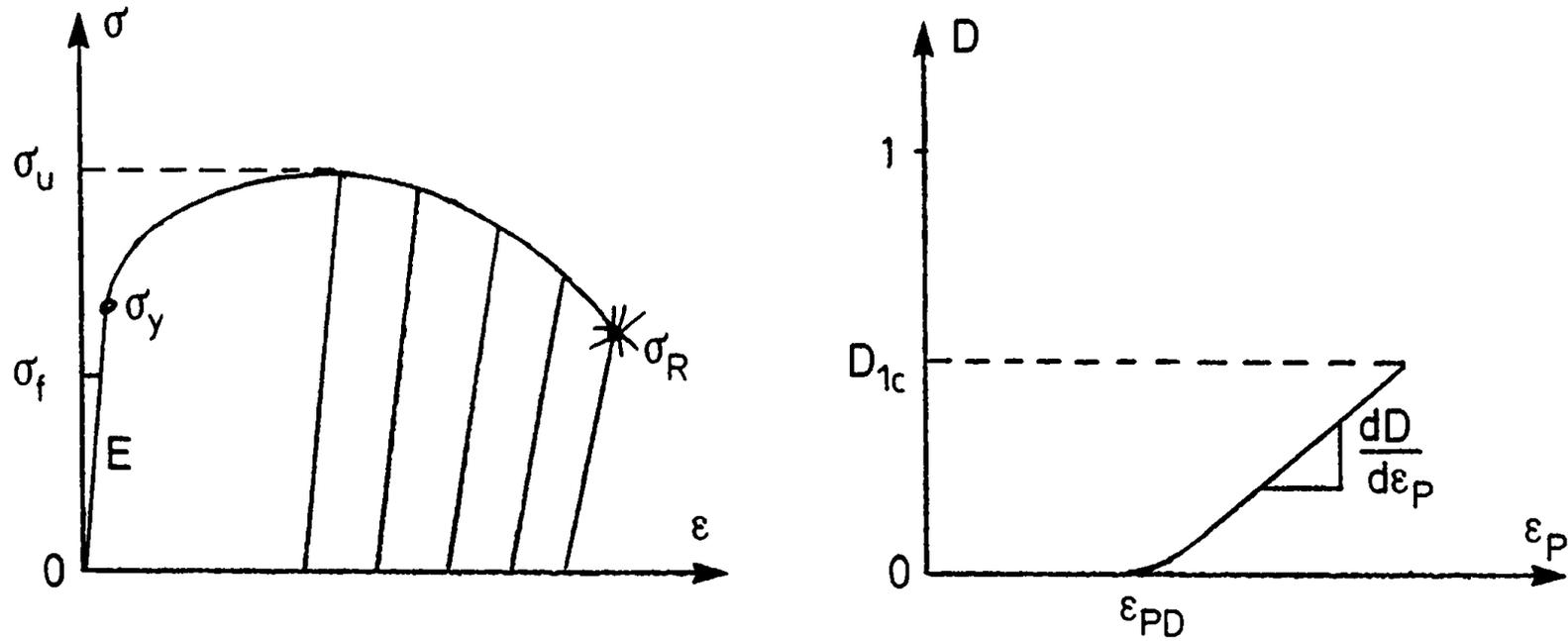


Fig. 3.4. Identification of damage coefficients

Hipótesis de equivalencia en deformación

	Material intacto	Material dañado	
Ley de elasticidad	$\sigma = E\varepsilon$	$\bar{\sigma} = E\varepsilon$	$\sigma = (1 - \omega)E\varepsilon$
Ley de elasticidad material dúctil	$\sigma = E(\varepsilon - \varepsilon^p)$	$\bar{\sigma} = E(\varepsilon - \varepsilon^p)$	$\sigma = (1 - \omega)E(\varepsilon - \varepsilon^p)$
Función de fluencia	$f = \sigma - \sigma_y$	$f = \bar{\sigma} - \sigma_y$	$f = \left \frac{\sigma}{1 - \omega} \right - \sigma_y$

Otra manera de ver la hipótesis de equivalencia en deformación

$$\sigma = (1 - \omega)E(\varepsilon - \varepsilon^p)$$

$$\varepsilon = \frac{\sigma}{(1 - \omega)E} + \varepsilon^p$$

$$\varepsilon = \frac{1}{E}\sigma + \frac{\omega}{(1 - \omega)E}\sigma + \varepsilon^p$$

$$\varepsilon = \varepsilon^e + \varepsilon^d + \varepsilon^p$$

$$\varepsilon^e = \frac{1}{E}\sigma$$

$$\varepsilon^d = \frac{\omega}{E(1 - \omega)}\sigma$$

Modelo de daño frágil

Ley de elasticidad:

Parte negativa da deformación

$$\sigma = (1 - \omega) E \langle \varepsilon \rangle_-$$

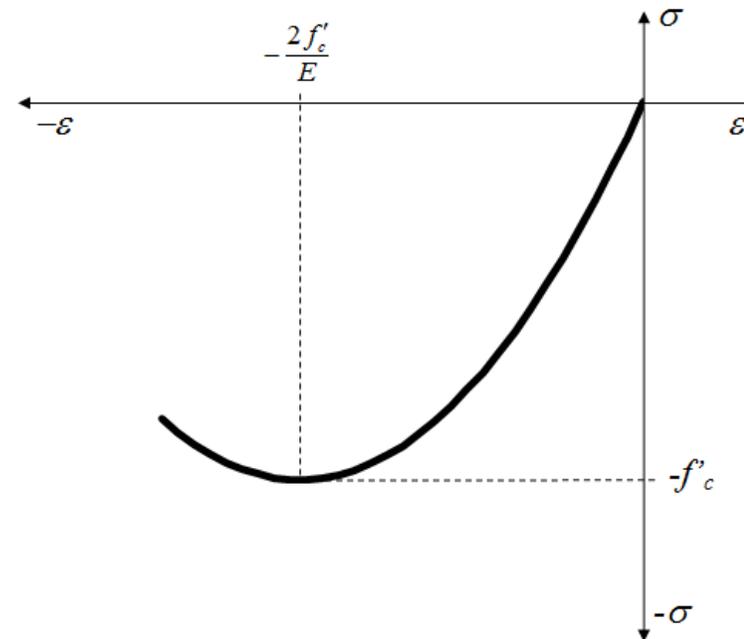
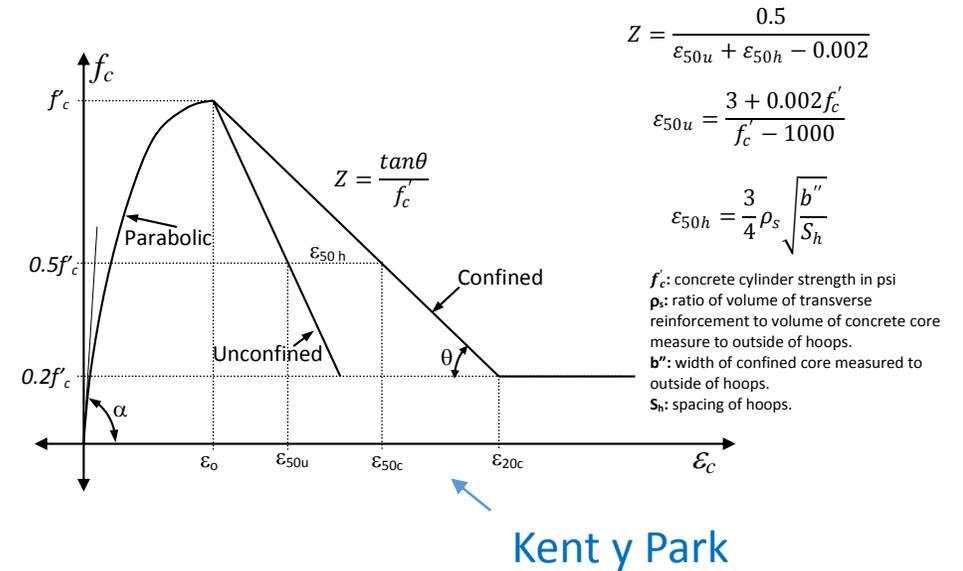
$$\langle \varepsilon \rangle_- = \begin{cases} \varepsilon & \text{if } \varepsilon \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

Ley de evolución del daño:

$$\begin{cases} d\omega = 0 & \text{if } G_m < R_m \\ G_m = R_m & \text{if } d\omega > 0 \end{cases}$$

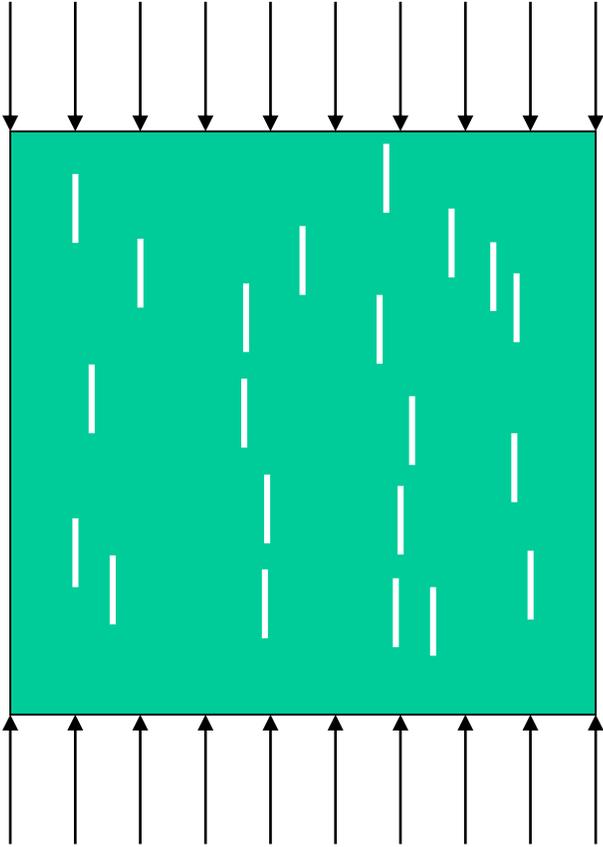
$$G_m = \frac{\partial W}{\partial \omega} = -\frac{\partial U}{\partial \omega} = \frac{1}{2} \frac{\sigma^2}{E(1-\omega)^2} = \frac{1}{2} E \varepsilon^2$$

$$R_m = \frac{8(f'_c)^2}{E} \omega^2$$

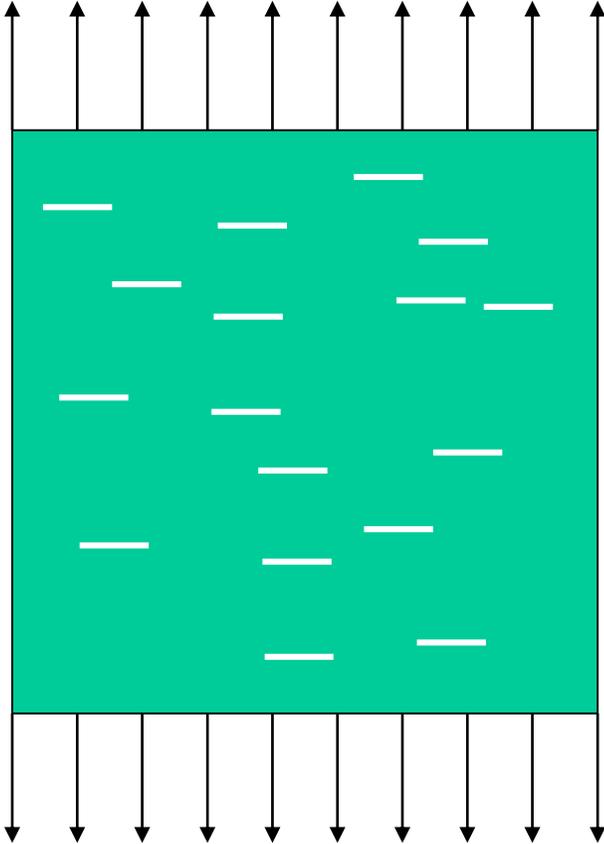


Modelo de daño con envolvente parabólica

Mecánica del daño continuo: daño unilateral (Ladeveze, Mazars 1986)



Daño en compresión: ω^-



Daño en tracción: ω^+

Modelo unilateral de daño frágil

$$\bar{\sigma} = \begin{cases} \frac{\sigma}{1-\omega^+} & \text{si } \sigma \geq 0 \\ \frac{\sigma}{1-\omega^-} & \text{si } \sigma < 0 \end{cases} = \frac{\langle \sigma \rangle_+}{1-\omega^+} + \frac{\langle \sigma \rangle_-}{1-\omega^-}$$

Esfuerzo efectivo unilateral

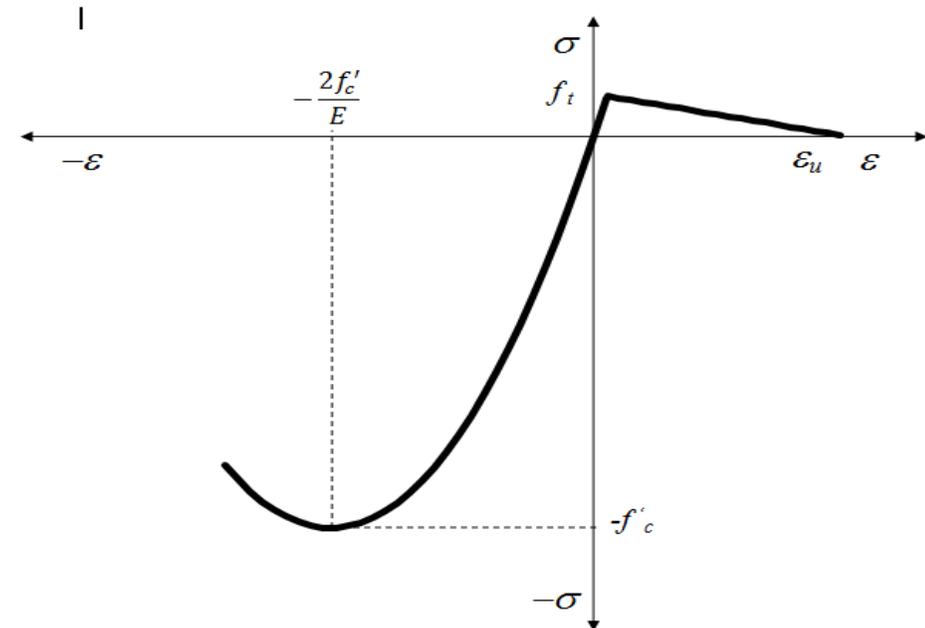
$$\varepsilon = \frac{\langle \sigma \rangle_+}{(1-\omega^+)E} + \frac{\langle \sigma \rangle_-}{(1-\omega^-)E}$$

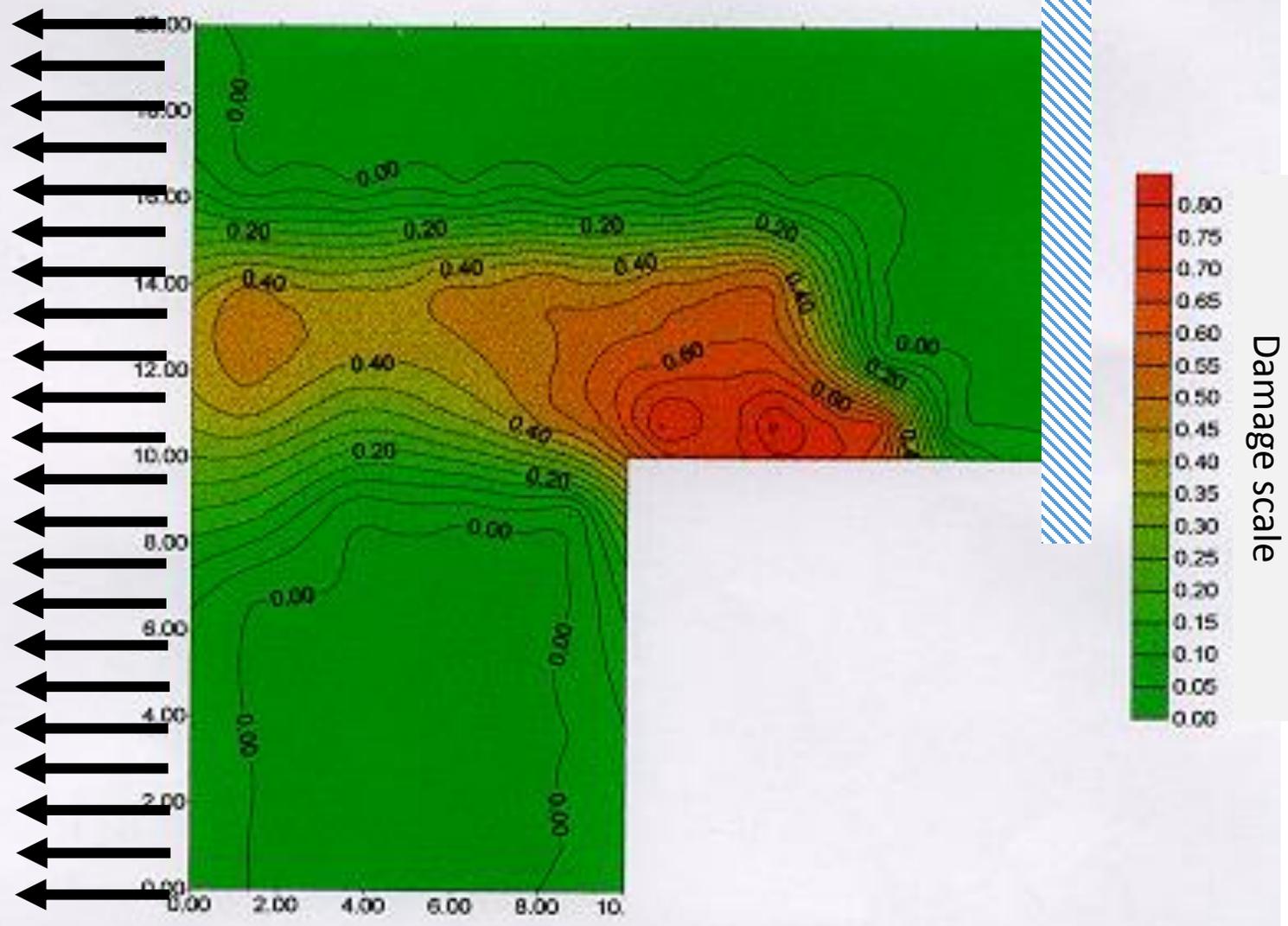
Ley de elasticidad

$$\begin{cases} d\omega^{+/-} = 0 & \text{if } G_m^{+/-} < R_m^{+/-} \\ G_m^{+/-} = R_m^{+/-} & \text{if } d\omega^{+/-} > 0 \end{cases}$$

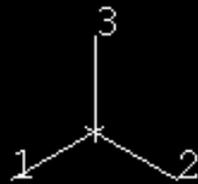
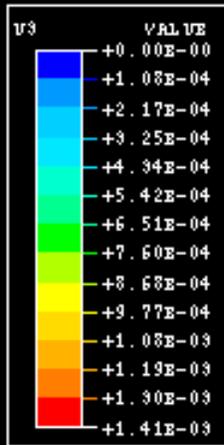
Leyes de evolución del daño:

$$G_m^{+/-} = \frac{1}{2} \frac{\langle \sigma \rangle_{+/-}^2}{(1-\omega^{+/-})^2 E}$$

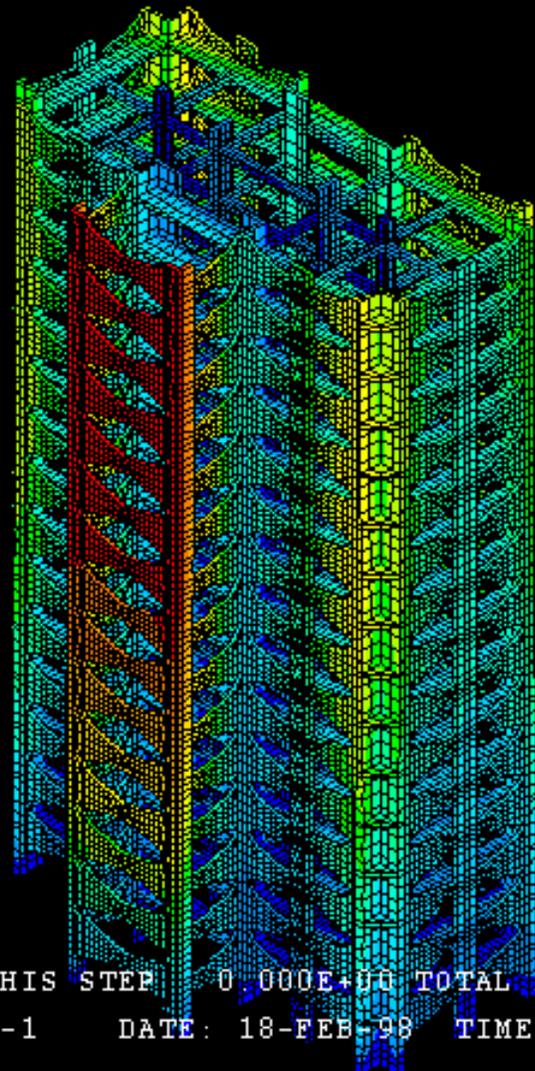


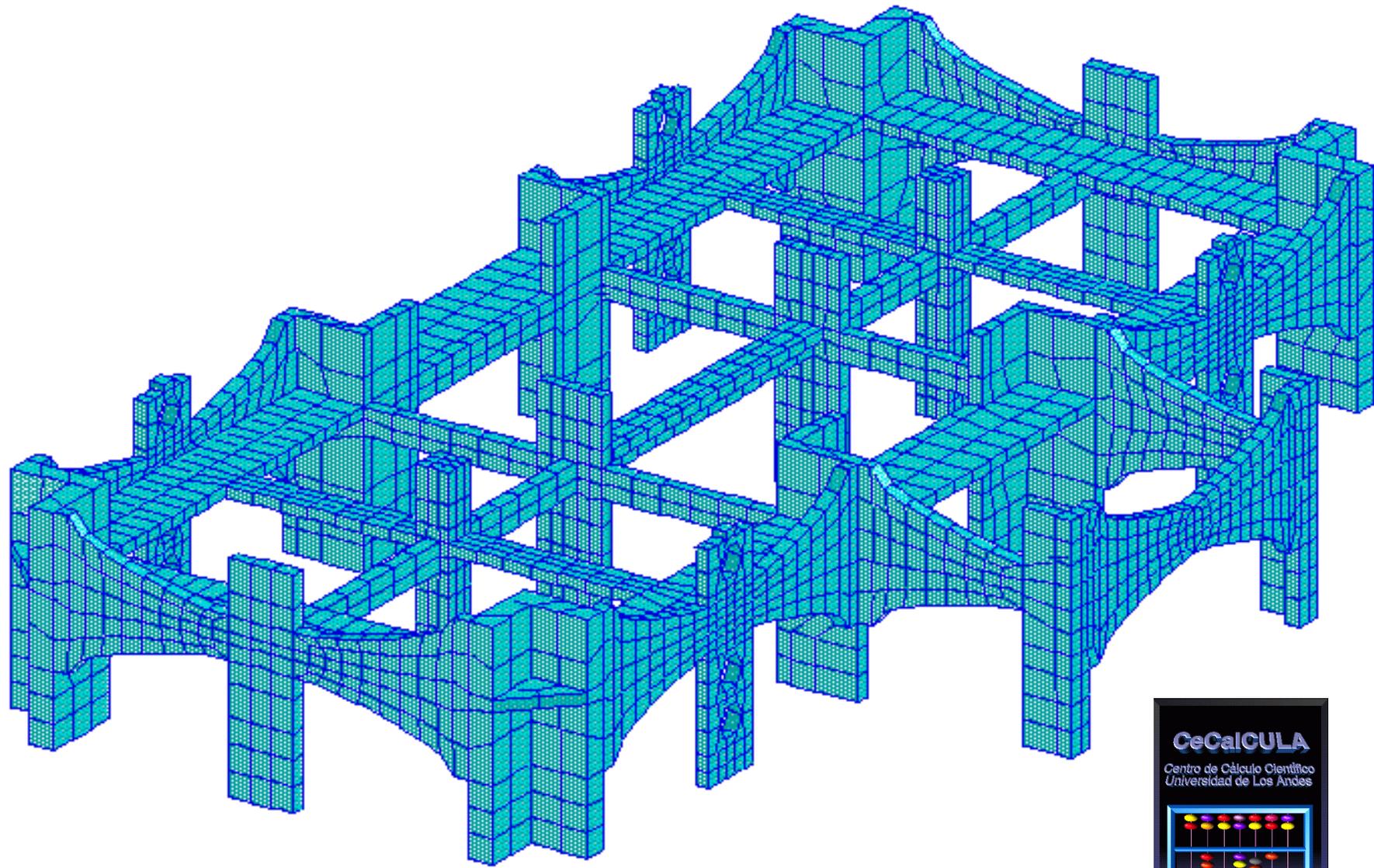


Mapa de daños



TIME COMPLETED IN THIS STEP: 0.000E+00 TOTAL ACCUMULATED TIME
ABAQUS VERSION: 5.5-1 DATE: 18-FEB-98 TIME: 17:38:36 1.00
STEP 3 INCREMENT 1

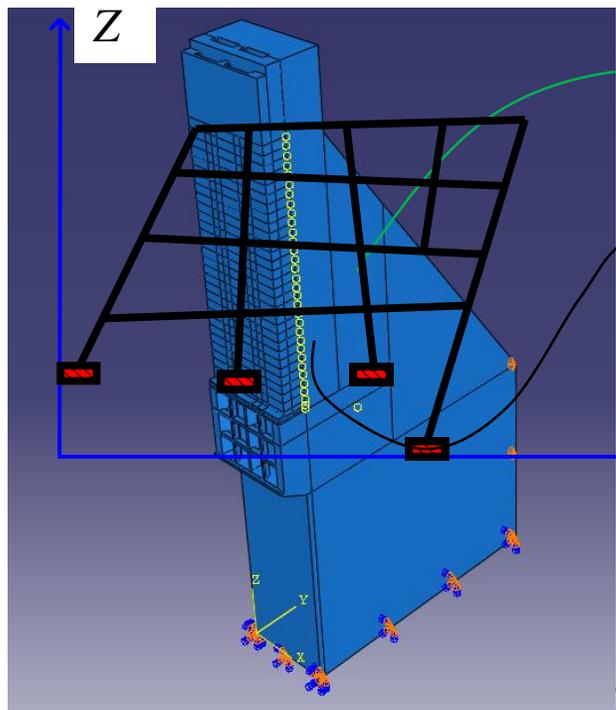




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Inconvenientes que hacen que la mecánica de la fractura y del daño continuo sean inadecuados para el análisis estructural de edificios

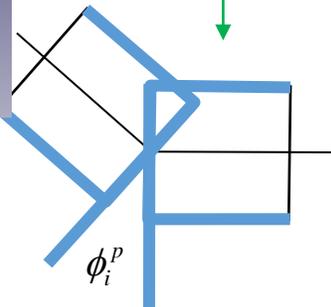
- El análisis de propagación fisura por fisura de la MF es inaplicable en el análisis de edificios
- El enfoque de la mecánica de los medios continuos es inadecuado para el análisis de edificios y otras estructuras de la ingeniería civil



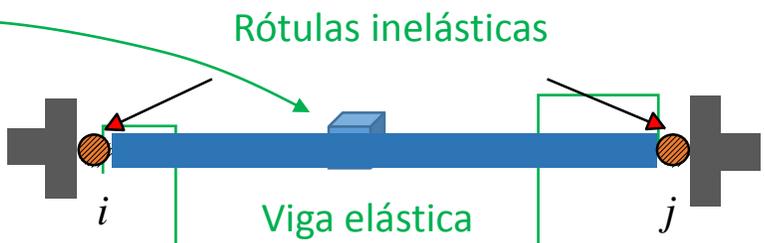
$$\{\boldsymbol{\varepsilon}^p\}_b = \begin{bmatrix} \phi_i^p \\ \phi_j^p \\ 0 \end{bmatrix}$$

$$(\mathbf{D})_b = (d_i, d_j)$$

Rotación plástica



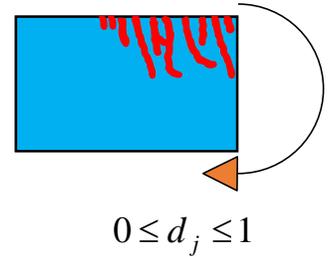
Mecánica del continuo



Rótulas inelásticas

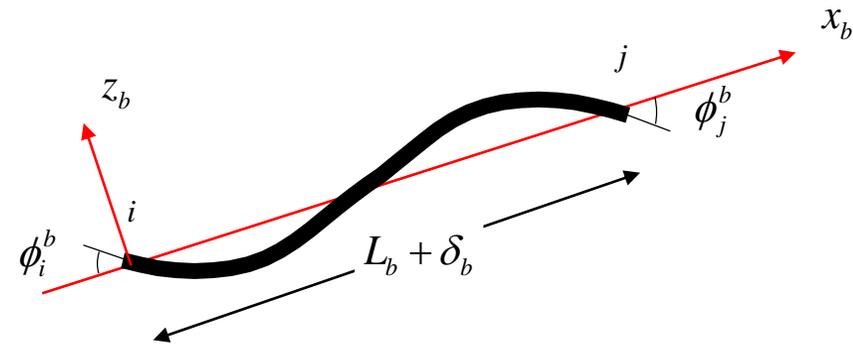
Viga elástica

agrietamiento, representado por una variable de daño

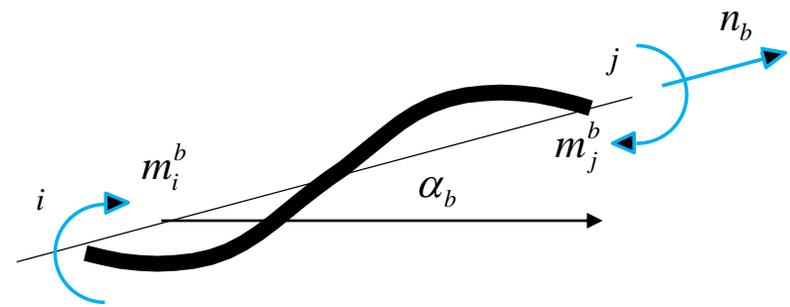


$$0 \leq d_j \leq 1$$

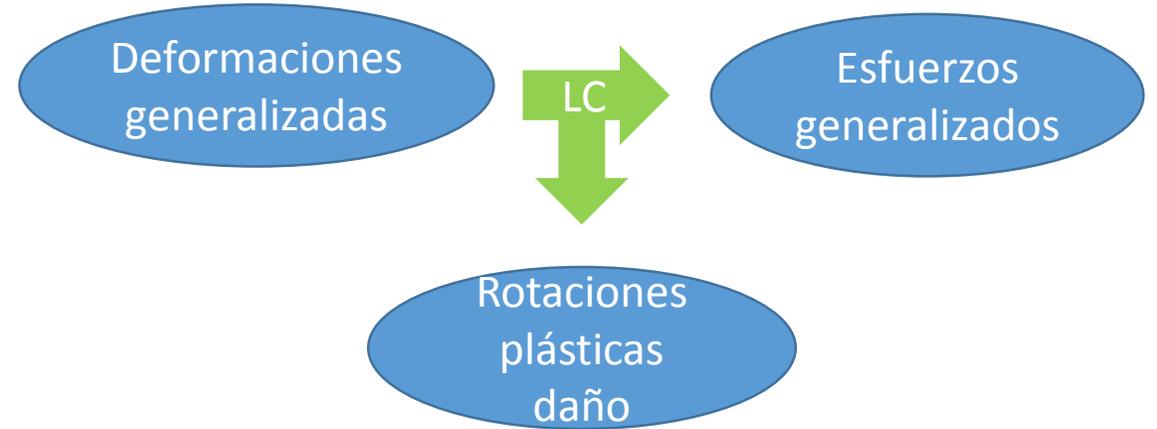
Mecánica estructural



$$\{\boldsymbol{\varepsilon}\}_b = \begin{bmatrix} \phi_i^b \\ \phi_j^b \\ \delta_b \end{bmatrix}$$

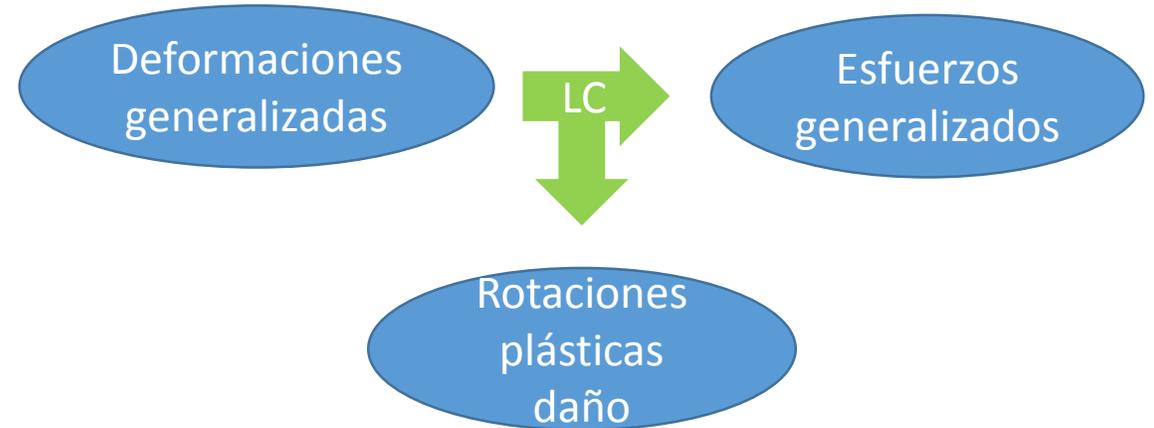


$$\{\boldsymbol{\sigma}\}_b = \begin{bmatrix} m_i^b \\ m_j^b \\ n_b \end{bmatrix}$$



Ley de comportamiento: {
Ley de elasticidad
Ley de daño
Ley de plasticidad

Rótula inelástica i



Ley de comportamiento: {
Ley de elasticidad
Ley de daño
Ley de plasticidad

Rótula inelástica i

Ley de elasticidad

Unidad estructural



Hipótesis de equivalencia

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^d + \boldsymbol{\varepsilon}^p$$

Deformación elástica

$$\boldsymbol{\varepsilon}^e = \frac{1}{E} \boldsymbol{\sigma}$$

Deformación de daño

$$\boldsymbol{\varepsilon}^d = \frac{\omega}{E(1-\omega)} \boldsymbol{\sigma}$$

Teoría del daño continuo:

Teoría del daño concentrado:



$$\{\boldsymbol{\varepsilon}\}_b = \{\boldsymbol{\varepsilon}^e\}_b + \{\boldsymbol{\varepsilon}^p\}_b + \{\boldsymbol{\varepsilon}^d\}_b \quad \{\boldsymbol{\varepsilon}^e\}_b = [\mathbf{F}_e]_b \{\boldsymbol{\sigma}\}_b$$

$$\phi_i^d = \frac{d_i F_{11}^0}{(1-d_i)} m_i$$

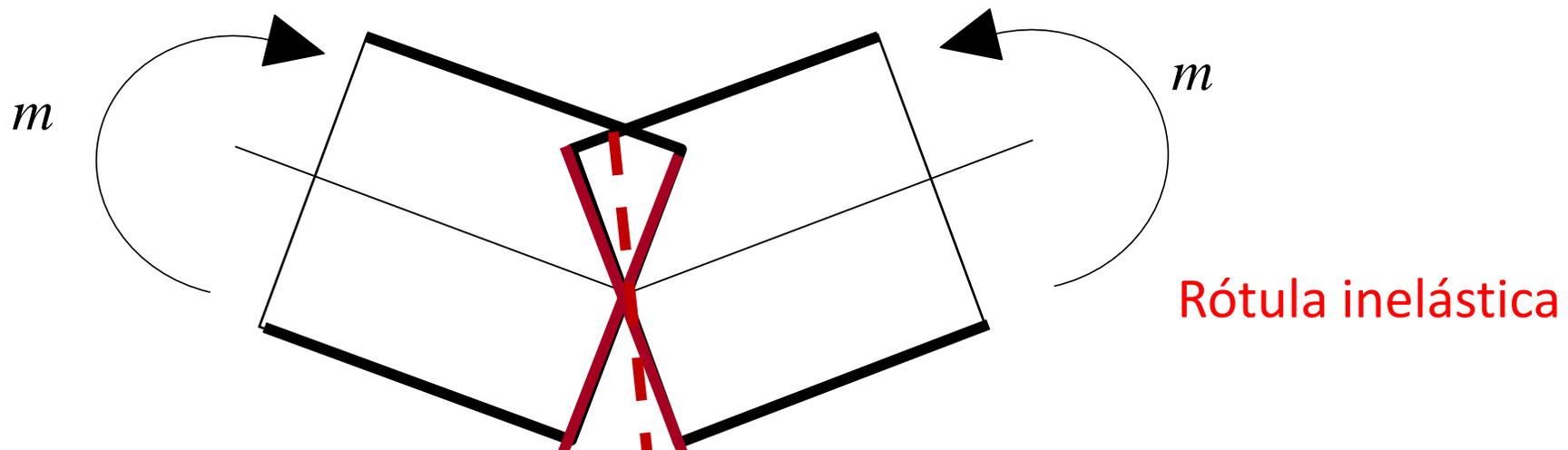
$$\phi_j^d = \frac{d_j F_{22}^0}{(1-d_j)} m_j$$

Teoría del
daño
continuo :

$$\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p = \frac{1}{E(1-\omega)} \boldsymbol{\sigma}$$

Teoría del
daño
concentrado :

$$\begin{bmatrix} \phi_i - \phi_i^p \\ \phi_j - \phi_j^p \\ \delta \end{bmatrix} = \begin{bmatrix} \frac{F_{11}^0}{(1-d_i)} & F_{12}^0 & F_{13}^0 \\ F_{21}^0 & \frac{F_{22}^0}{(1-d_j)} & F_{23}^0 \\ F_{31}^0 & F_{32}^0 & F_{33}^0 \end{bmatrix} \begin{bmatrix} m_i \\ m_j \\ n_i \end{bmatrix}$$



Rotación plástica ϕ_p Rotación por daño ϕ_d

$$\phi_d = \frac{d L}{3EI(1-d)} m$$

Ley de elasticidad para un elemento recto:

$$\{\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p\} = [\mathbf{F}(\mathbf{D})] \{\boldsymbol{\sigma}\}$$

$$[\mathbf{F}(\mathbf{D})] =$$

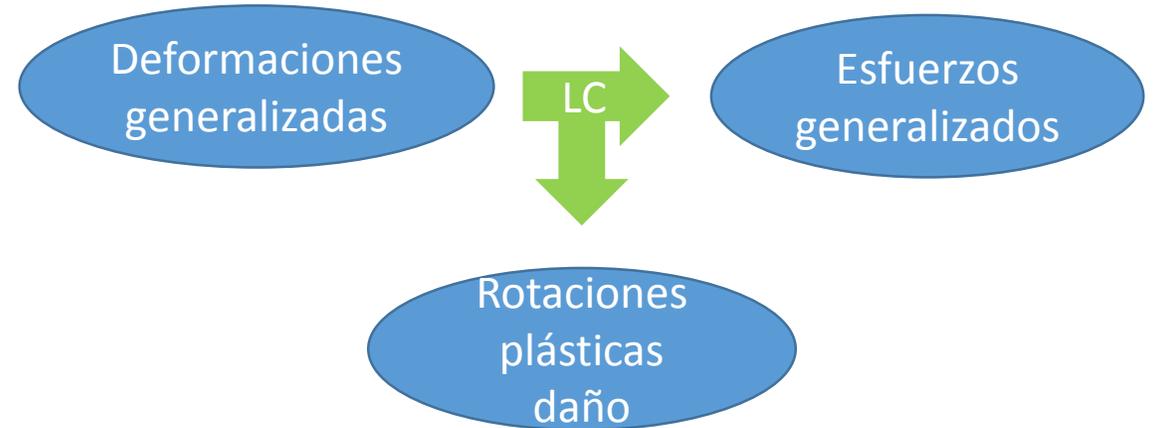
$$\begin{bmatrix} \frac{L}{3EI(1-d_i)} & -\frac{L}{6EI} & 0 \\ -\frac{L}{6EI} & \frac{L}{3EI(1-d_j)} & 0 \\ 0 & 0 & \frac{L}{AE} \end{bmatrix}$$

Matriz de flexibilidad
Matriz de rigidez

$$\{\boldsymbol{\sigma}\} = [\mathbf{E}(\mathbf{D})] \{\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p\}$$

$$[\mathbf{E}(\mathbf{D})] = k \begin{bmatrix} 12(1-d_i) & 6(1-d_i)(1-d_j) & 0 \\ & 12(1-d_j) & 0 \\ & & \frac{EA}{kL} \end{bmatrix}$$

$$k = \frac{1}{4 - (1-d_i)(1-d_j)} \frac{EI}{L}$$



Ley de comportamiento: {
Ley de elasticidad
Ley de daño
Ley de plasticidad

Rótula inelástica i

Ley de daño

$$W = \frac{1}{2} \{ \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p \}^t \{ \boldsymbol{\sigma} \} = \frac{1}{2} \{ \boldsymbol{\sigma} \}^t [\mathbf{F}(\mathbf{D})] \{ \boldsymbol{\sigma} \}$$

Energía de deformación
complementaria

$$G_i = \frac{\partial W}{\partial d_i} = \frac{Lm_i^2}{12EI(1-d_i)^2}$$

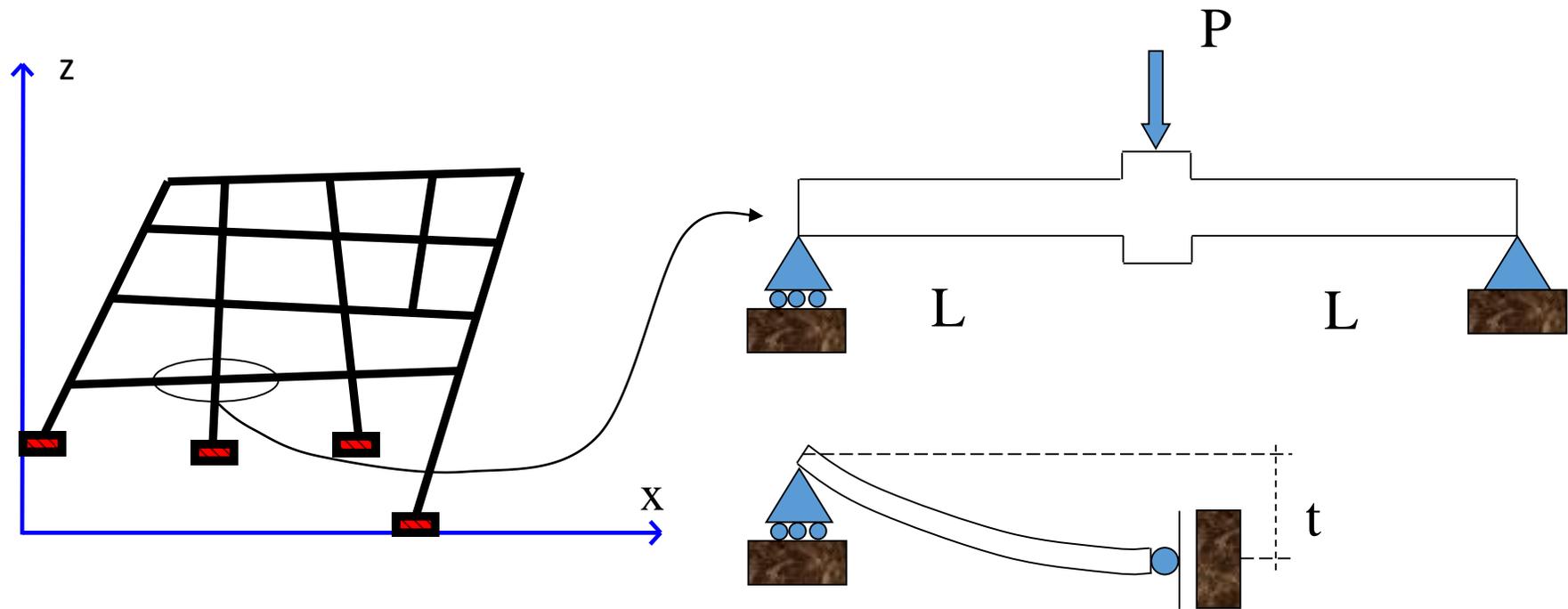
Tasa de restitución de energía de una
rótula inelástica o momento extensor
del agrietamiento

$$G_i \leq R(d_i)$$

Criterio de Griffith generalizado

 Función de resistencia al agrietamiento

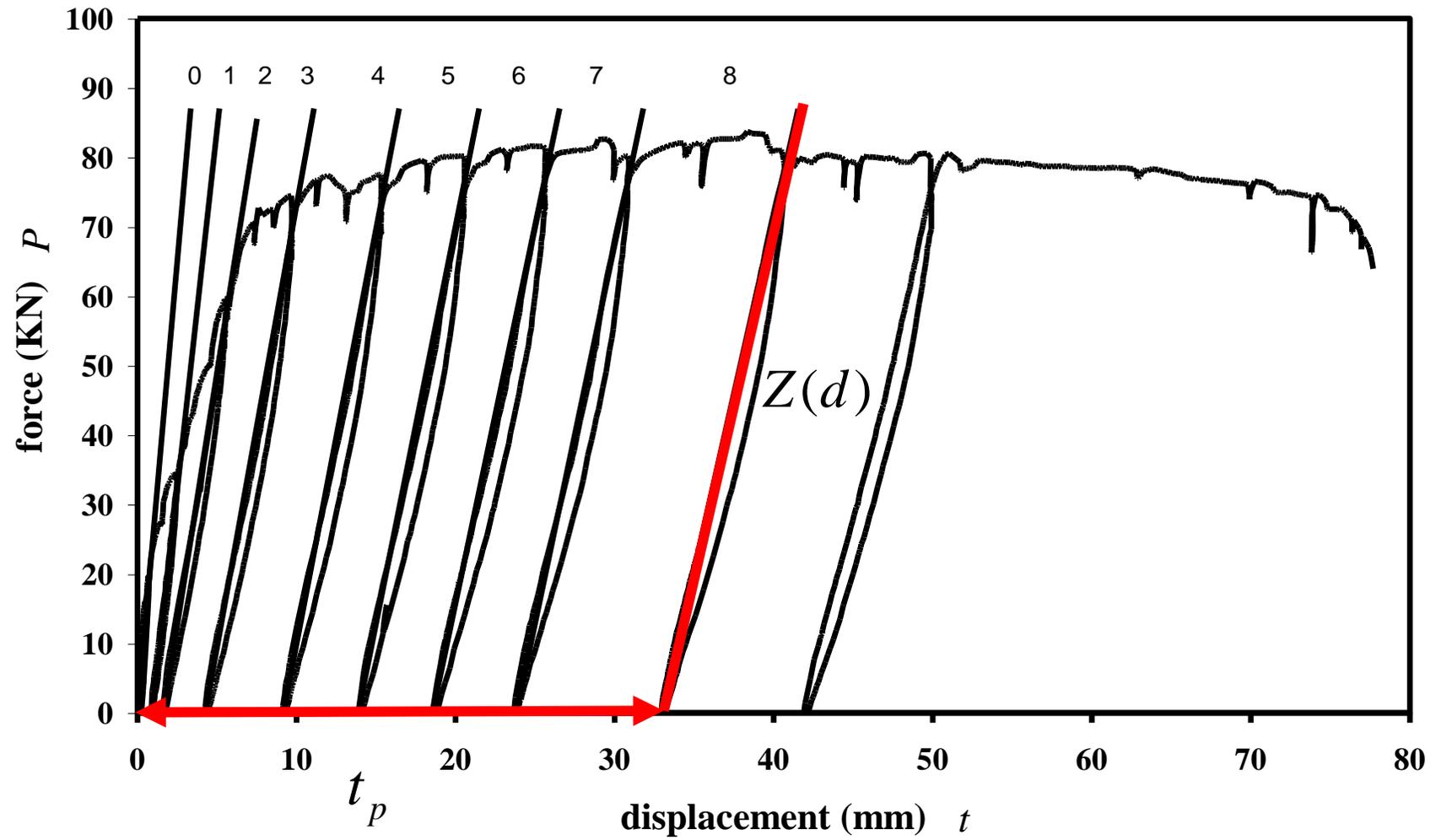
Identificación experimental de la función de resistencia



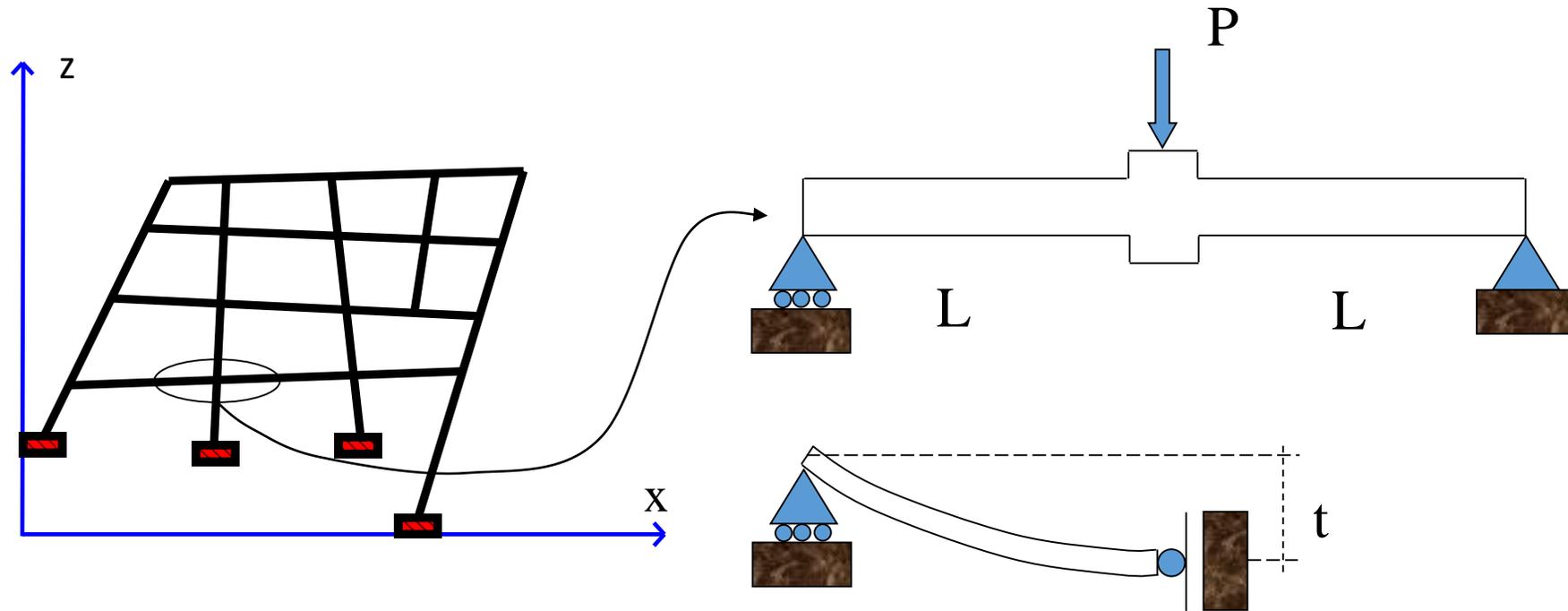
$$P = Z(d)(t - t_p)$$

$$Z(d) = (1 - d)Z_0$$

$$Z_0 = \frac{6EI}{L^3}$$



Identificación experimental de la función de resistencia



$$P = Z(d)(t - t_p)$$

$$Z(d) = (1 - d)Z_0$$

$$Z_0 = \frac{6EI}{L^3}$$

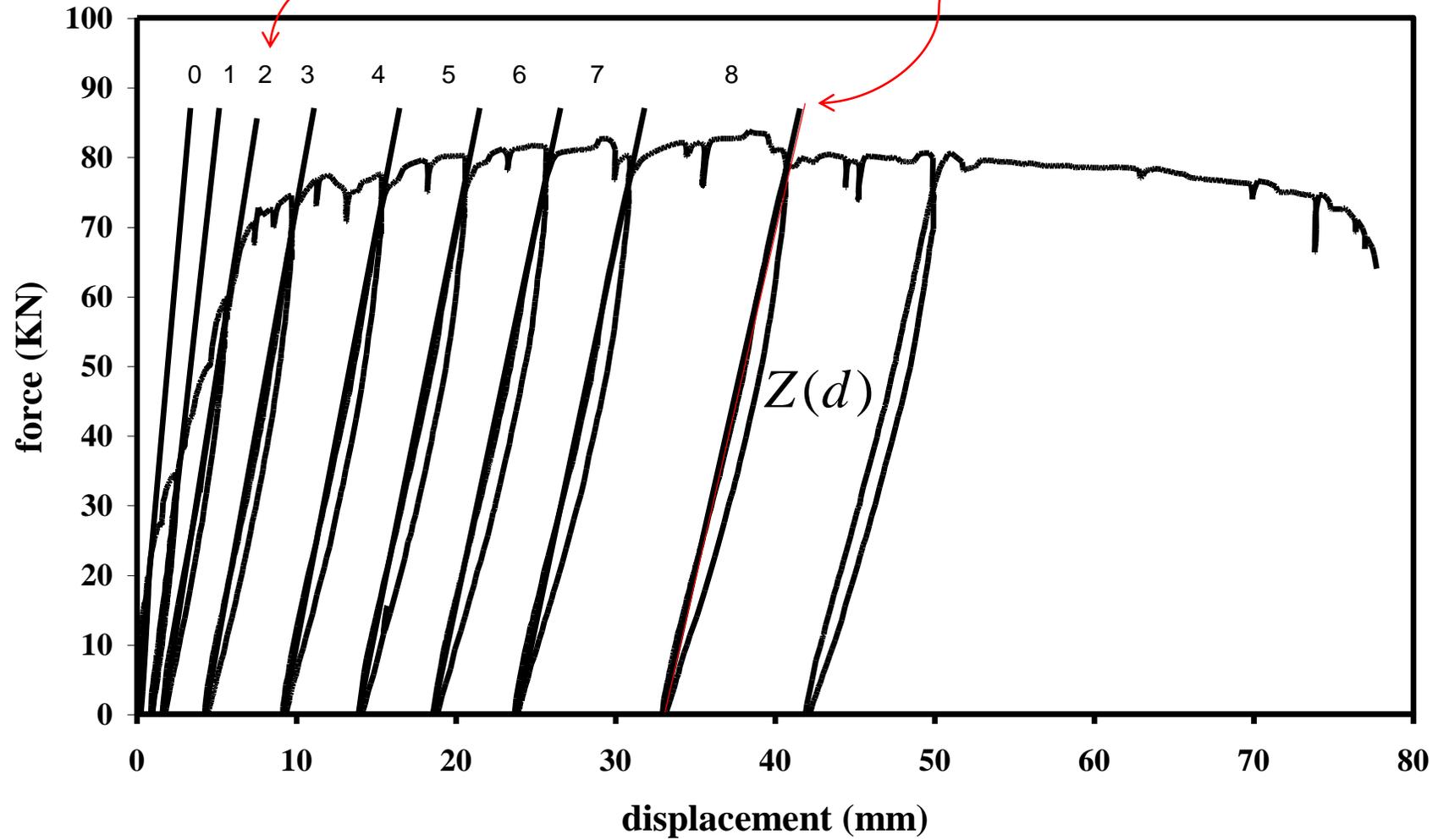
$$d = 1 - Z(d) / Z_0$$



$d = 0.3$

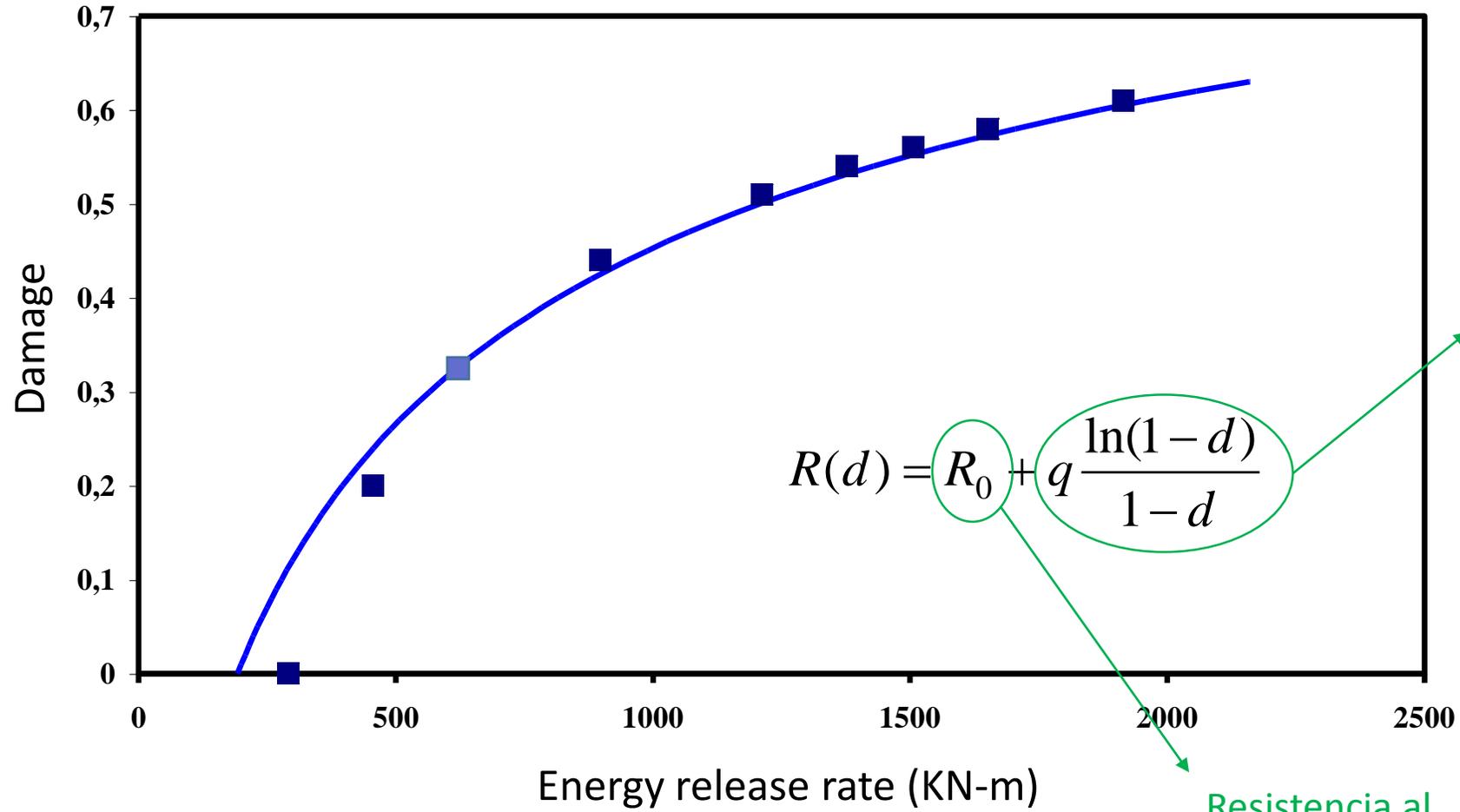


$d = 0.6$



■ Ensayo

— Modelo



Incremento de resistencia por la acción de la armadura

$$R(d) = R_0 + q \frac{\ln(1-d)}{1-d}$$

Resistencia al agrietamiento del concreto

Determinación de los parámetros R_0 y q

$$G = R \quad \Rightarrow \quad m^2 = \frac{2(1-d)^2}{F^0} R_0 + \frac{2q}{F^0} (1-d) \ln(1-d)$$

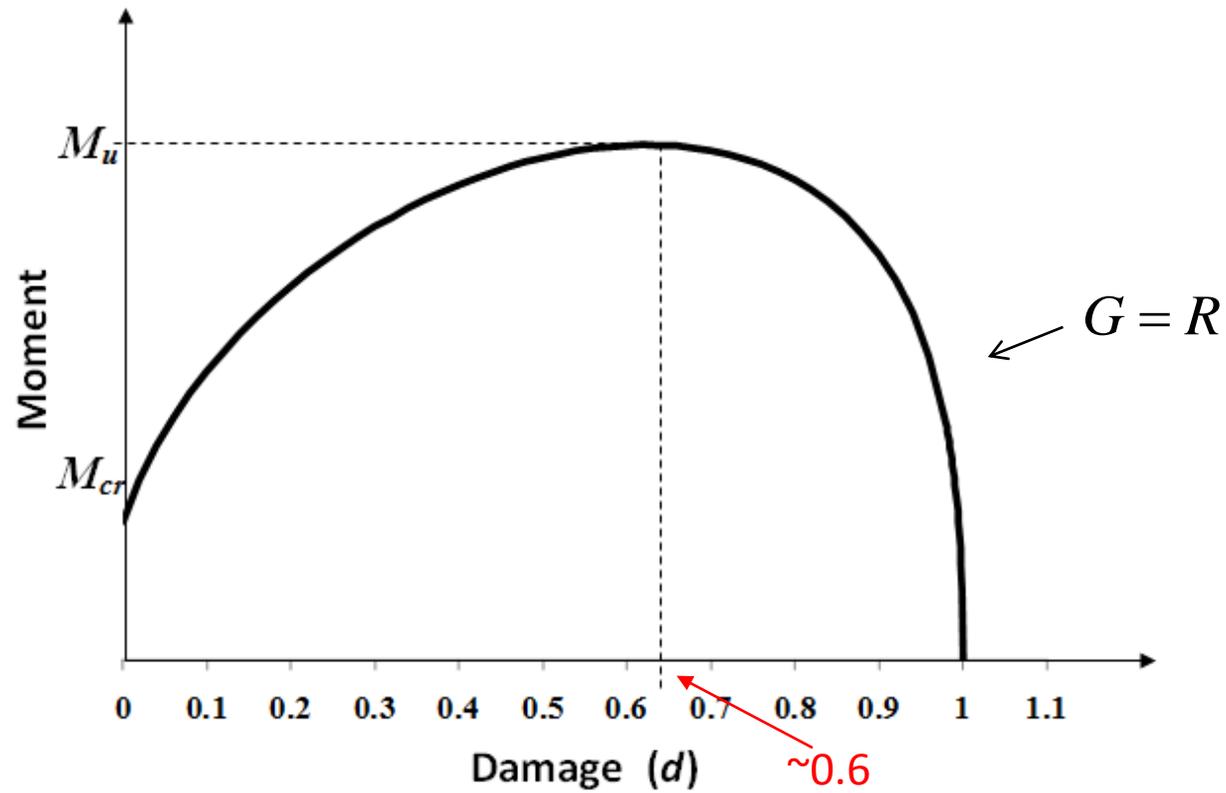
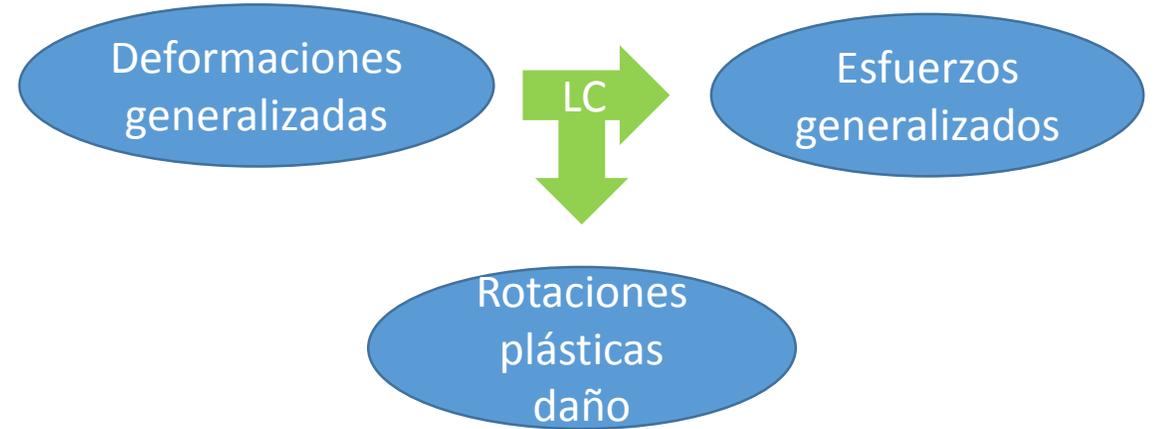


Table 13.2 Performance Level for flexural damage

Performance level	Max expected damage in Beams	Max expected damage in Columns	Description
1	$0.30 \approx d_p$	0.10	The elements do not require any intervention after the event.
2	0.40	$0.30 \approx d_p$	Some minor repairs may be needed.
3	0.50	0.40	The element requires reparation at reasonable costs.
4	$0.6 \approx d_u$	0.5	The structure requires a major rehabilitation process.
5	> 0.7	$> 0.6 \approx d_u$	Inacceptable structural behavior.



Ley de comportamiento: {
Ley de elasticidad
Ley de daño
Ley de plasticidad

Rótula inelástica i

Ley de plasticidad

Unidad estructural

Esfuerzo efectivo

Función de fluencia sin daño

Hipótesis de equivalencia

Teoría del daño continuo:



$$\bar{\sigma} = \frac{\sigma}{1-\omega}$$

$$f = |\sigma| - \sigma_y$$

$$f = \left| \frac{\sigma}{1-\omega} \right| - \sigma_y$$

Teoría del daño concentrado:



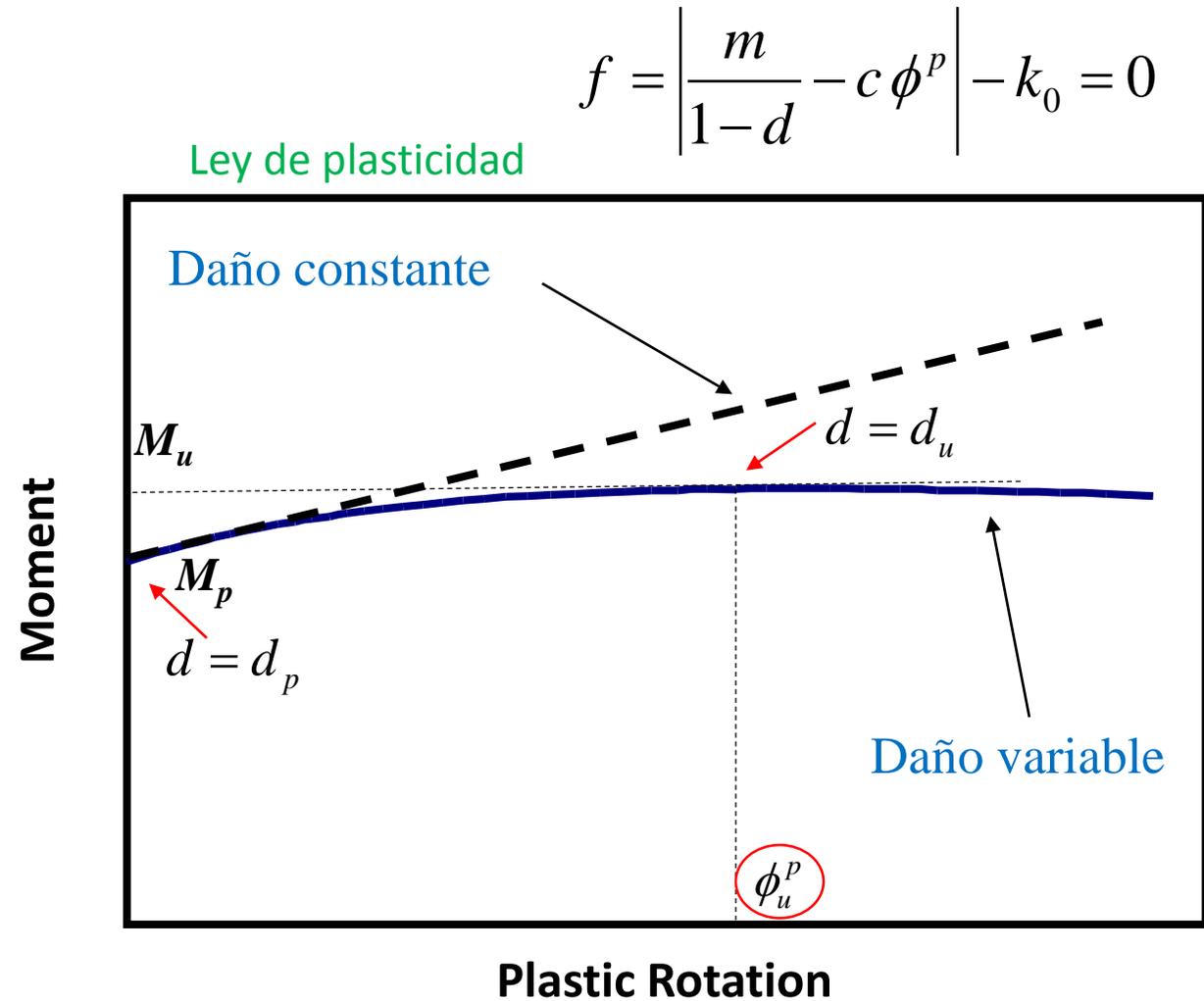
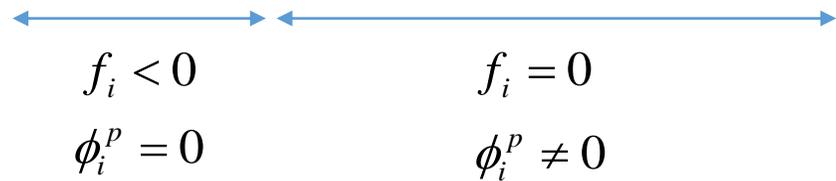
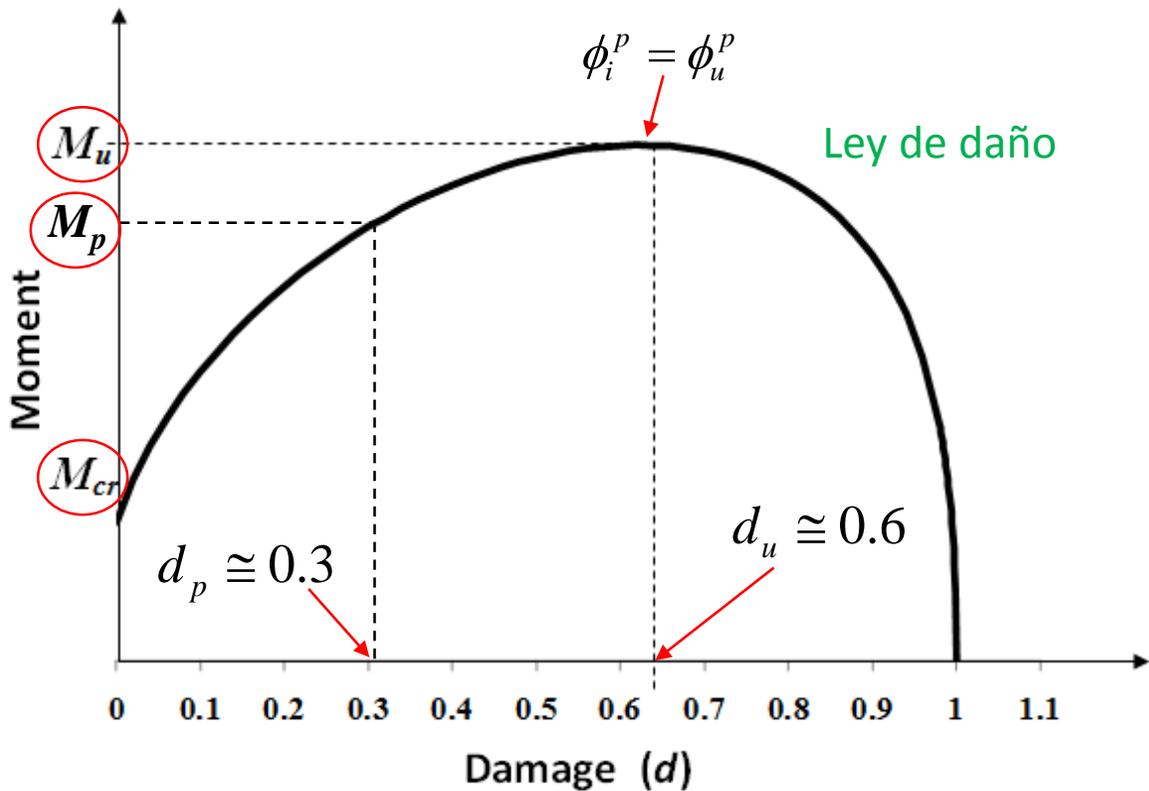
$$\bar{m}_i = \frac{m_i}{1-d_i}$$

$$f_i = |m_i - c\phi_i^p| - M_u \leq 0$$

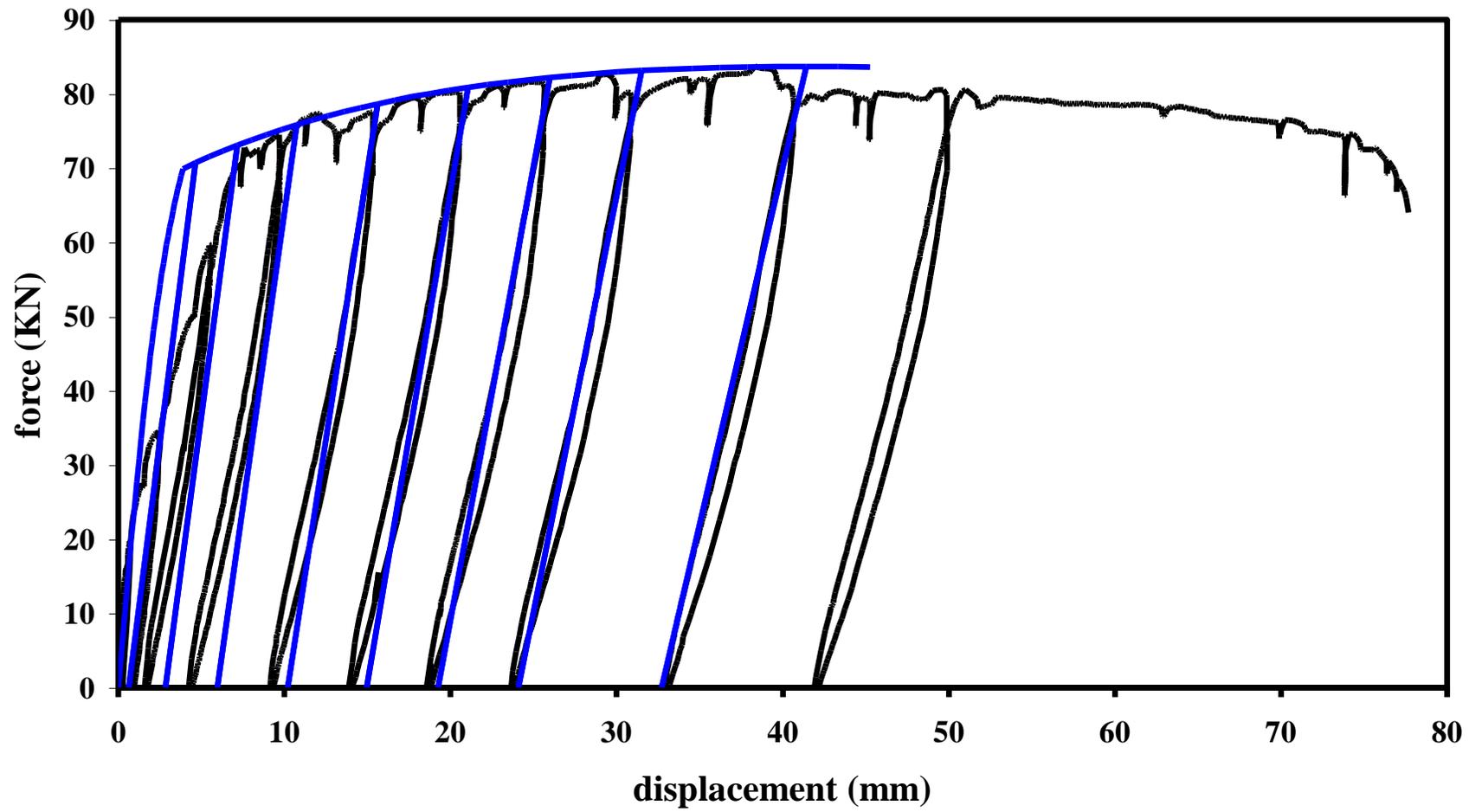
$$f_i = \left| \frac{m_i}{1-d_i} - c\phi_i^p \right| - M_u \leq 0$$

$$\begin{cases} d\phi_i^p = 0 & \text{if } f_i(m_i, d_i) < 0 \text{ (rótula } i \text{ bloqueada)} \\ f_i(m_i, d_i) = 0 & \text{if } d\phi_i^p \neq 0 \text{ (rótula } i \text{ activa)} \end{cases}$$

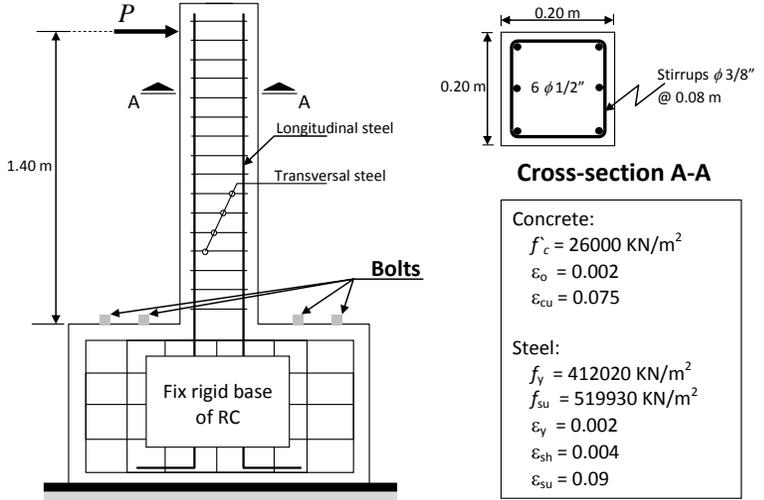
Determinación de los parámetros k_0 y c



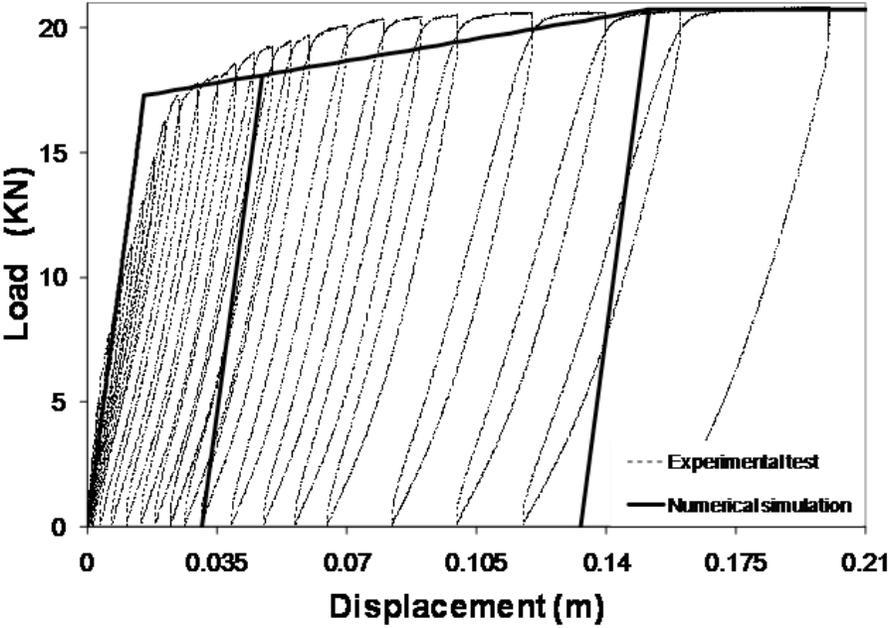
— Ensayo — modelo



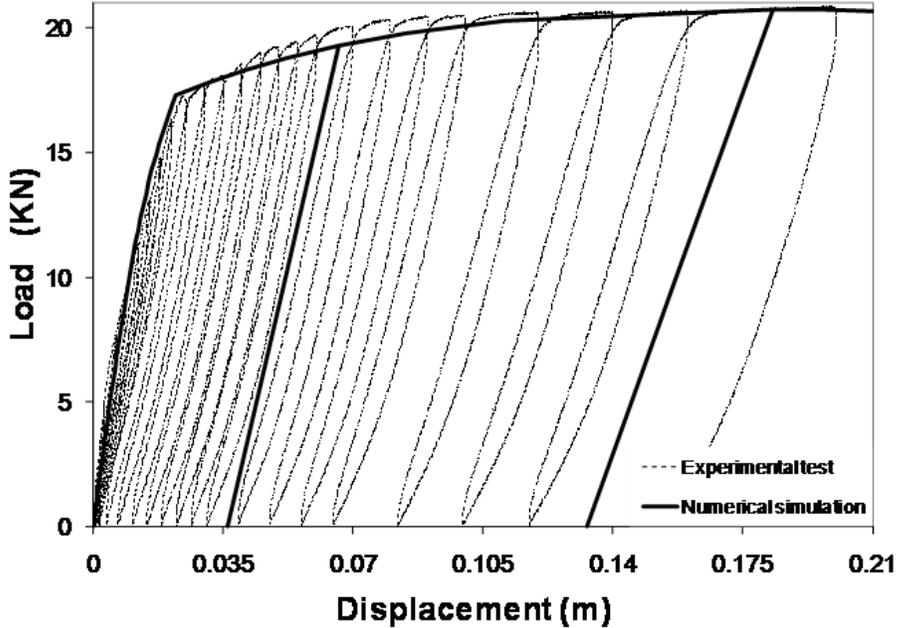
Simulaciones numéricas con dos modelos

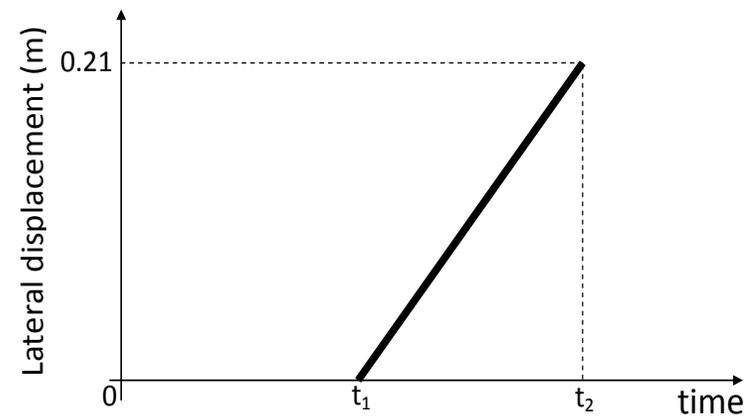
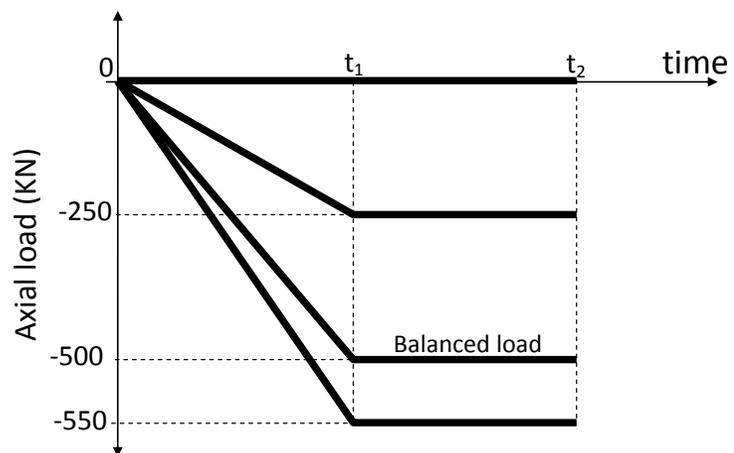
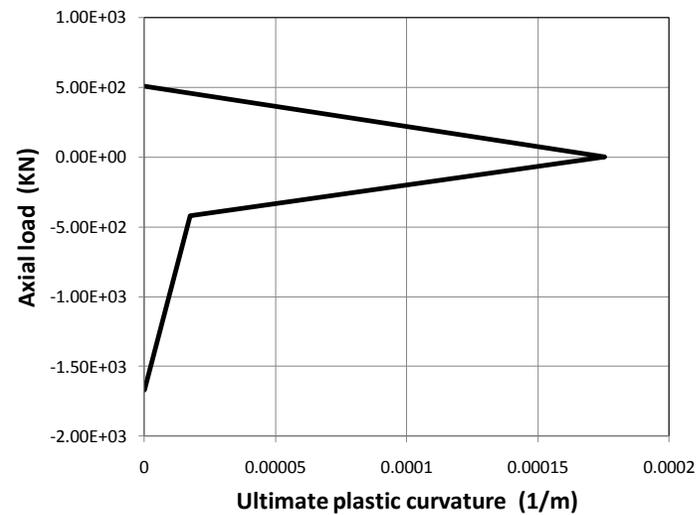
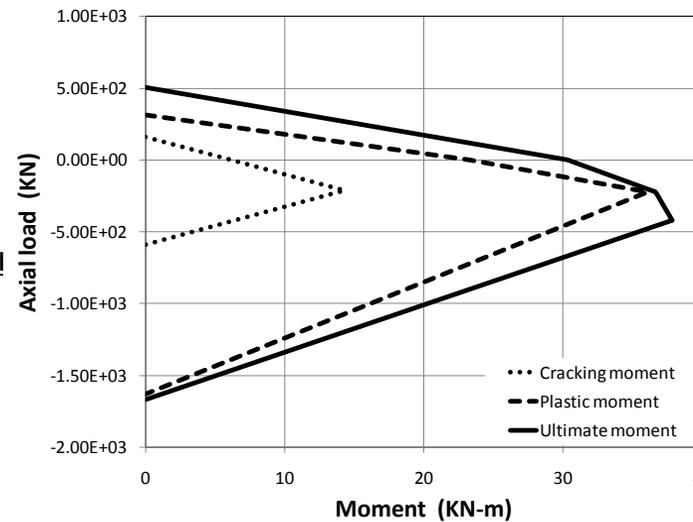
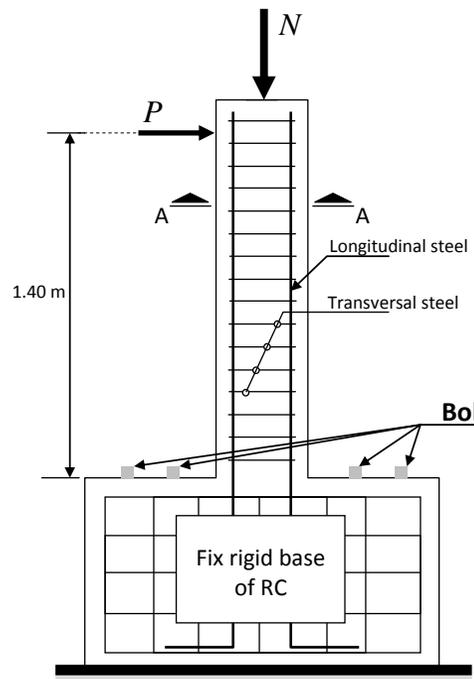


Modelo "tri linear" de plasticidad

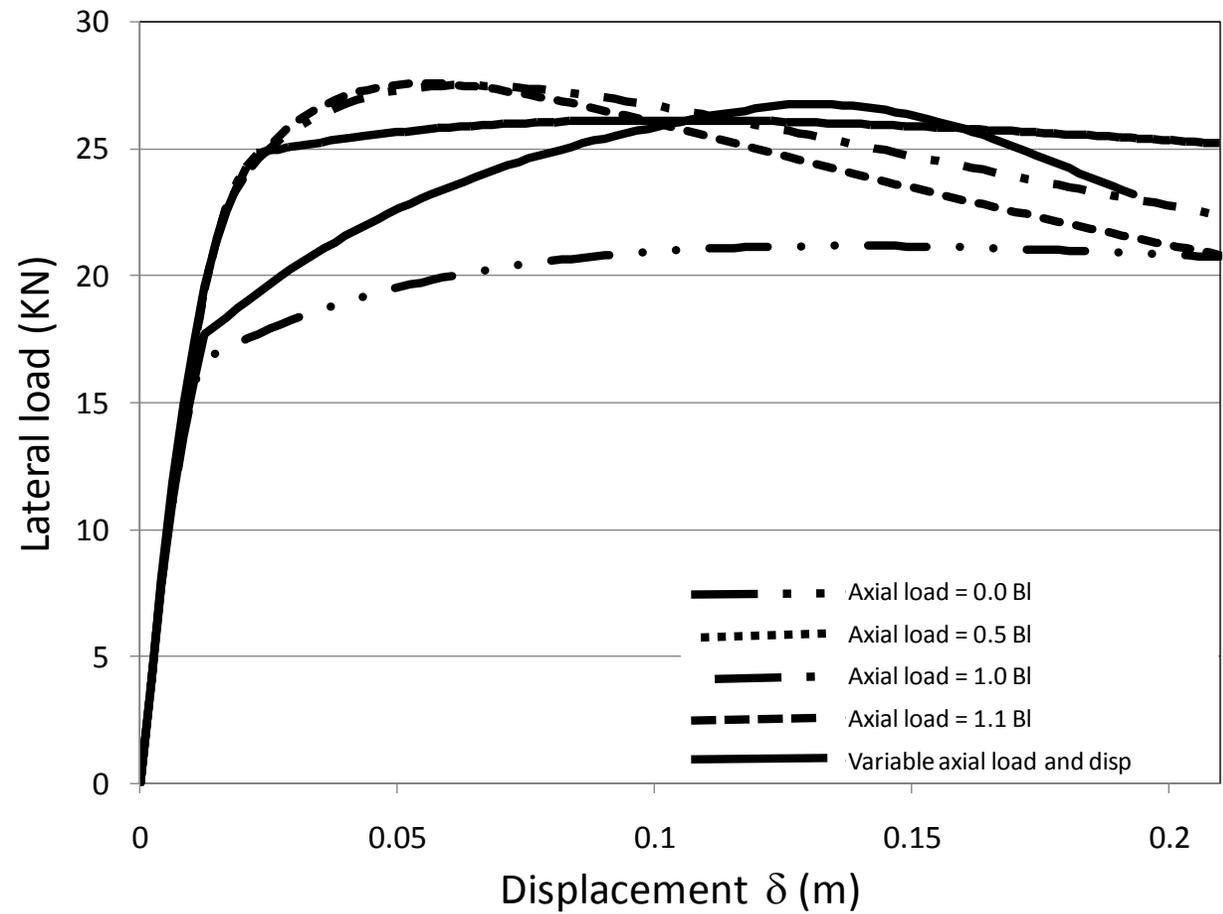


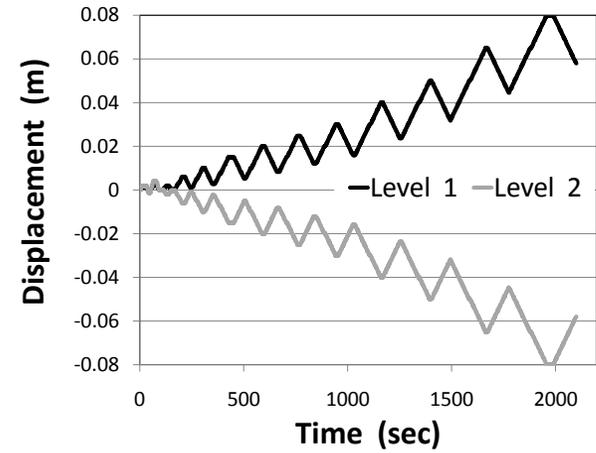
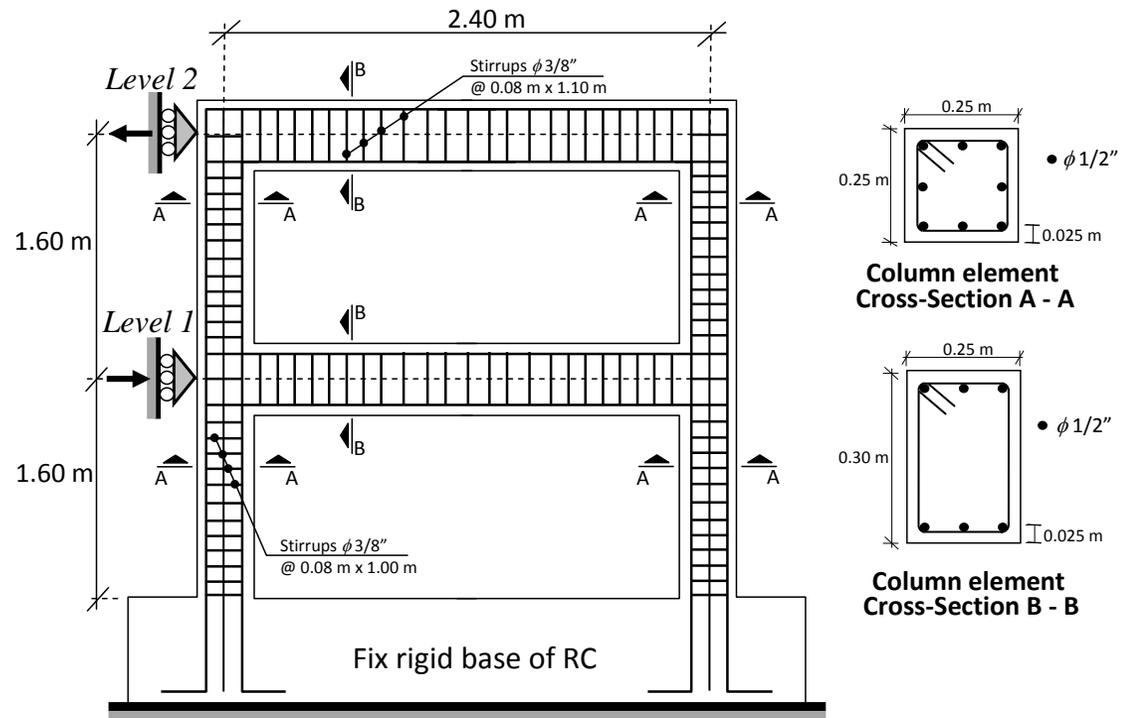
TDC





Análisis con fuerza axial variable

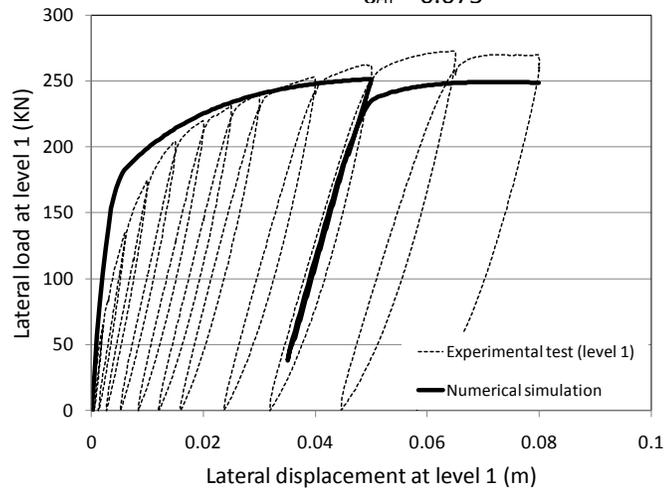




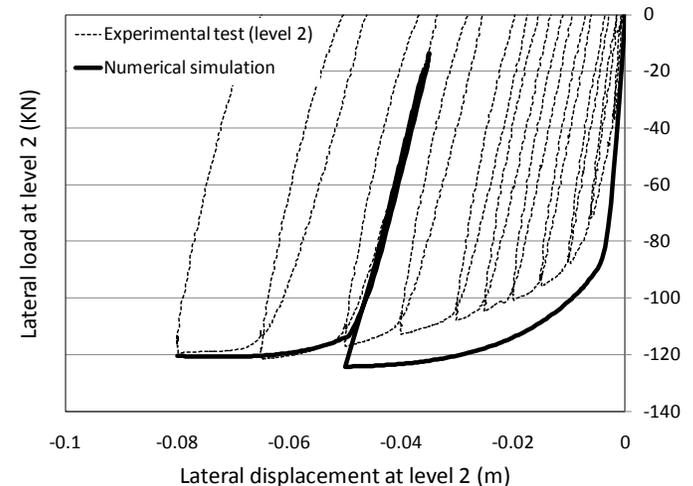
Material Properties:

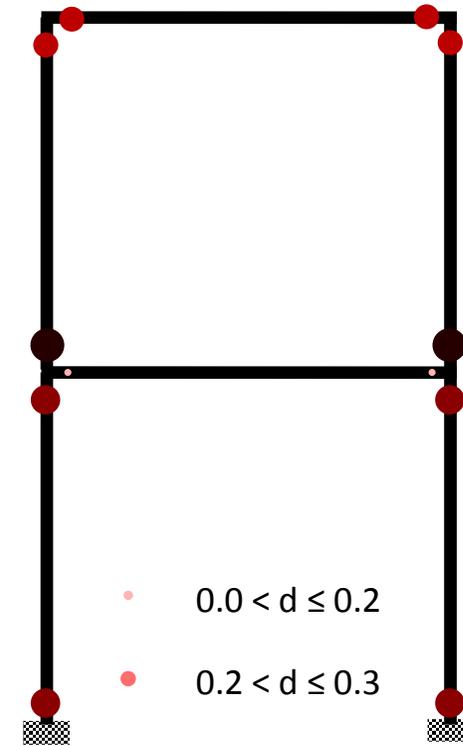
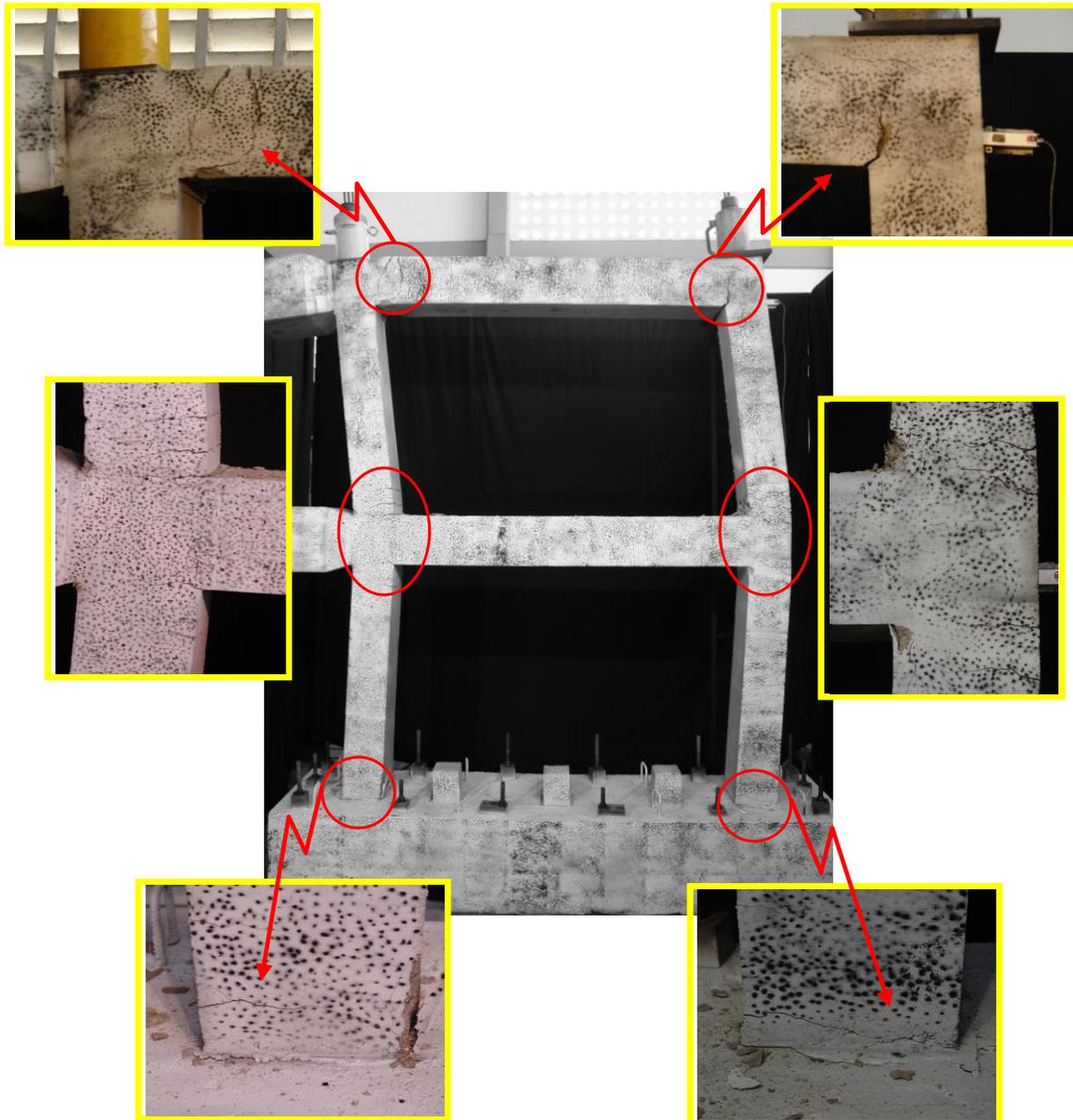
Concrete:
 $f_c = 24517 \text{ KN/m}^2$
 $\epsilon_o = 0.002$
 $\epsilon_{cu} = 0.075$

Steel:
 $f_y = 411879 \text{ KN/m}^2$
 $f_{su} = 617819 \text{ KN/m}^2$
 $\epsilon_y = 0.002$
 $\epsilon_{sh} = 0.004$
 $\epsilon_{su} = 0.09$



Ensayo en. un
 pórtico de
 concreto armado

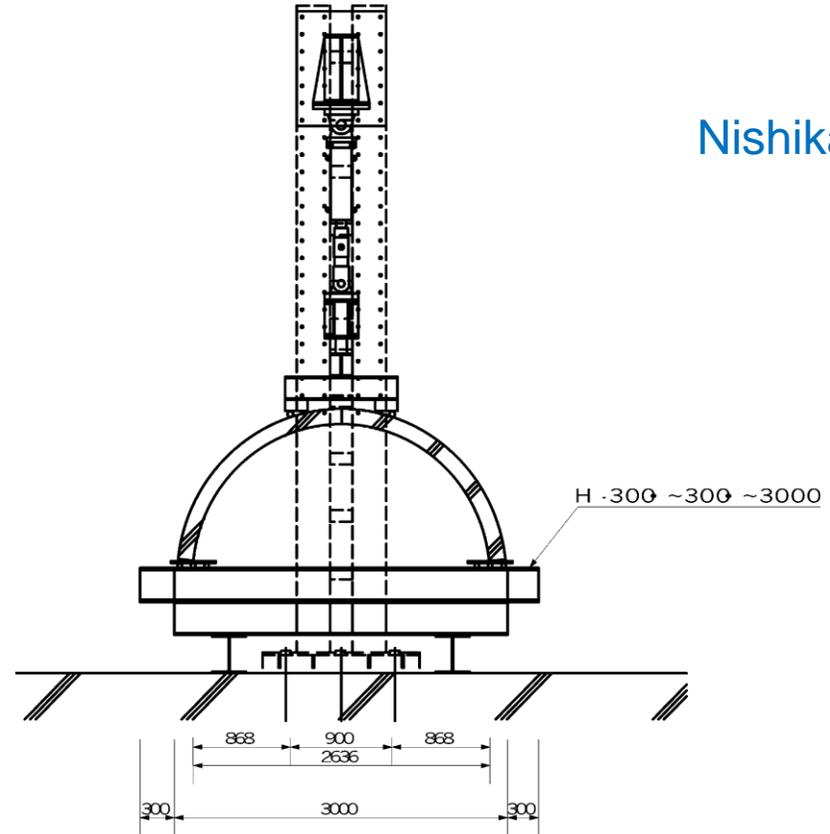
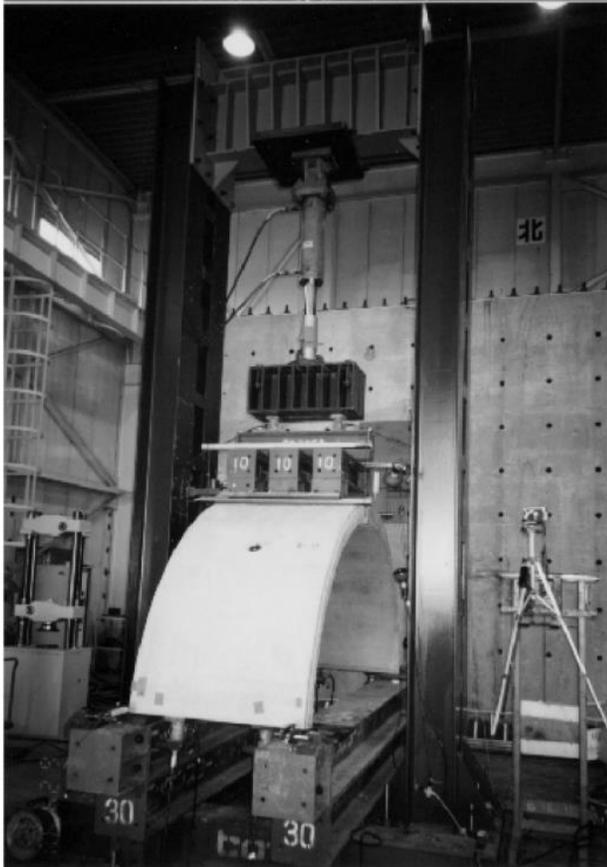




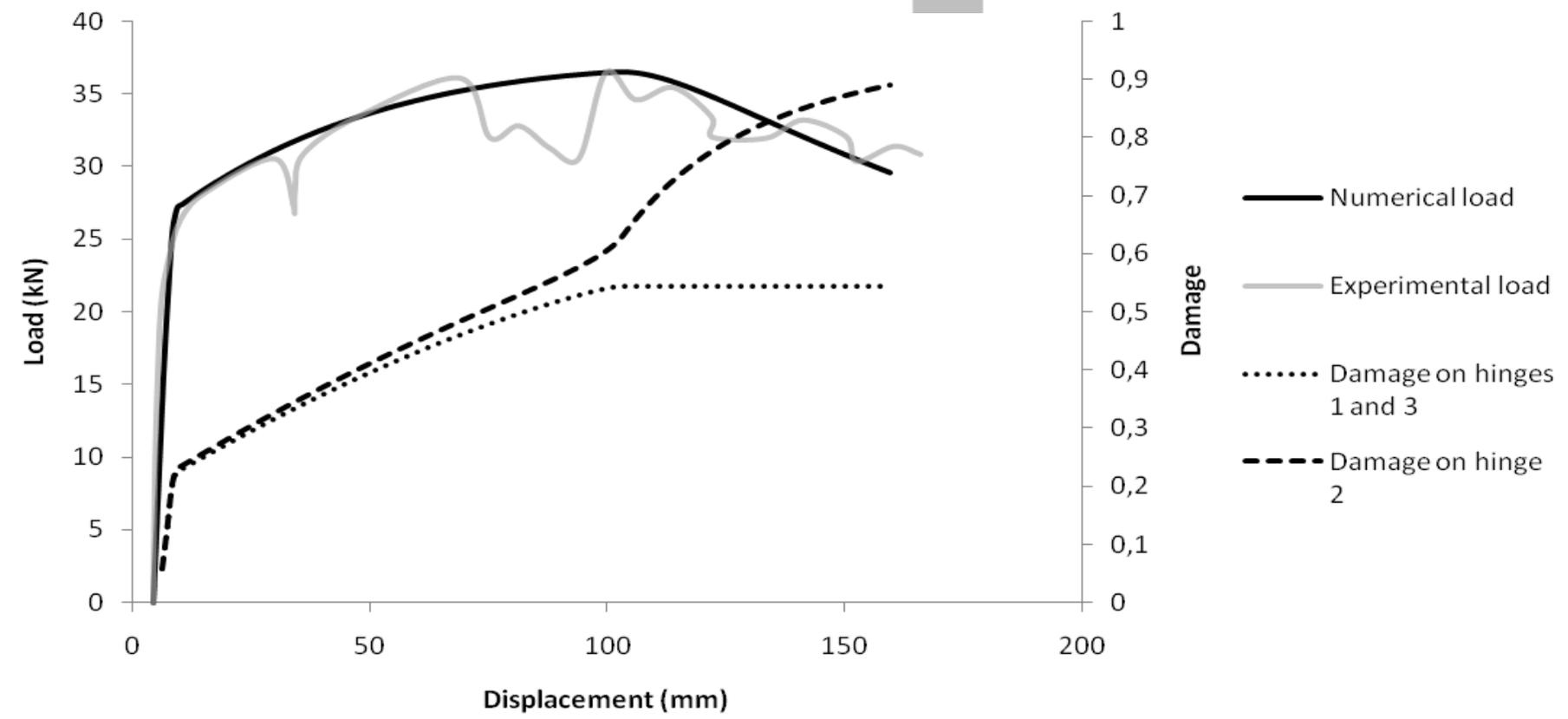
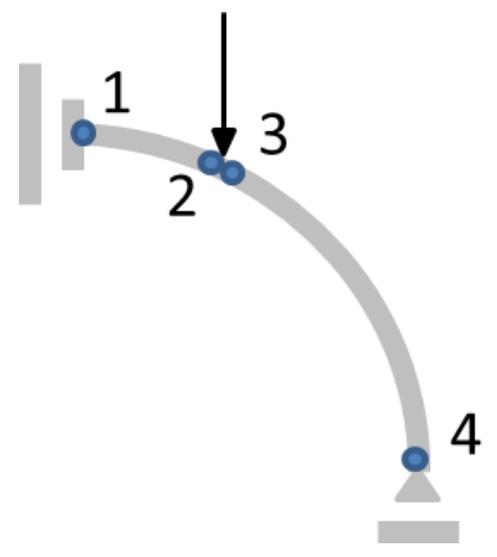
- $0.0 < d \leq 0.2$
- $0.2 < d \leq 0.3$
- $0.3 < d \leq 0.4$
- $0.4 < d \leq 0.5$
- $0.5 < d \leq 0.6$
- $0.6 < d \leq 0.7$
- $d > 0.7$

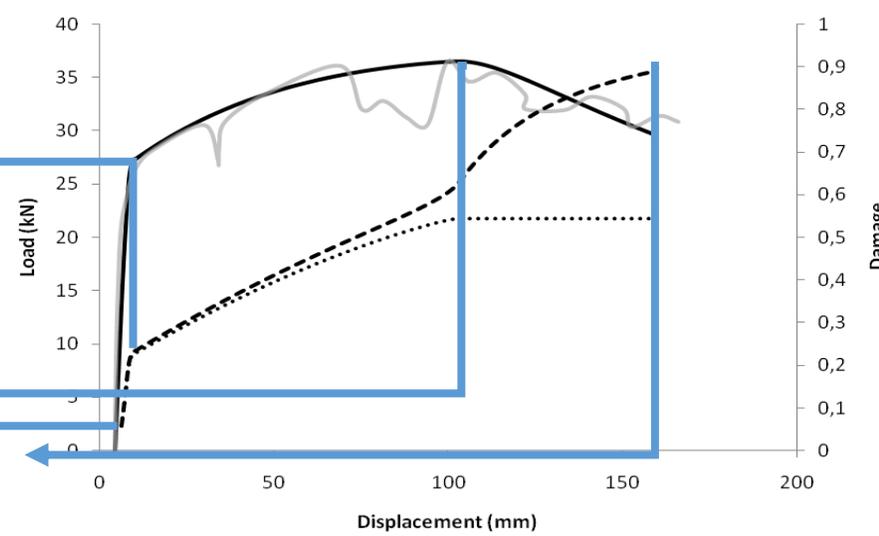
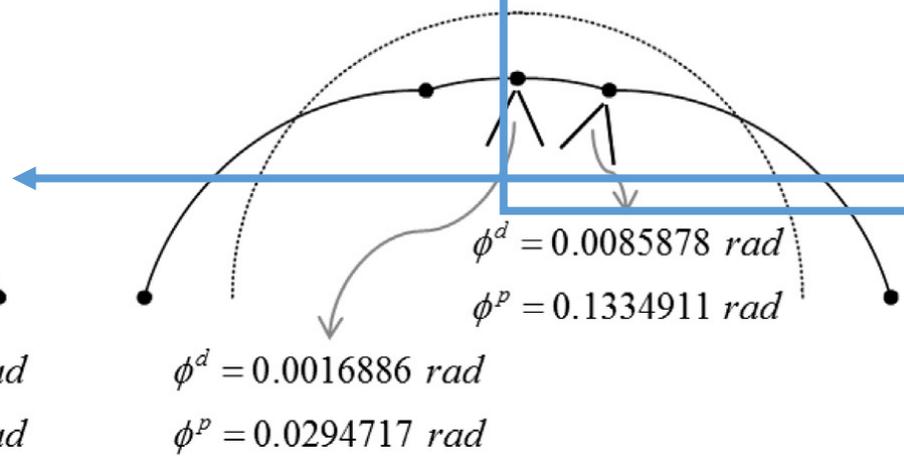
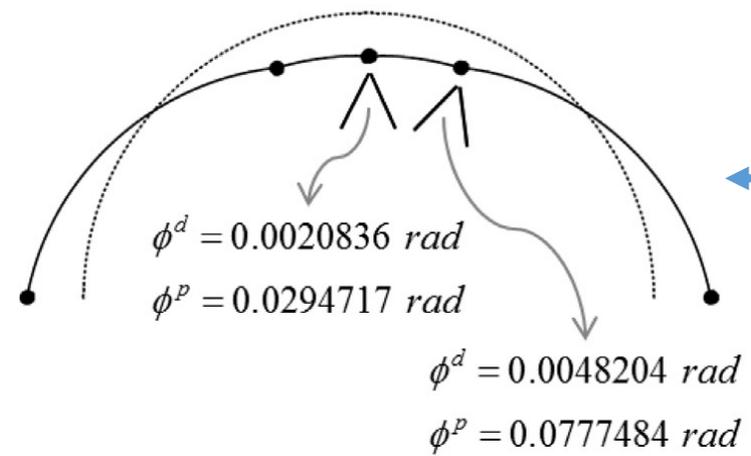
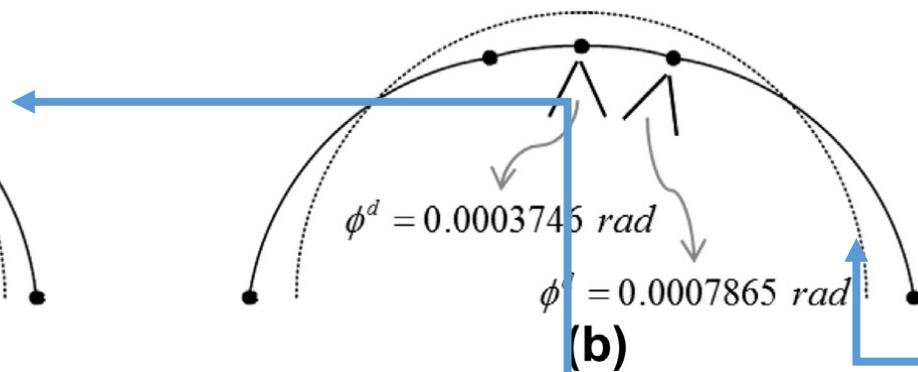
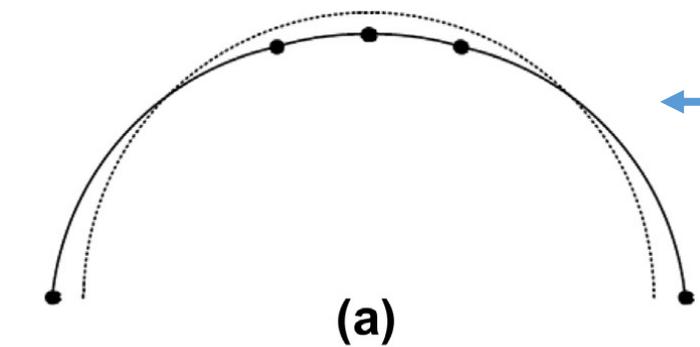
Mapa de daños

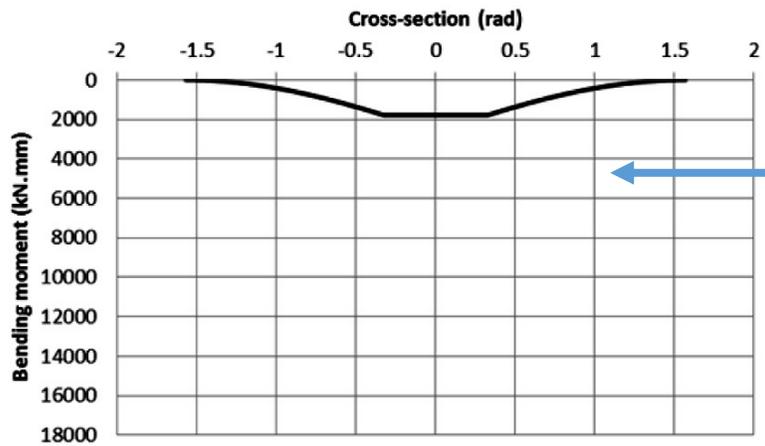
Simulación numérica
de arcos de concreto
armado



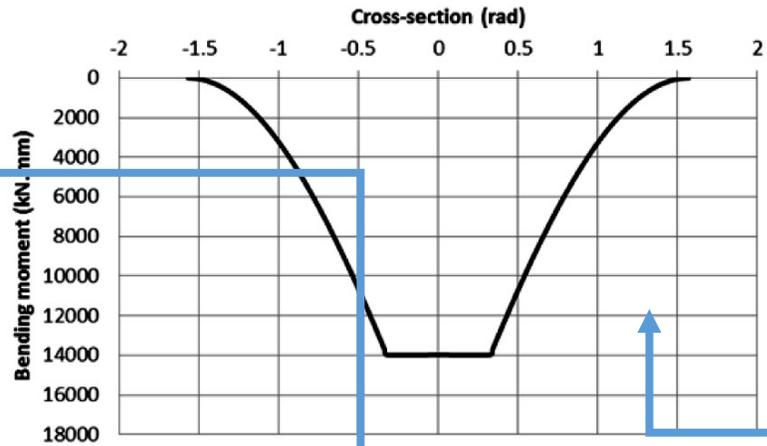
Nishikawa (2003)



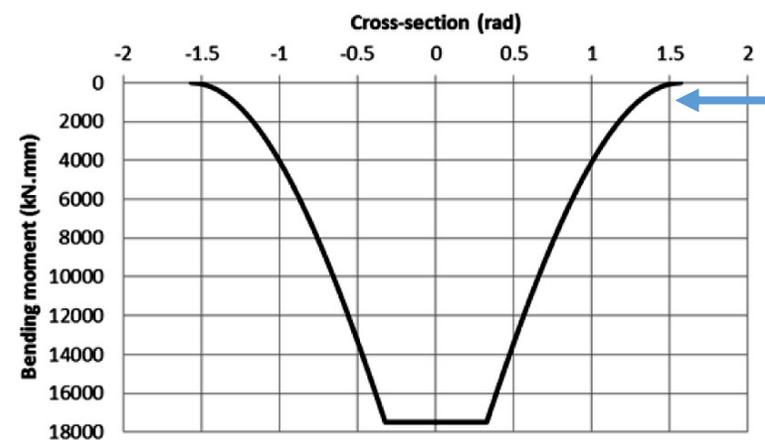




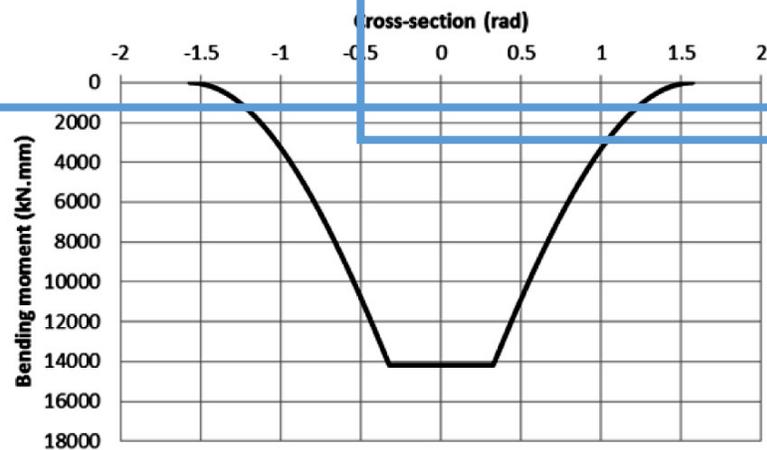
(a)



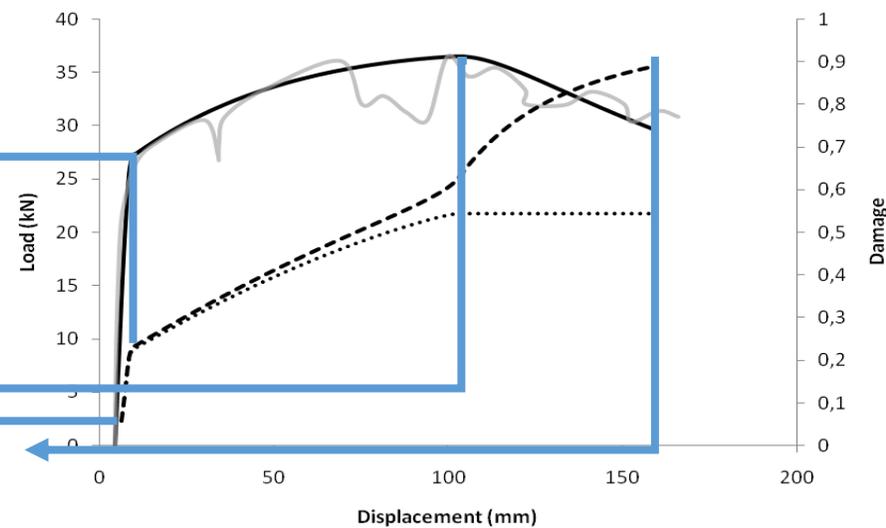
(b)



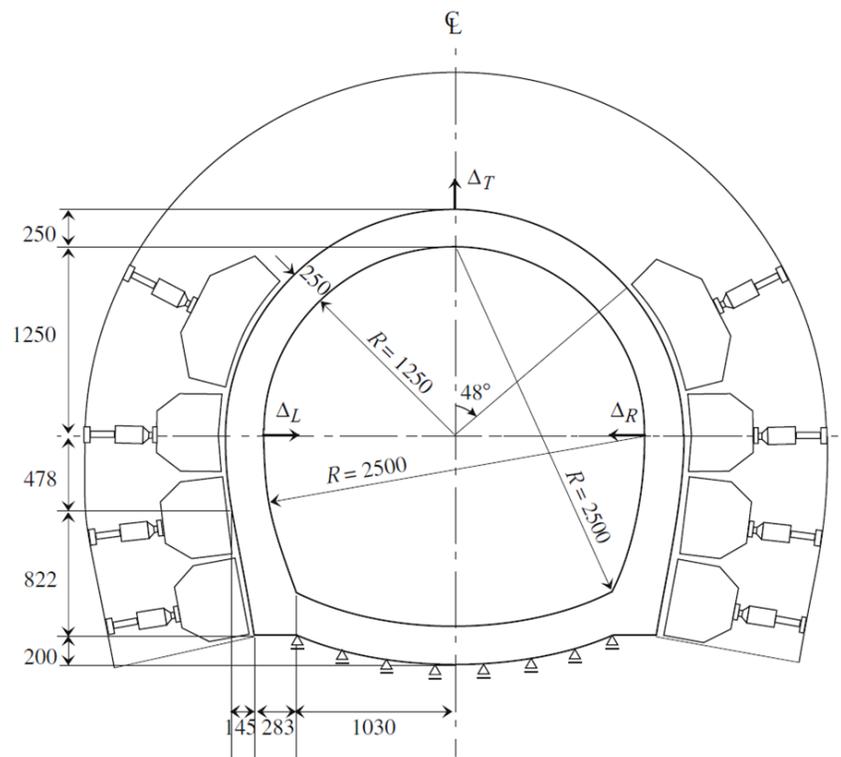
(c)



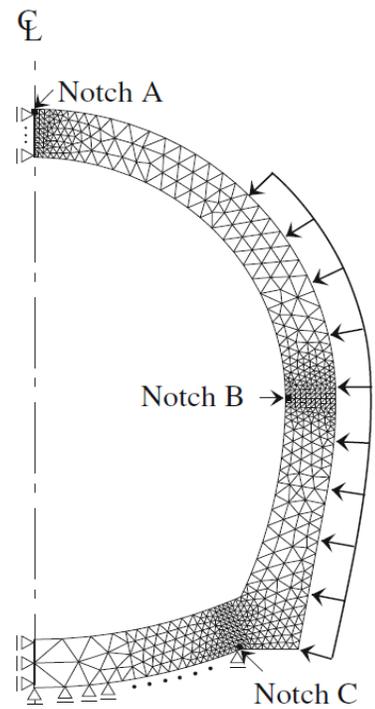
(d)



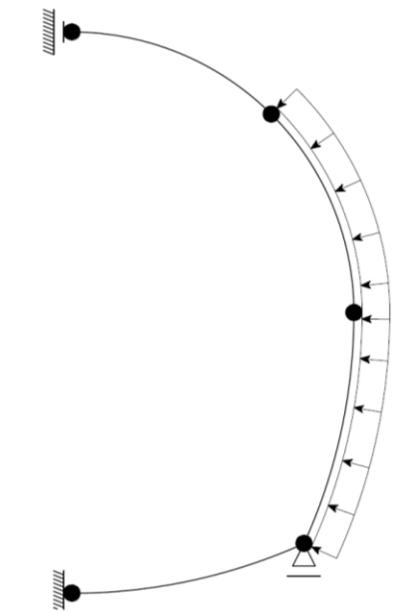
- Conceptos fundamentales de Mecánica da Fractura
- Conceptos fundamentales de la Teoría del Daño Continuo
- TDC: Modelado de elementos esbeltos de concreto armado sometidos a solicitaciones monotónicas
- **Modelado de estructuras de concreto no reforzado**
- Modelado de sistemas duales y de elementos de concreto armado con cualquier relación de aspecto
- Modelado de elementos estructurales bajo solicitaciones generales
- Aplicaciones prácticas
- Conclusiones



Shi et al

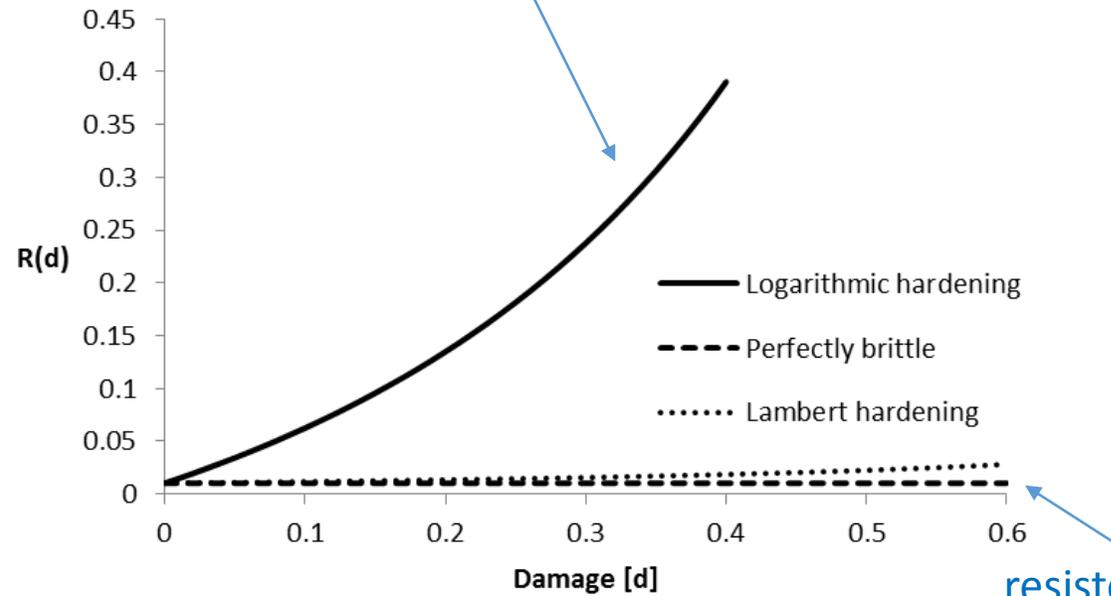


Malla de elementos finitos



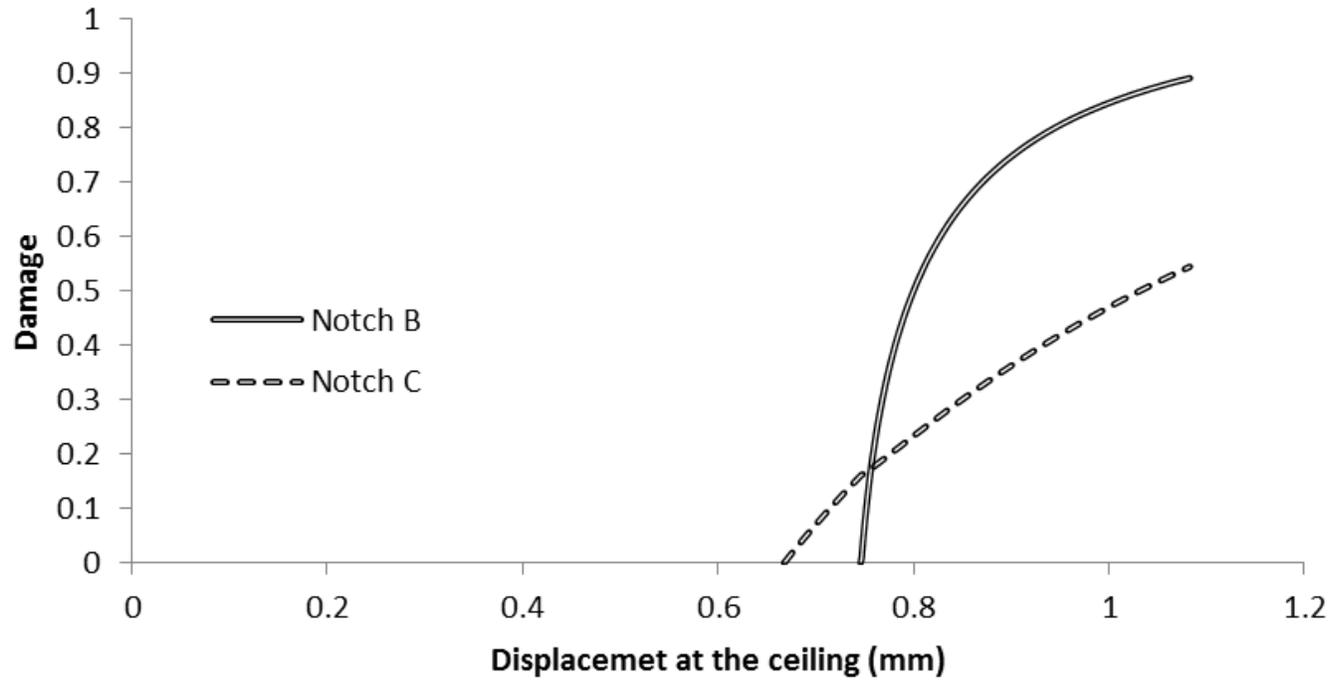
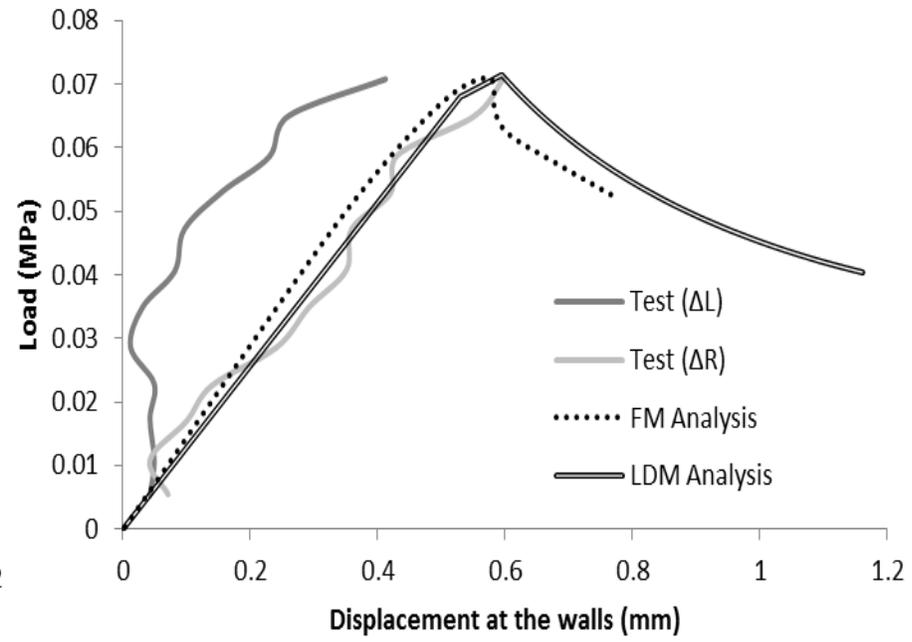
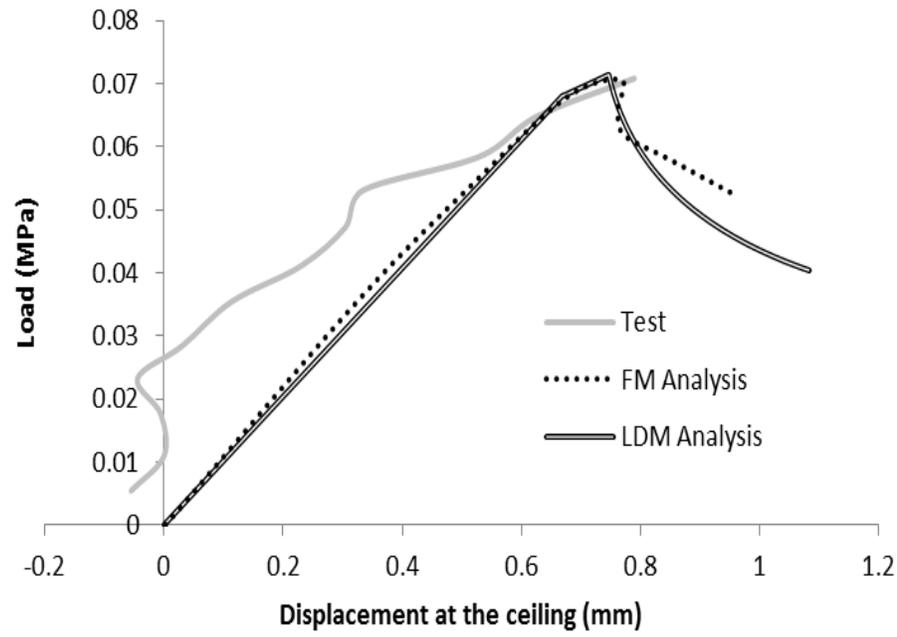
malla LDM

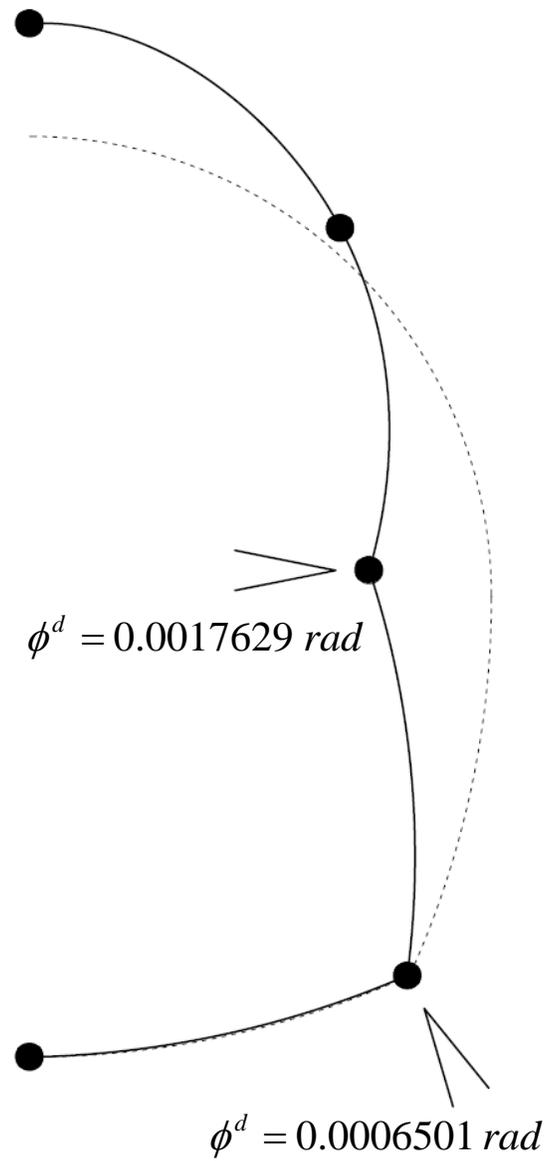
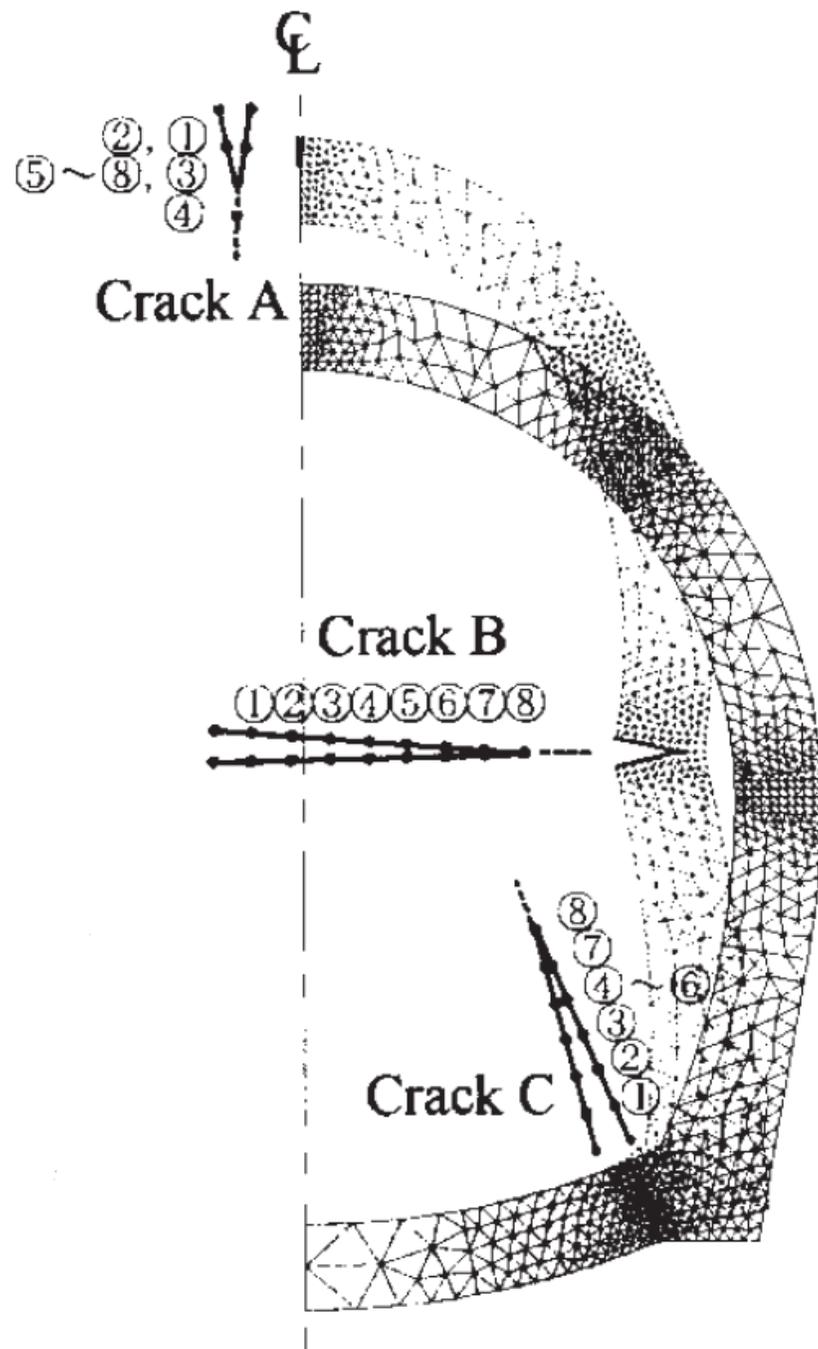
Función de resistencia al agrietamiento para concreto armado

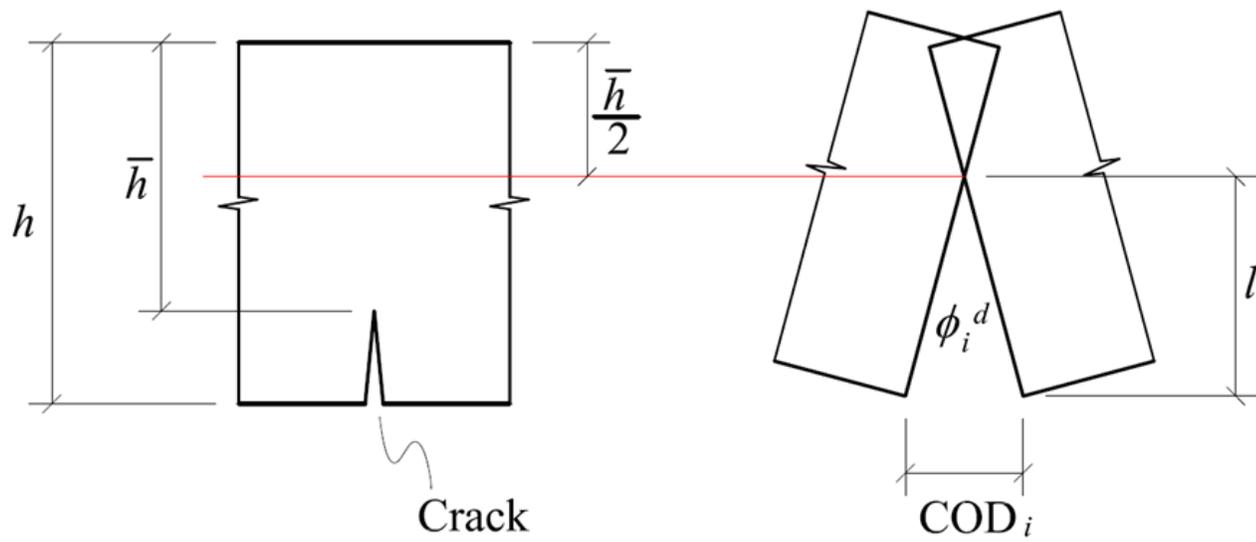


resistencia al agrietamiento para concreto no reforzado

$$\{\boldsymbol{\varepsilon}^P\} = 0$$

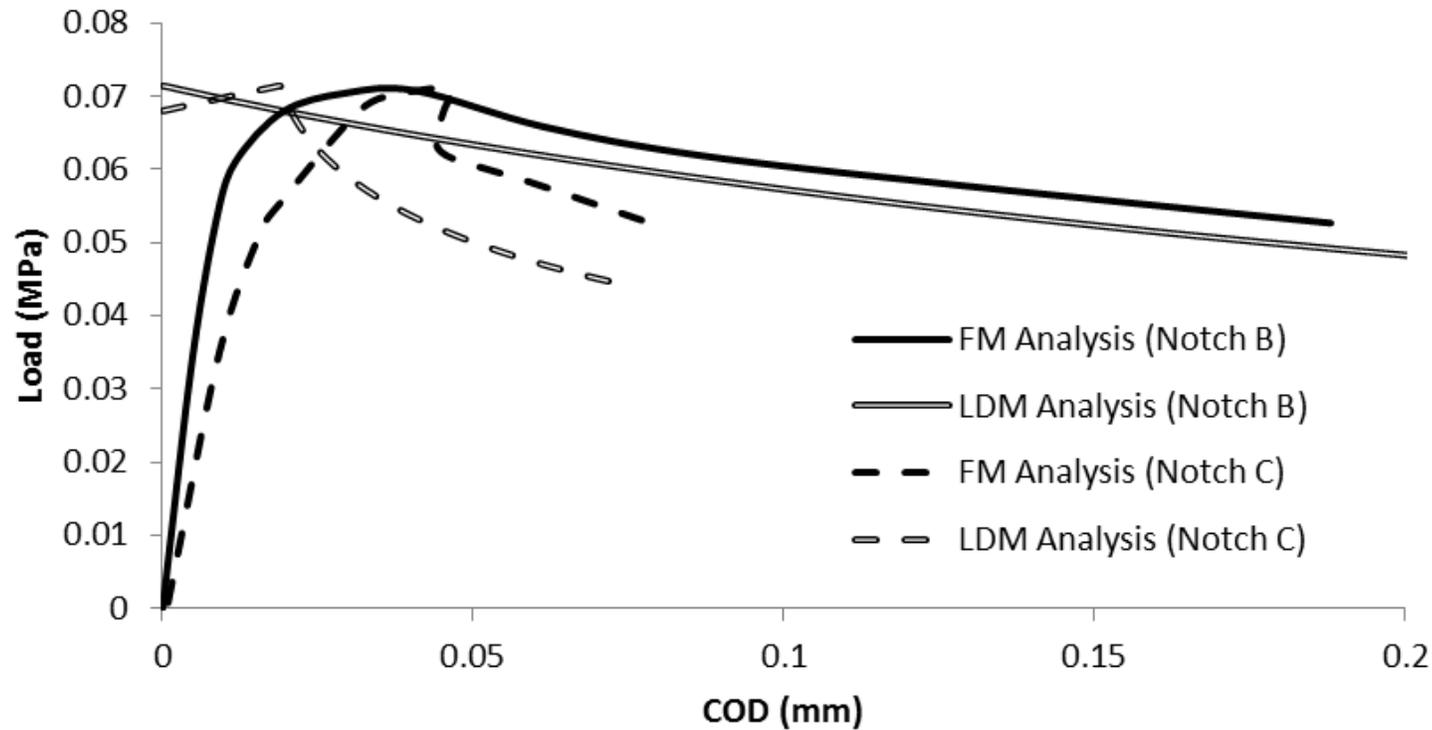




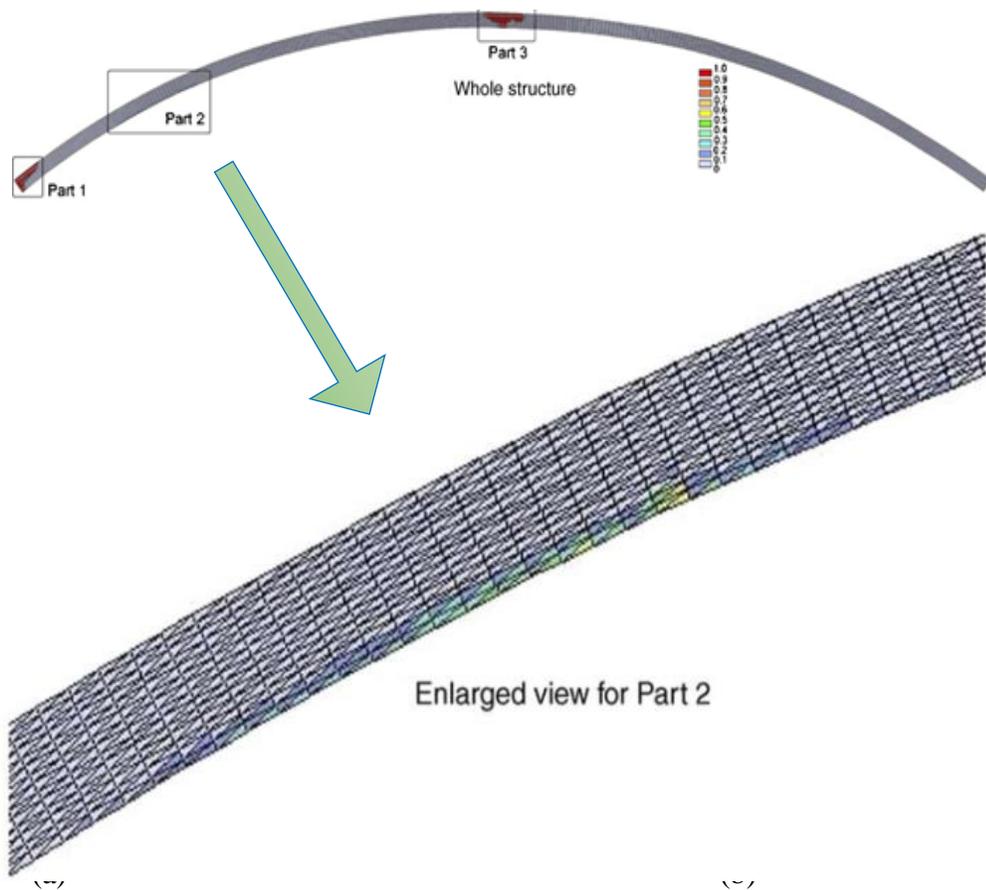


Cálculo de la abertura de la fisura (COD)

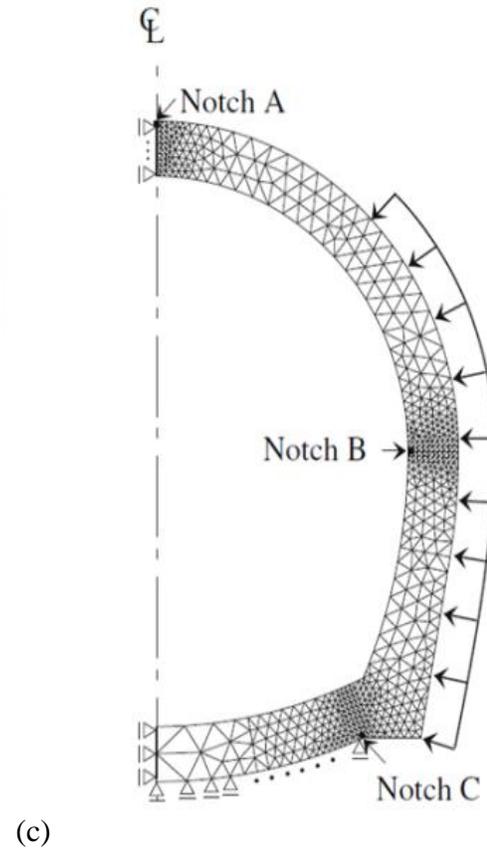
$$COD_i = h\sqrt{2R(d_i)F_{ii}^0}d_i\left(1 - \frac{\sqrt[3]{1-d_i}}{2}\right)$$



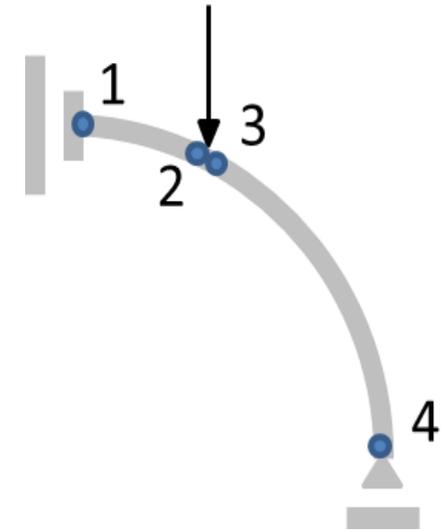
Análisis del daño estructural en un arco, 3 enfoques diferentes



Teoría del daño continuo



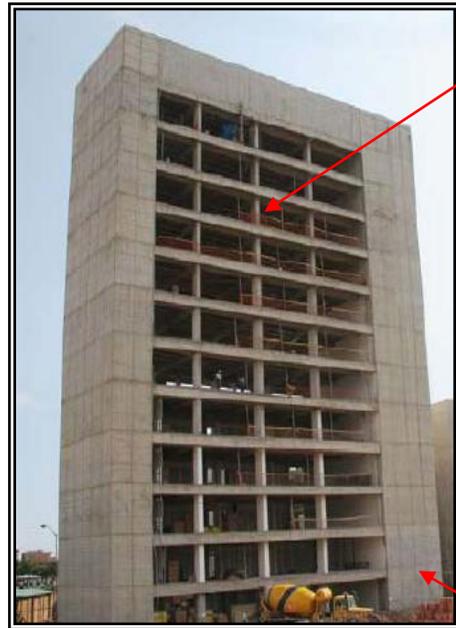
Mecánica de la fractura



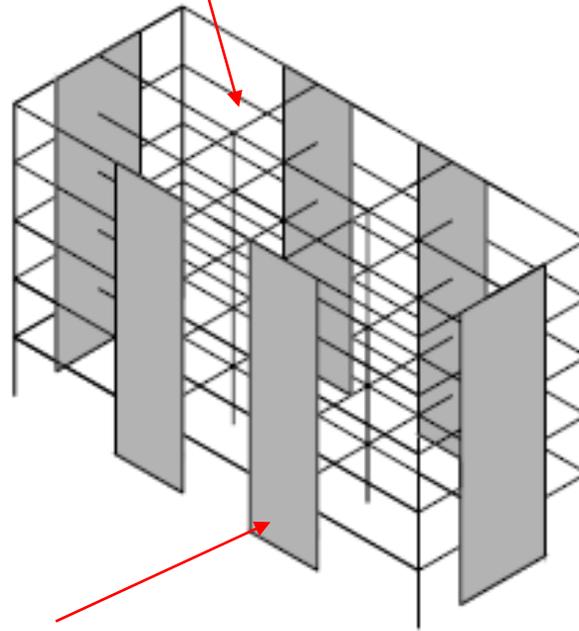
Teoría do daño concentrado

- Conceptos fundamentales de Mecánica da Fractura
- Conceptos fundamentales de la Teoría del Daño Continuo
- TDC: Modelado de elementos esbeltos de concreto armado sometidos a solicitaciones monotónicas
- Modelado de estructuras de concreto no reforzado
- **Modelado de sistemas duales y de elementos de concreto armado con cualquier relación de aspecto**
- Modelado de elementos estructurales bajo solicitaciones generales
- Aplicaciones prácticas
- Conclusiones

Sistemas duales y estructuras con elementos de cualquier relación de aspecto



Elementos
esbeltos



Muros

Columnas
cortas



Variación del patrón de agrietamiento con la relación de aspecto

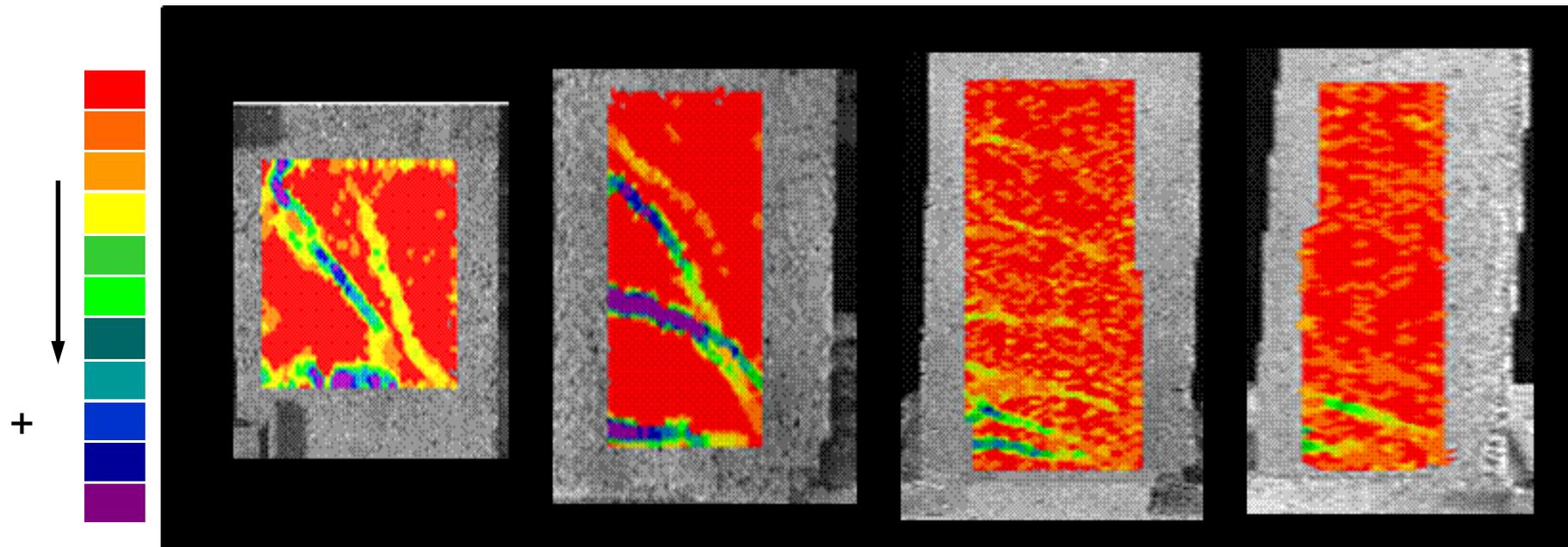
Relación de Aspecto

Baja: 1.78

intermedia: 3.55

intermedia: 5.33

alta: 7.110



B1

B2

B3

B4

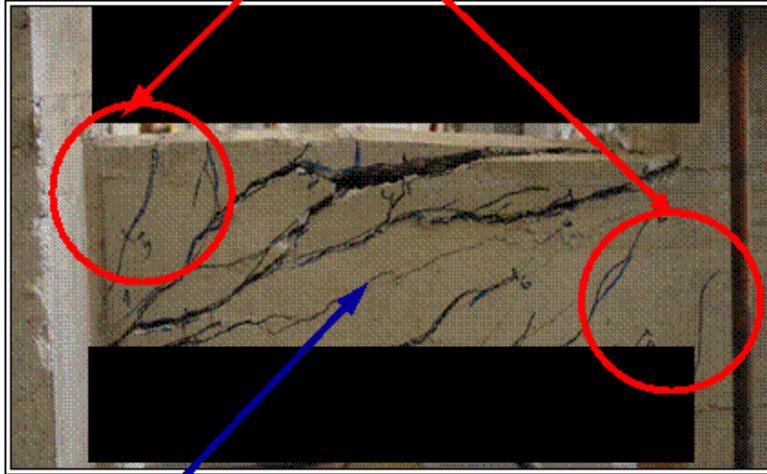
Baja: tracción diagonal

Alta: flexión

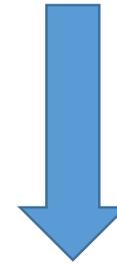
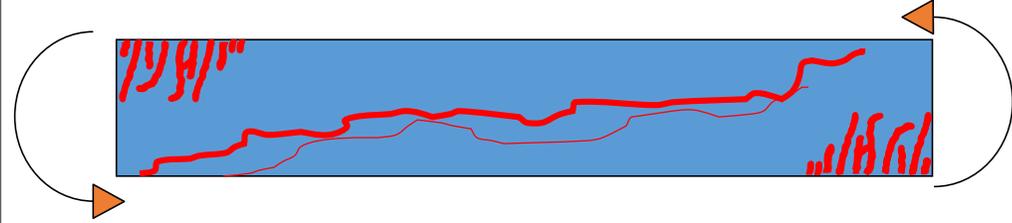
3

6

Fisuras de flexión

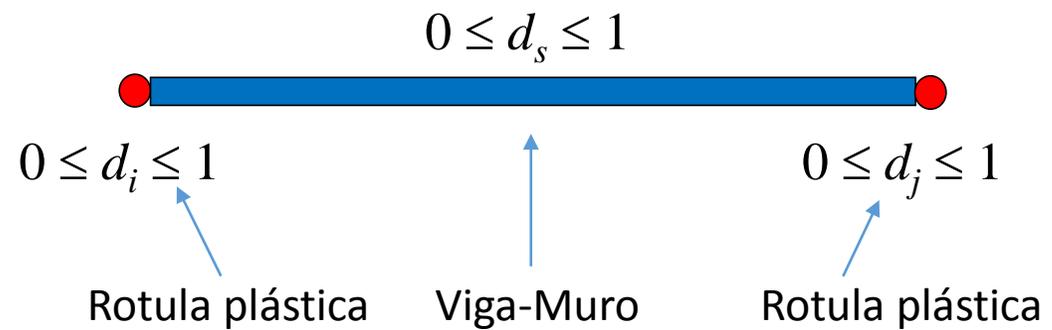


Fisuras de cortante



$\{\mathbf{D}\}=(d_i, d_j)$: Daño por flexión

d_s : Daño por cortante



Ley de comportamiento

Ley de elasticidad

Criterios de Griffith generalizados para la descripción de la evolución del daño de flexión

Criterio de Griffith generalizado para a descripción de la evolución del daño por cortante

Funciones de fluencia para la plasticidad de flexión (fluencia del refuerzo longitudinal)

Función de fluencia para la plasticidad por cortante (fluencia del refuerzo transversal)

Hipótesis de equivalencia en deformaciones

Deformaciones por agrietamiento de flexión

Deformaciones por fluencia del refuerzo longitudinal

$$\{\boldsymbol{\varepsilon}\} = \{\boldsymbol{\varepsilon}^e\} + \{\boldsymbol{\varepsilon}^d\} + \{\boldsymbol{\gamma}^d\} + \{\boldsymbol{\varepsilon}^p\} + \{\boldsymbol{\gamma}^p\}$$

Deformaciones elásticas

Deformaciones por agrietamiento de tracción diagonal

Deformaciones por fluencia del refuerzo transversal



Hipótesis de equivalencia en deformaciones

Deformaciones por agrietamiento de flexión

Deformaciones por fluencia del refuerzo longitudinal

$$\{\boldsymbol{\varepsilon}\} = \{\boldsymbol{\varepsilon}^e\} + \{\boldsymbol{\varepsilon}^d\} + \{\boldsymbol{\gamma}^d\} + \{\boldsymbol{\varepsilon}^p\} + \{\boldsymbol{\gamma}^p\}$$

Deformaciones elásticas

Deformaciones por agrietamiento de tracción diagonal

Deformaciones por fluencia del refuerzo transversal



Deformaciones elásticas

$$\{\boldsymbol{\varepsilon}^e\} = [\mathbf{F}_o] \cdot \{\boldsymbol{\sigma}\}$$

$$[\mathbf{F}_o] = \begin{bmatrix} \frac{L}{3EI} & -\frac{L}{6EI} & 0 \\ -\frac{L}{6EI} & \frac{L}{3EI} & 0 \\ 0 & 0 & \frac{L}{EA} \end{bmatrix} + \begin{bmatrix} \frac{1}{GAL} & \frac{1}{GAL} & 0 \\ \frac{1}{GAL} & \frac{1}{GAL} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Matriz de flexibilidad
elástica de la Teoría de
Euler-Bernoulli

Matriz de flexibilidad
elástica adicional de la
Teoría de Timoshenko

Hipótesis de equivalencia en deformaciones

Deformaciones por agrietamiento de flexión

Deformaciones por fluencia del refuerzo longitudinal

$$\{\boldsymbol{\varepsilon}\} = \{\boldsymbol{\varepsilon}^e\} + \{\boldsymbol{\varepsilon}^d\} + \{\boldsymbol{\gamma}^d\} + \{\boldsymbol{\varepsilon}^p\} + \{\boldsymbol{\gamma}^p\}$$

Deformaciones elásticas

Deformaciones por agrietamiento de tracción diagonal

Deformaciones por fluencia del refuerzo transversal

The diagram illustrates the decomposition of total strain into five components. The equation is $\{\boldsymbol{\varepsilon}\} = \{\boldsymbol{\varepsilon}^e\} + \{\boldsymbol{\varepsilon}^d\} + \{\boldsymbol{\gamma}^d\} + \{\boldsymbol{\varepsilon}^p\} + \{\boldsymbol{\gamma}^p\}$. Red arrows point from the following text labels to the corresponding terms in the equation: 'Deformaciones elásticas' to $\{\boldsymbol{\varepsilon}^e\}$, 'Deformaciones por agrietamiento de flexión' to $\{\boldsymbol{\varepsilon}^d\}$, 'Deformaciones por agrietamiento de tracción diagonal' to $\{\boldsymbol{\gamma}^d\}$, 'Deformaciones por fluencia del refuerzo longitudinal' to $\{\boldsymbol{\varepsilon}^p\}$, and 'Deformaciones por fluencia del refuerzo transversal' to $\{\boldsymbol{\gamma}^p\}$.

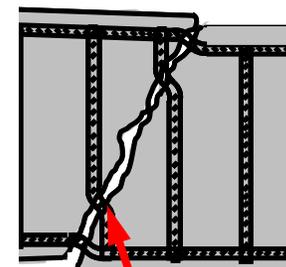
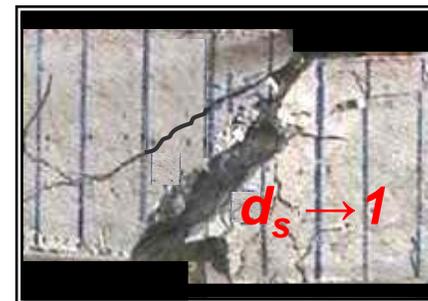
Deformaciones por agrietamiento de tracción diagonal

$$\{\gamma^d\} = [\mathbf{C}_s(\mathbf{d}_s)]\{\mathbf{M}\}$$

$$[\mathbf{C}_s(d_s)] = \begin{bmatrix} \frac{d_s}{GAL(1-d_s)} & \frac{d_s}{GAL(1-d_s)} & 0 \\ \frac{d_s}{GAL(1-d_s)} & \frac{d_s}{GAL(1-d_s)} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Matriz de flexibilidad adicional por agrietamiento de tracción diagonal

Agrietamiento por cortante



ϕ_s^d

Ley de elasticidad

$$\{\Phi\} = \{\Phi^e\} + \{\Phi^d\} + \{\gamma^d\} + \{\Phi^p\} + \{\gamma^p\}$$



$$\{\Phi - \Phi^p - \gamma^p\} = ([F(D, d_s)]) \{\mathbf{M}\}$$

$$[F(D, d_s)] = [F_o] + [C(D)] + [C_s(d_s)]$$

Matriz de flexibilidad
incluyendo daños por
flexión y cortante

$$[F(D, d_s)] = \begin{bmatrix} \frac{L}{3EI(1-di)} + \frac{1}{GAL(1-ds)} & -\frac{L}{6EI} + \frac{1}{GAL(1-ds)} & 0 \\ -\frac{L}{6EI} + \frac{1}{GAL(1-ds)} & \frac{L}{3EI(1-dj)} + \frac{1}{GAL(1-ds)} & 0 \\ 0 & 0 & \frac{L}{EA} \end{bmatrix}$$

Leyes de plasticidad

Unidad estructural

Esfuerzo efectivo

Función de fluencia

Teoría del daño continuo:



$$\bar{\sigma} = \frac{\sigma}{1-\omega}$$

$$f = \left| \frac{\sigma}{1-\omega} \right| - \sigma_y$$

Rotaciones plásticas:



$$\bar{m}_i = \frac{m_i}{1-d_i}$$

$$f_i = \left| \frac{m_i}{1-d_i} - c\phi_i^p \right| - M_u \leq 0$$

Distorsiones plásticas:



$$\bar{v} = \frac{v}{1-d_s} \cong \frac{m_i + m_j}{L(1-d_s)}$$

$$f_s = \left| \frac{m_i + m_j}{L(1-d_s)} - c\gamma^p \right| - V_u \leq 0$$

Leyes de daño

$$W = \frac{1}{2} \{ \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p - \boldsymbol{\gamma}^p \}^t \{ \boldsymbol{\sigma} \} = \frac{1}{2} \{ \boldsymbol{\sigma} \}^t [\mathbf{F}(\mathbf{D}, d_s)] \{ \boldsymbol{\sigma} \}$$

Energía potencial complementaria

$$G_i = \frac{\partial W}{\partial d_i} = \frac{Lm_i^2}{12EI(1-d_i)^2}$$

Tasa de restitución de energía por flexión

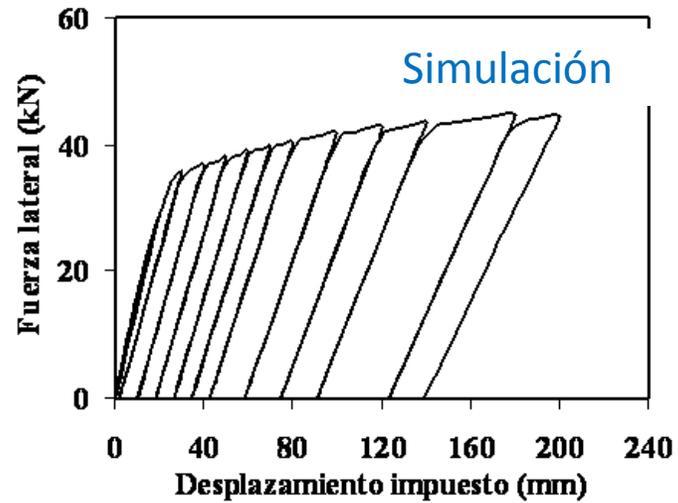
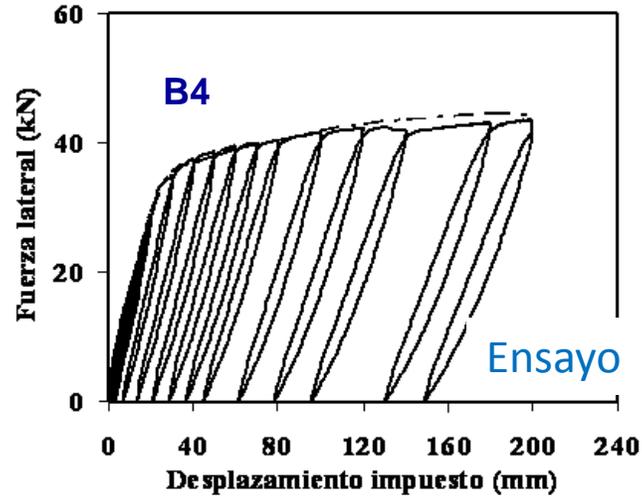
$$G_i = R(d_i)$$

$$G_s = \frac{\partial W}{\partial d_s} = \frac{Lv^2}{2GA(1-d_s)^2}$$

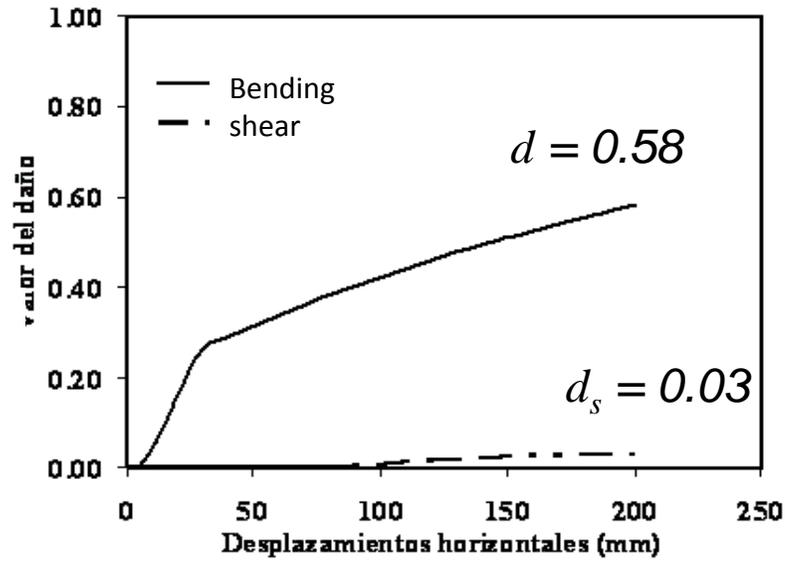
Tasa de restitución de energía por cortante

$$G_s = R(d_s)$$

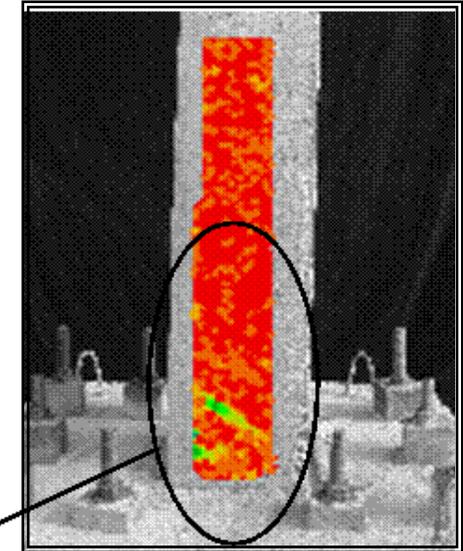
Viga esbelta ($a/d = 7.11$)



Historia de daños

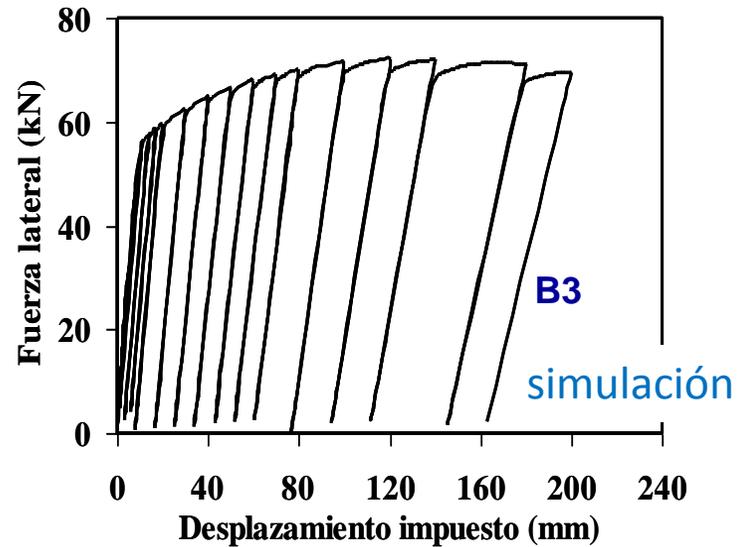
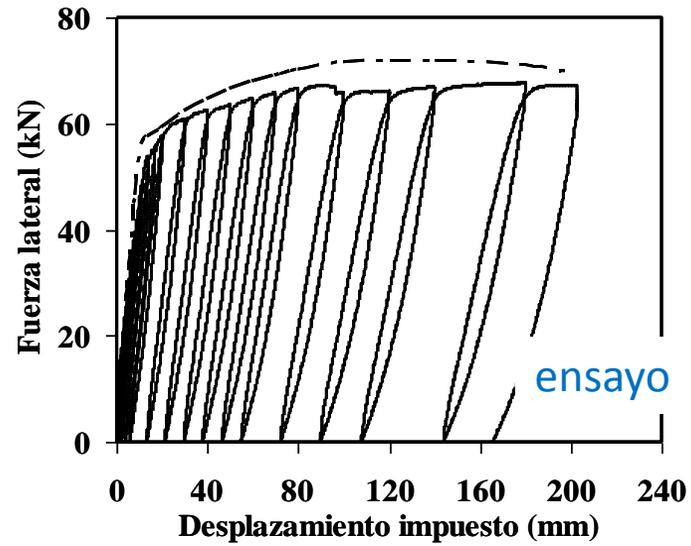


$\Delta = 30$ mm

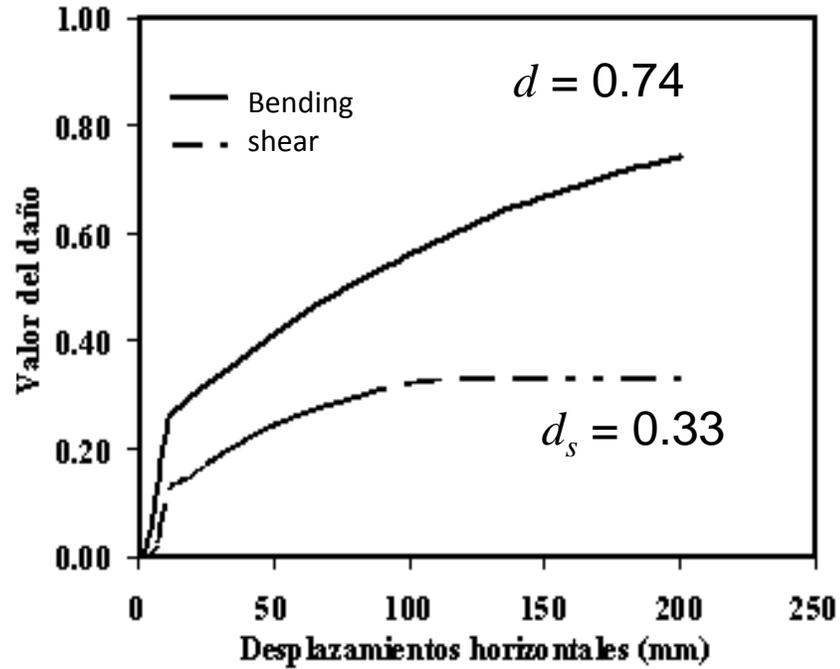


Fisuras de flexión

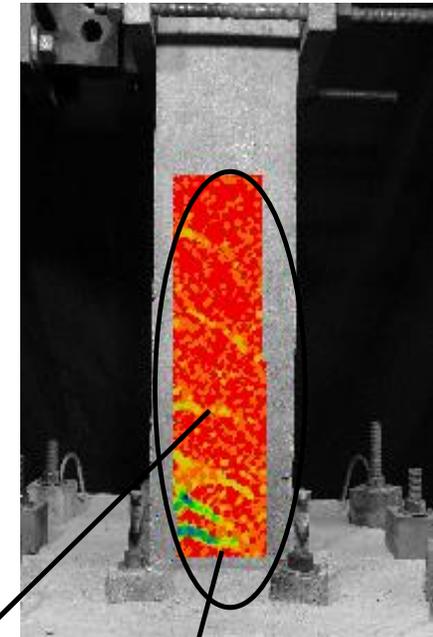
Viga intermedia (a/d = 5.36)



Historia de daños



$\Delta = 50$ mm

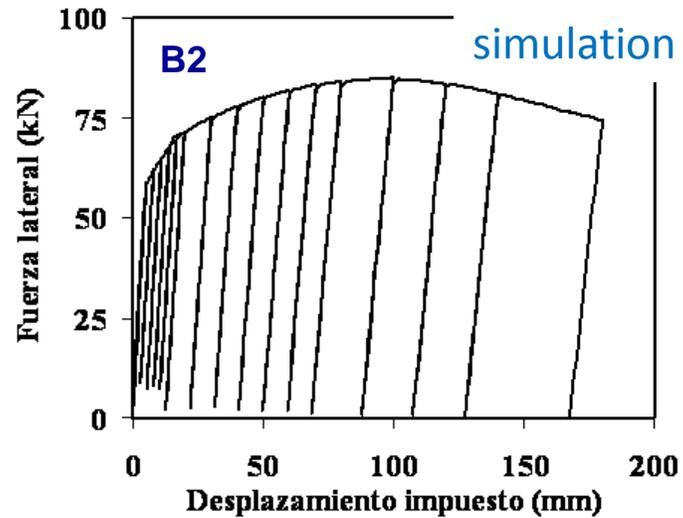
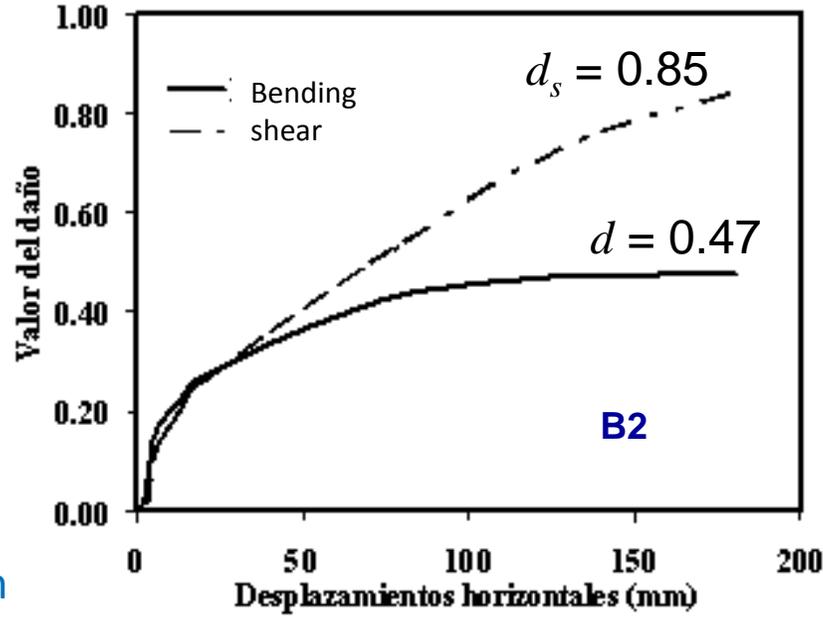
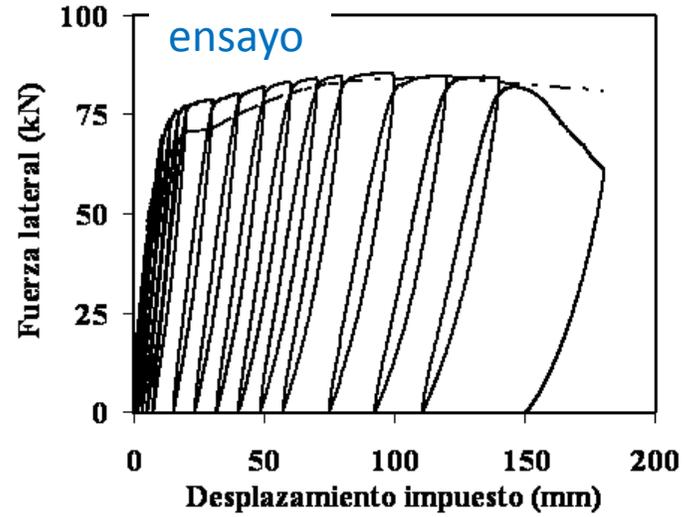


Grietas de corte

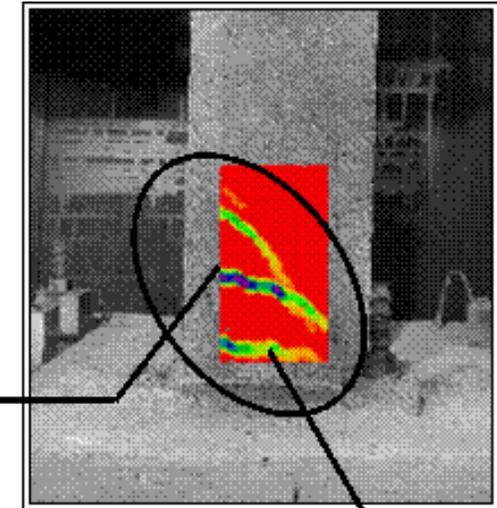
Grietas de flexión

Viga intermedia (a/d = 3.56)

Historia de daños



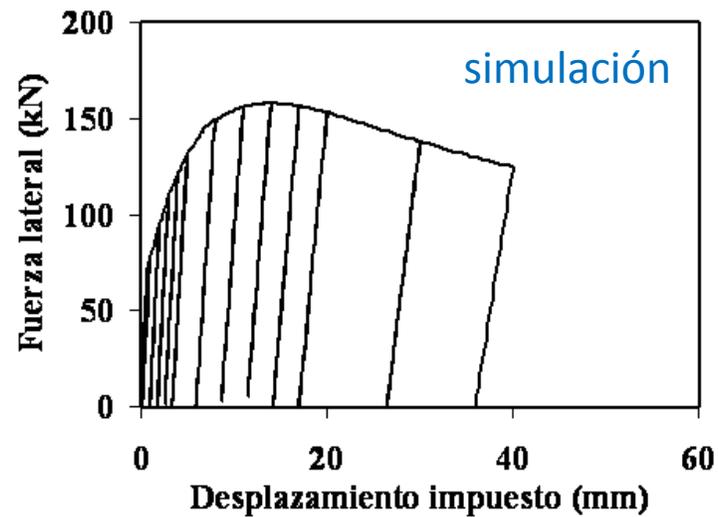
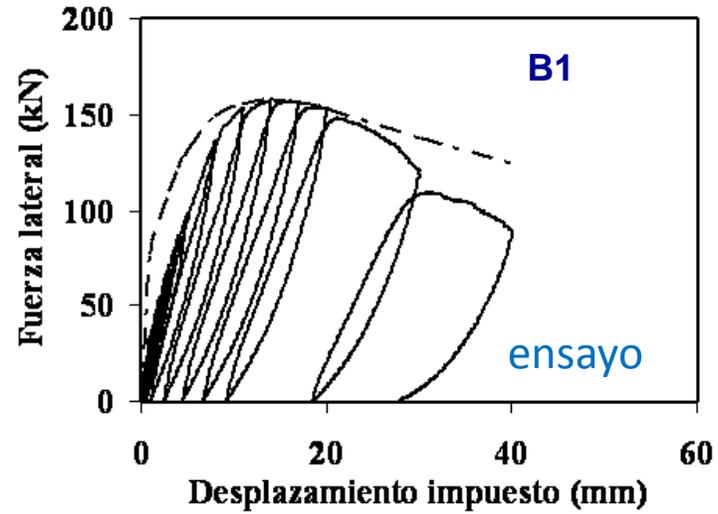
$\Delta = 50$ mm



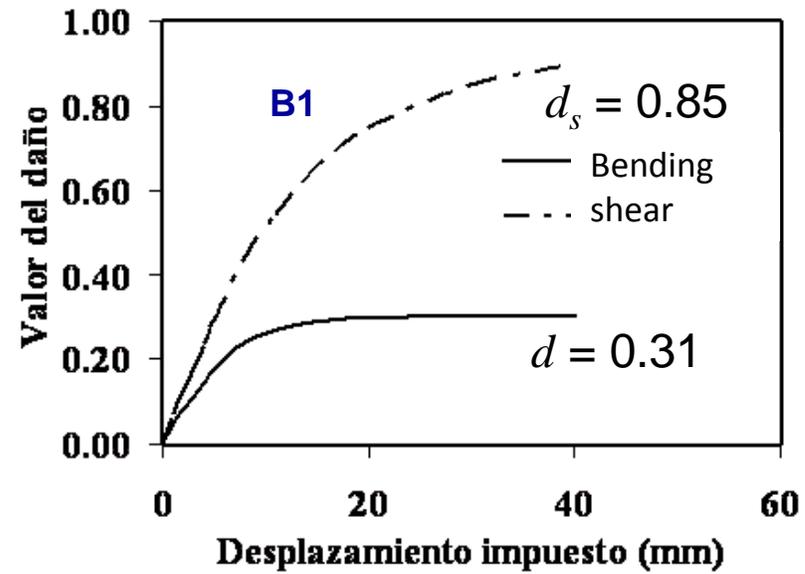
Shear cracks

Bending cracks

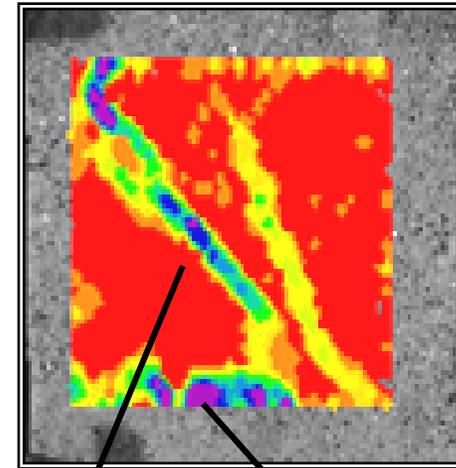
Short beam ($a/d = 1.78$)



Historia de daños

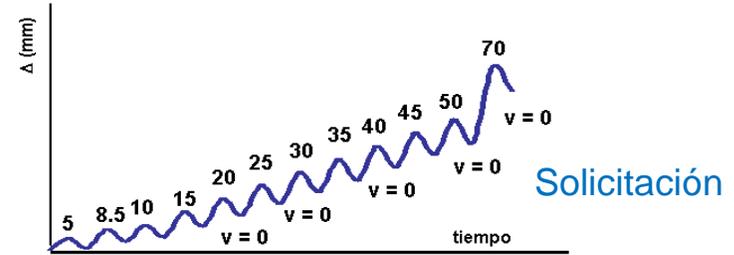
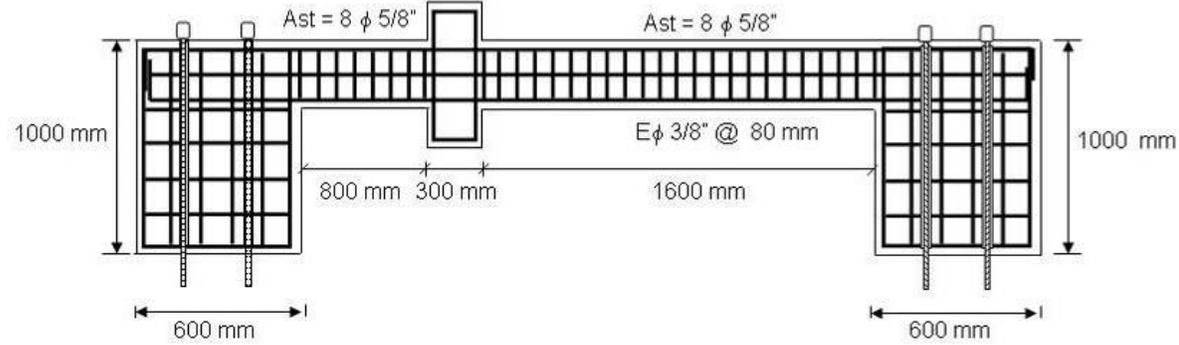


$\Delta = 14$ mm



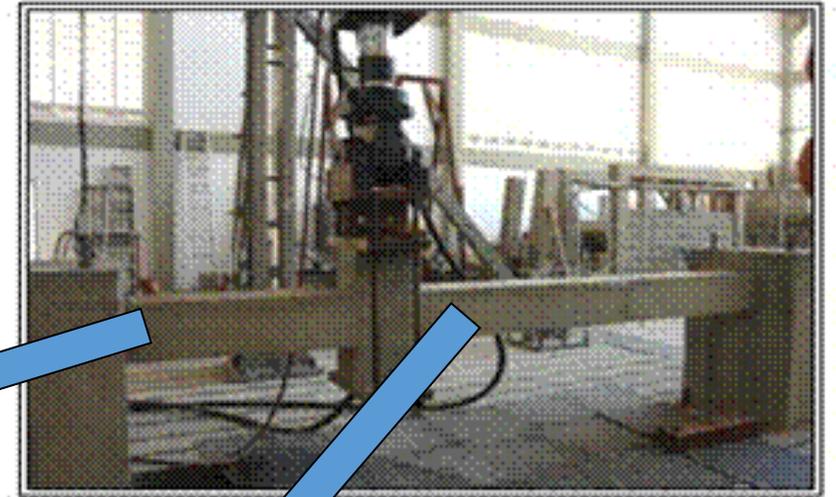
Grietas de corte

Grietas de flexión

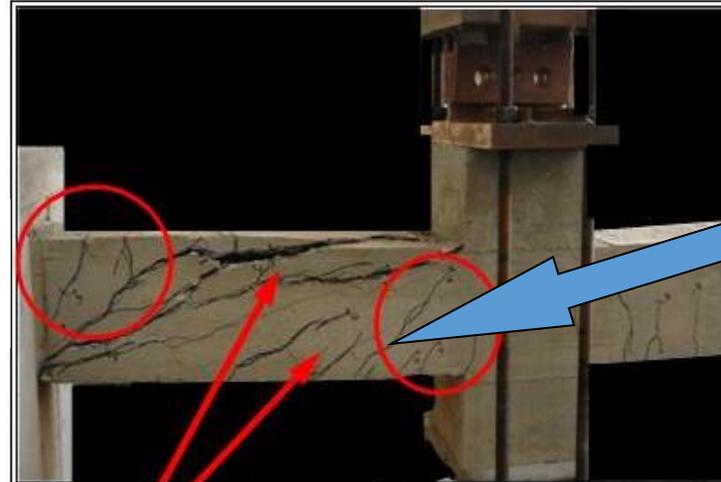


Probeta	w (mm)	t (mm)	a (mm)	d (mm)	a/d	ρ_v %	ρ_s %	R_{cr} (MPa)
BTC	250	250	1750	225	8.22	2.53	1.03	21.4
BTL	250	250	815	225	3.55	2.53	1.03	21.4

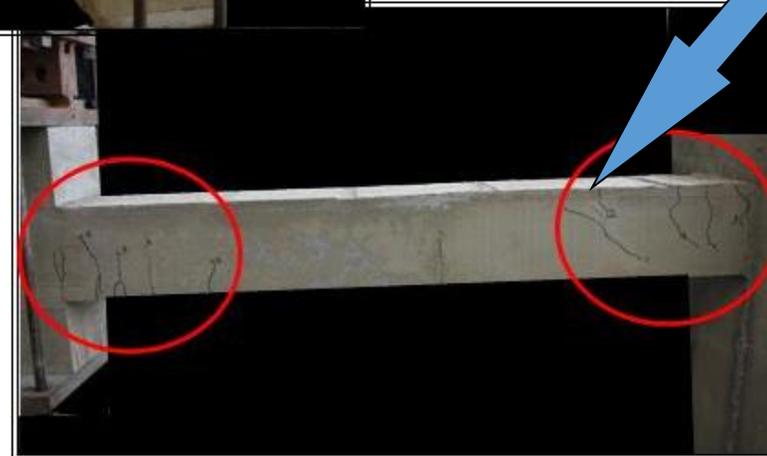
Short beam ($a/d = 3.55$)



Bending cracks



Grietas de corte

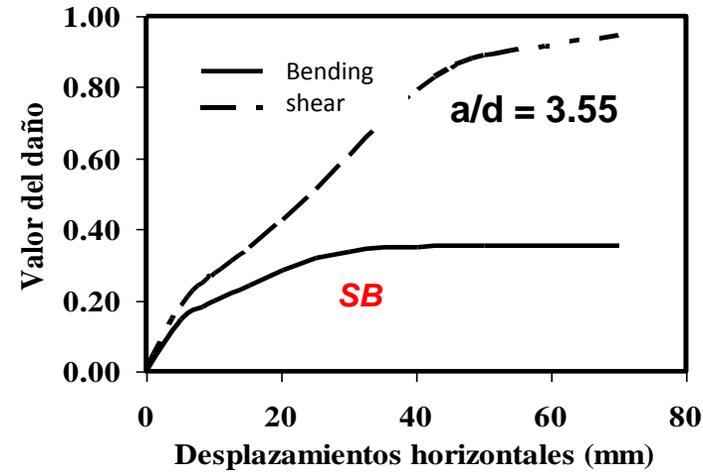
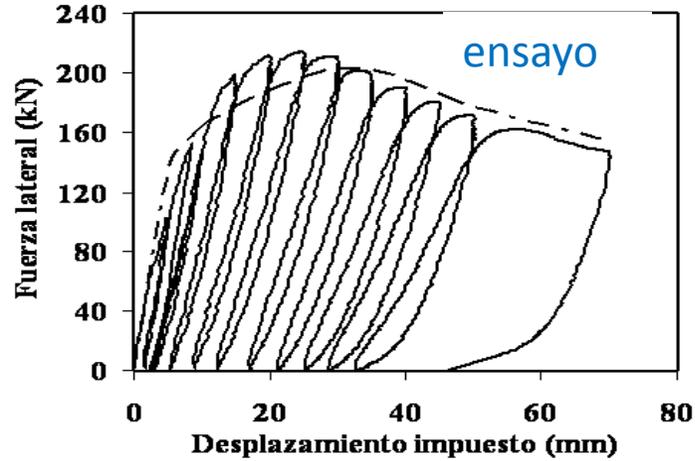


Grietas de flexión

long beam ($a/d = 8.22$)



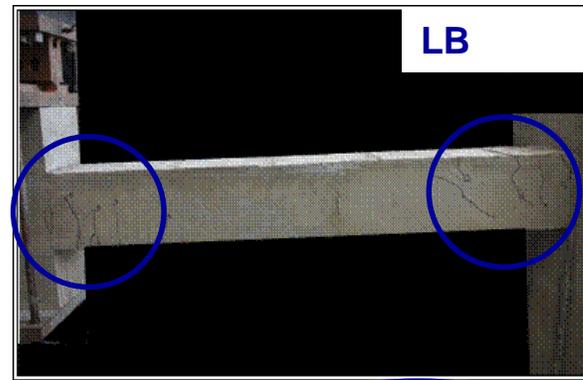
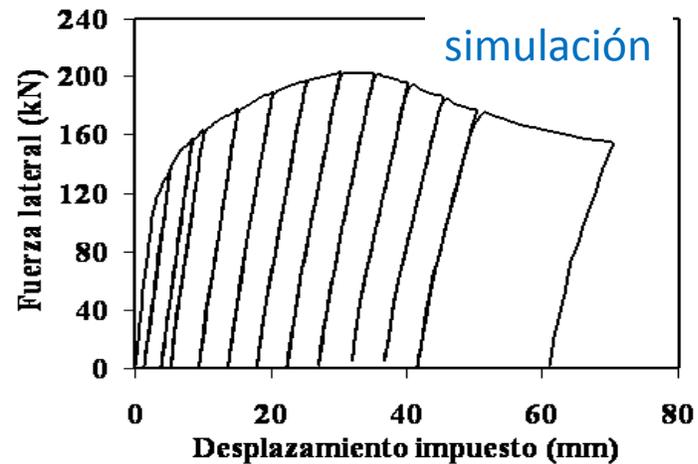
$\Delta = 70 \text{ mm}$



$d_s = 0.94$

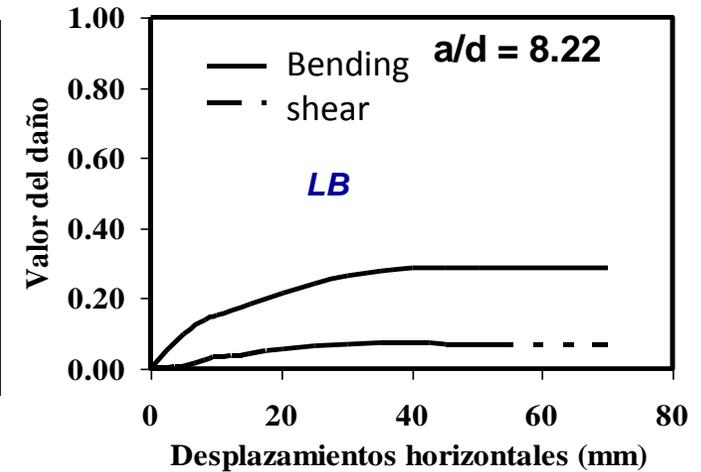
$d = 0.35$

$\Delta = 70 \text{ mm}$

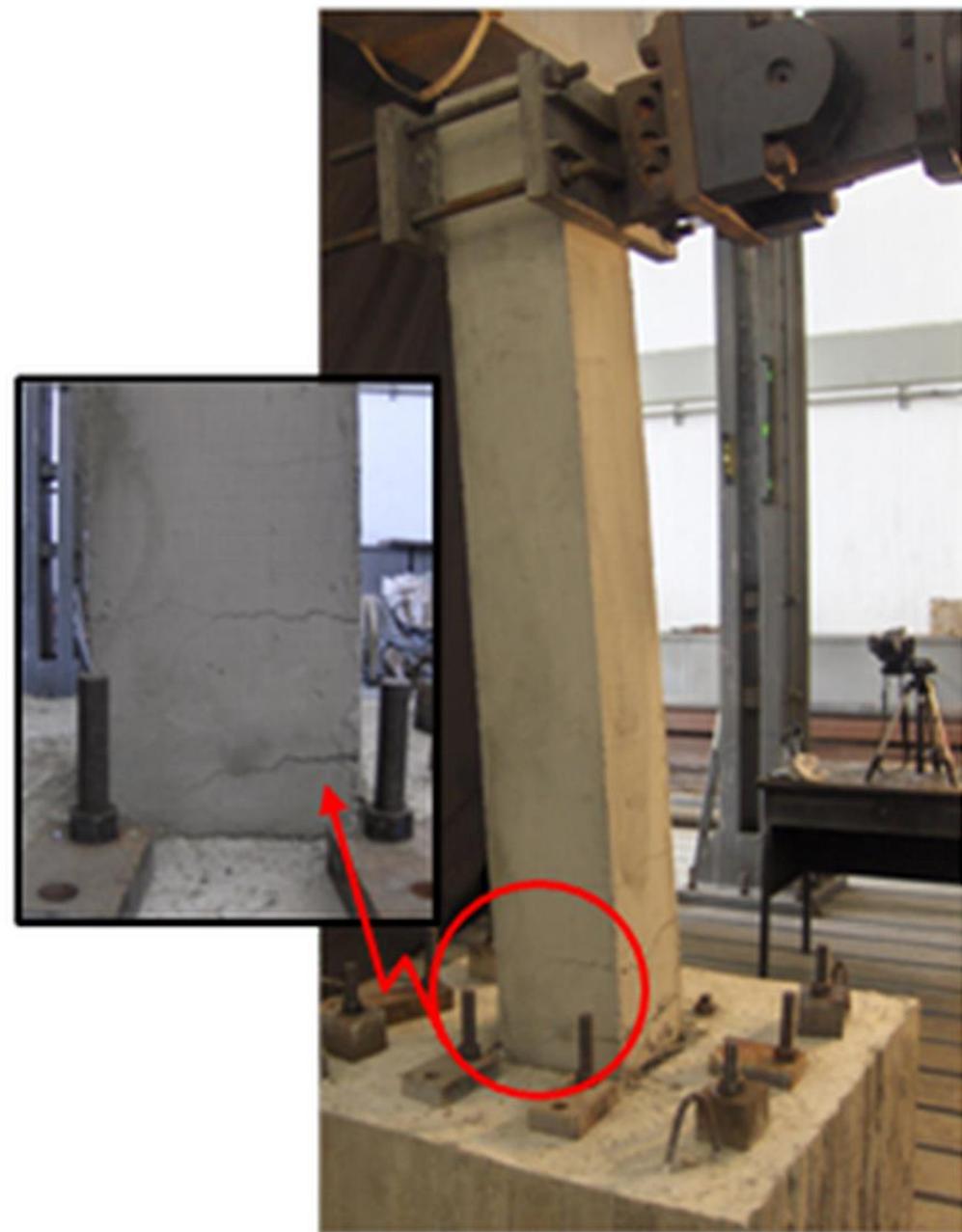
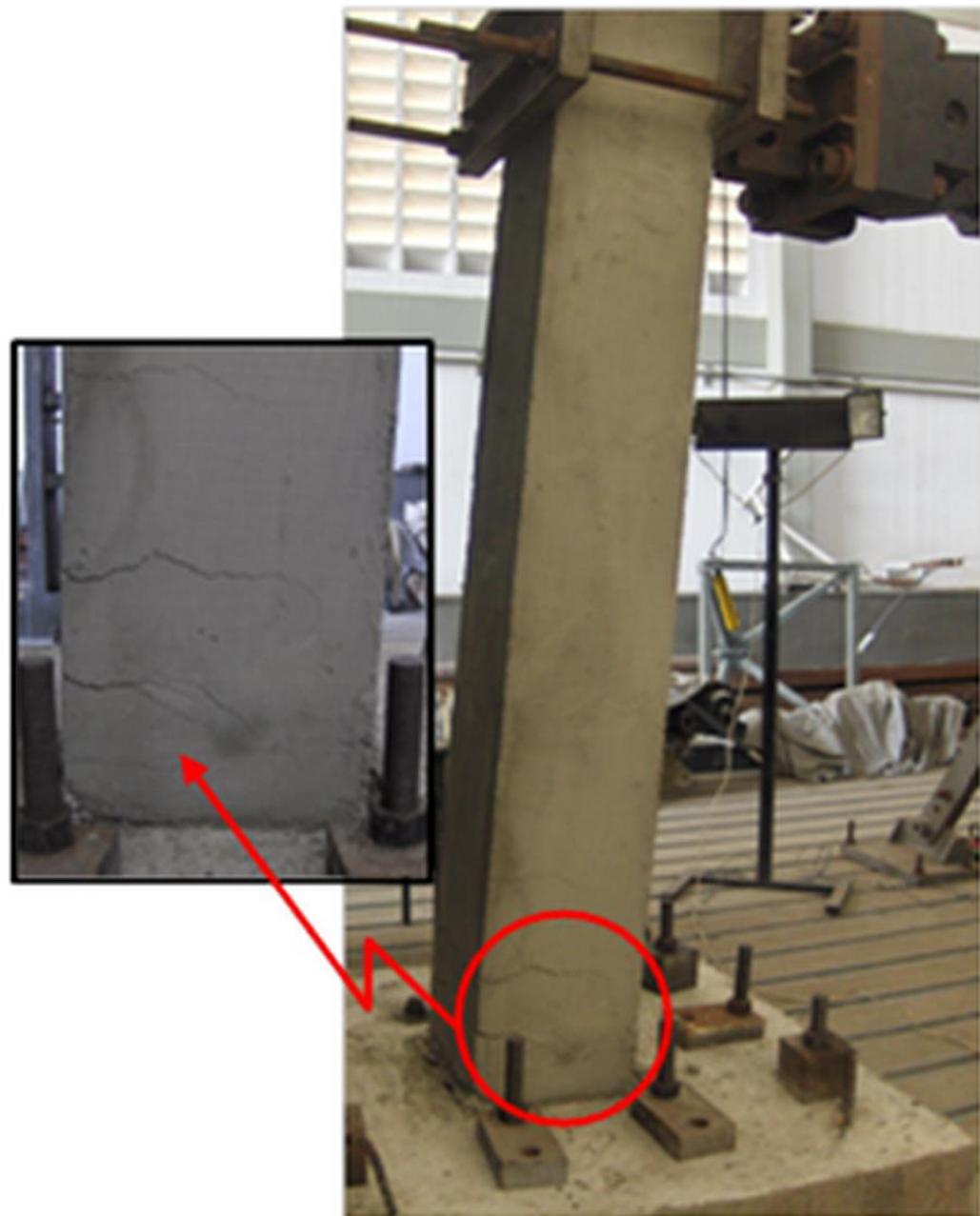


$d_s = 0.07$

$d = 0.27$

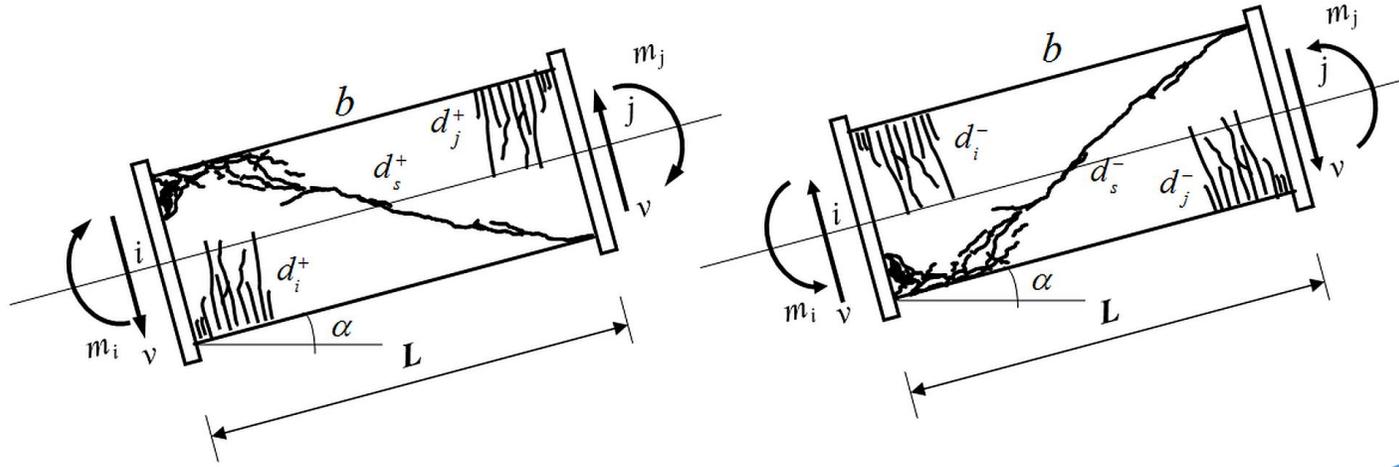


- Conceptos fundamentales de Mecánica da Fractura
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$$(\mathbf{D}^+)_b = (d_i^+, d_j^+)$$

$$d_s^+$$



$$(\mathbf{D}^-)_b = (d_i^-, d_j^-)$$

$$d_s^-$$

Ley de elasticidad con daño unilateral

$$\{\boldsymbol{\varepsilon}\}_b - \{\boldsymbol{\varepsilon}^p\}_b - \{\boldsymbol{\gamma}^p\}_b = [\mathbf{F}_f(\mathbf{D}^+)] \langle \boldsymbol{\sigma} \rangle_b^+ + [\mathbf{F}_f(\mathbf{D}^-)] \langle \boldsymbol{\sigma} \rangle_b^- + \frac{1}{GA_b(1-d_s^+)} \langle \mathbf{V} \rangle_b^+ + \frac{1}{GA_b(1-d_s^-)} \langle \mathbf{V} \rangle_b^-$$

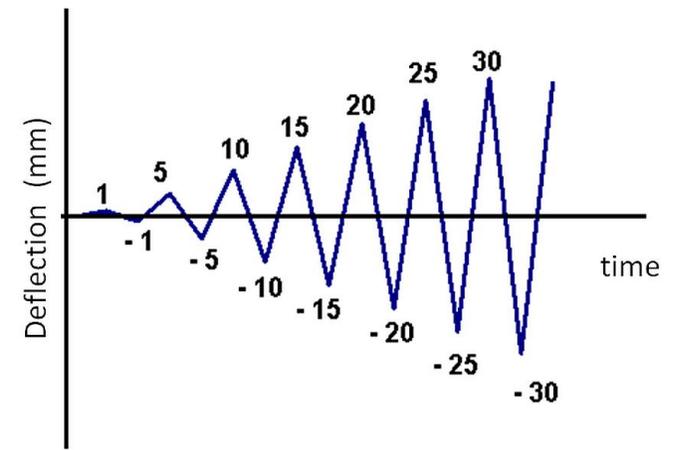
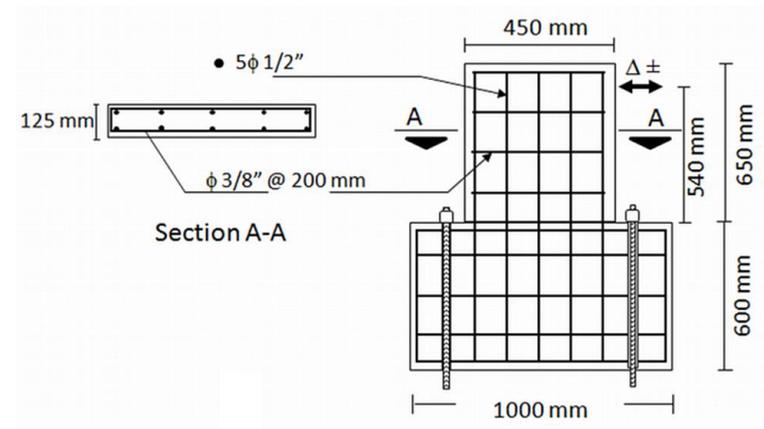
Fuerzas cortantes

$$\langle \mathbf{V} \rangle_b^{+/-} = \begin{bmatrix} \left\langle \frac{m_i + m_j}{L_b} \right\rangle_{+/-} \\ \left\langle \frac{m_i + m_j}{L_b} \right\rangle_{+/-} \\ 0 \end{bmatrix}$$

Función de fluencia (ley de plasticidad)

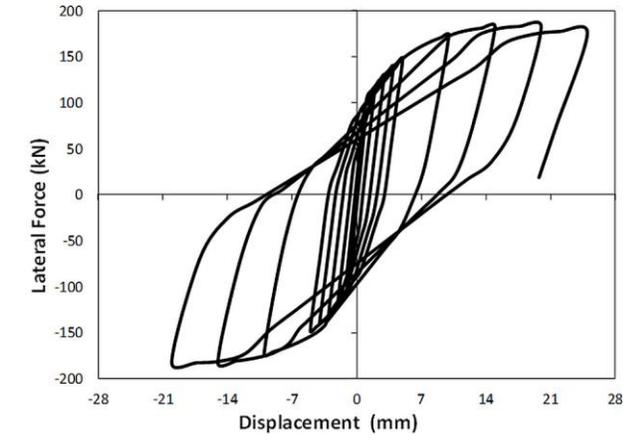
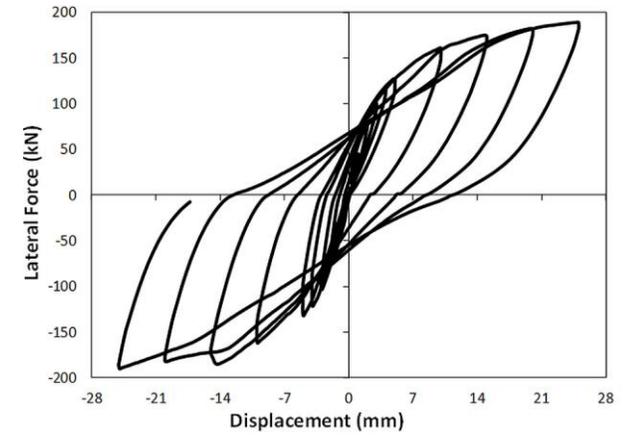
Criterio de Griffith (ley de daño)

$$f_i = \text{Max} \left(\frac{m_i}{1-d_i^+} - c_i^+ \phi_i^p - k_{0i}^+, -\frac{m_i}{1-d_i^-} + c_i^- \phi_i^p - k_{0i}^- \right) \leq 0 \quad \begin{cases} \Delta d_{i/j/s}^{+/-} = 0 & \text{if } G_{i/j/s}^{+/-} < R_{i/j/s}^{+/-} \\ G_{i/j/s}^{+/-} = R_{i/j/s}^{+/-} & \text{if } \Delta d_{i/j/s}^{+/-} > 0 \end{cases}$$



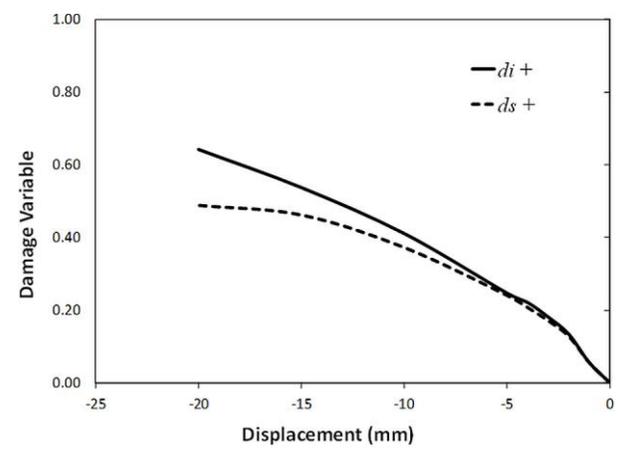
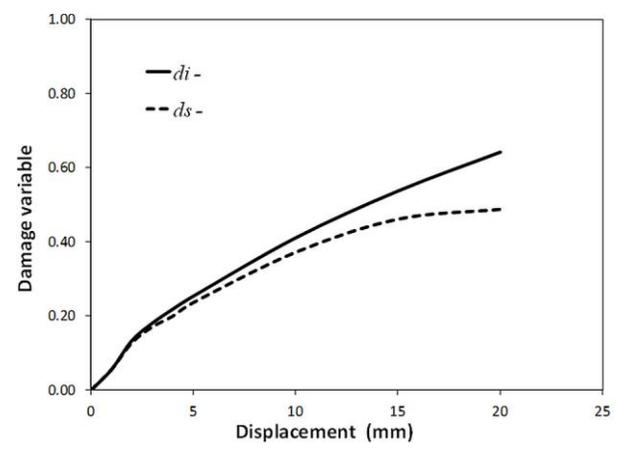
Ensayo de un muro

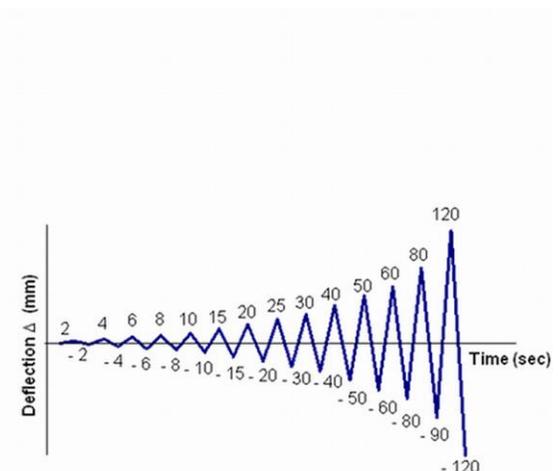
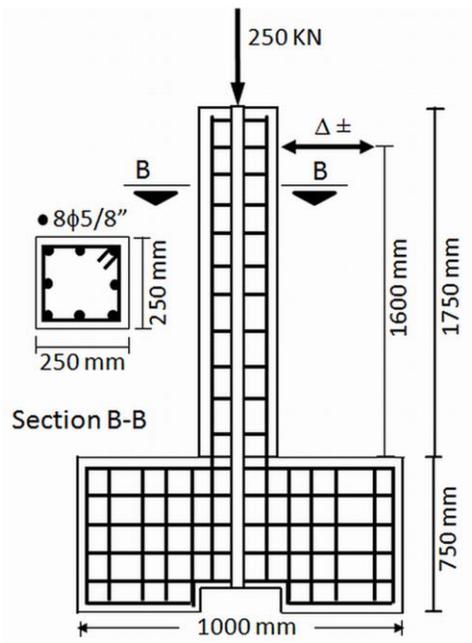
Historias de daño



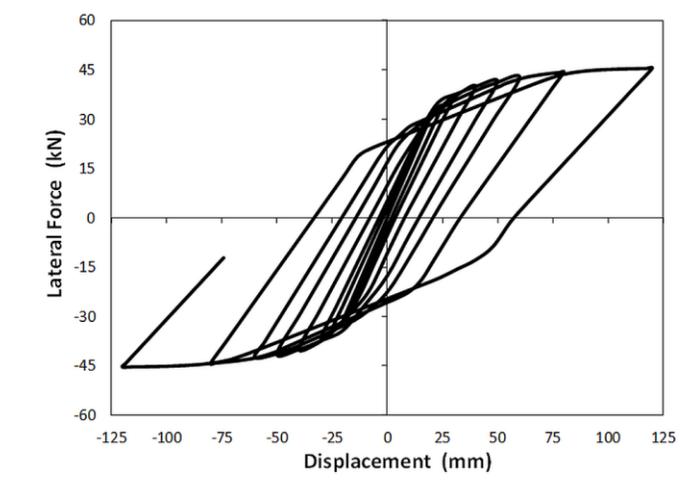
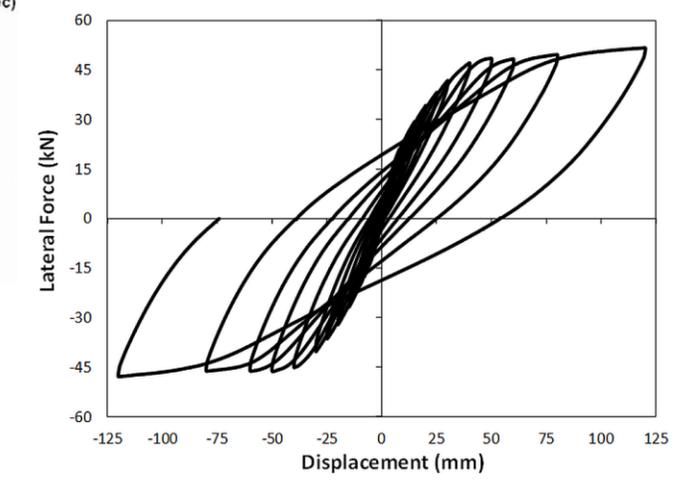
Ensayo

Simulación numérica

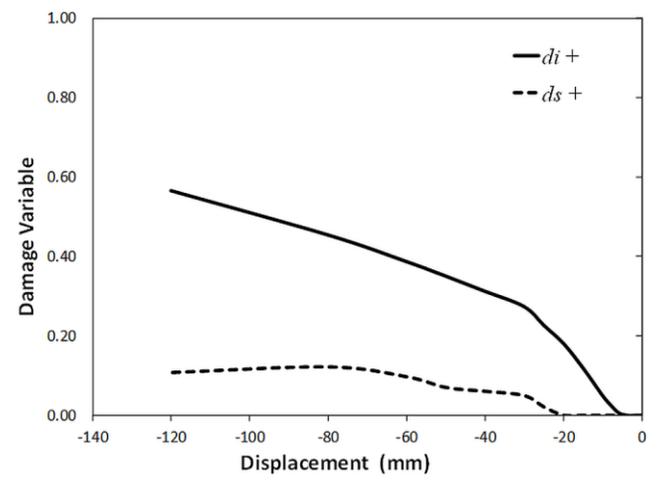
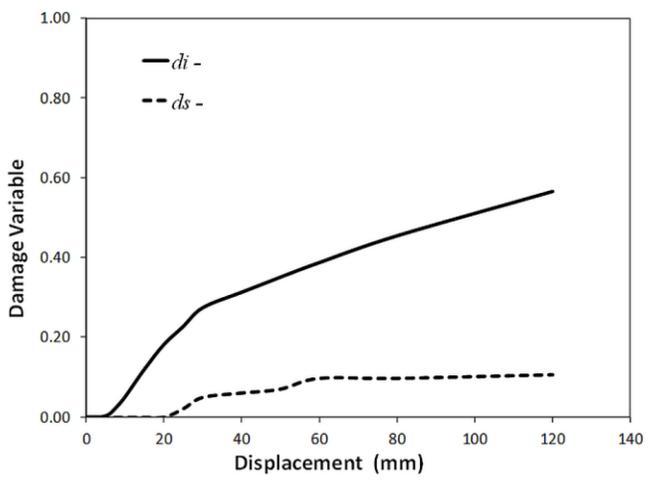




Ensayo en una columna esbelta



Historias de daño

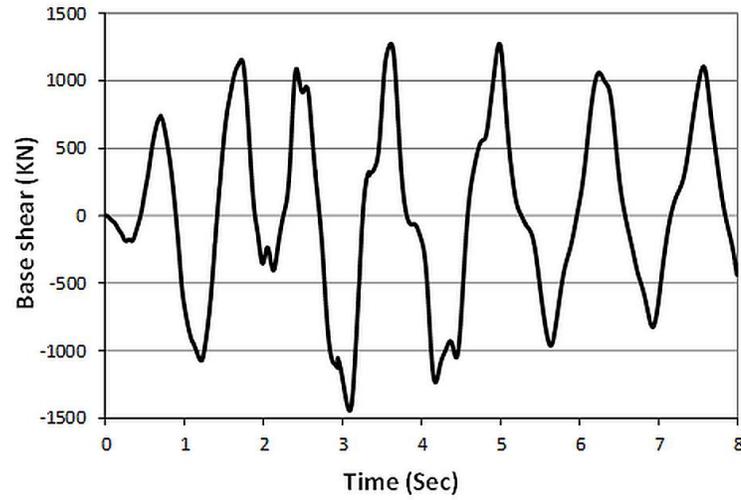


Ensayo

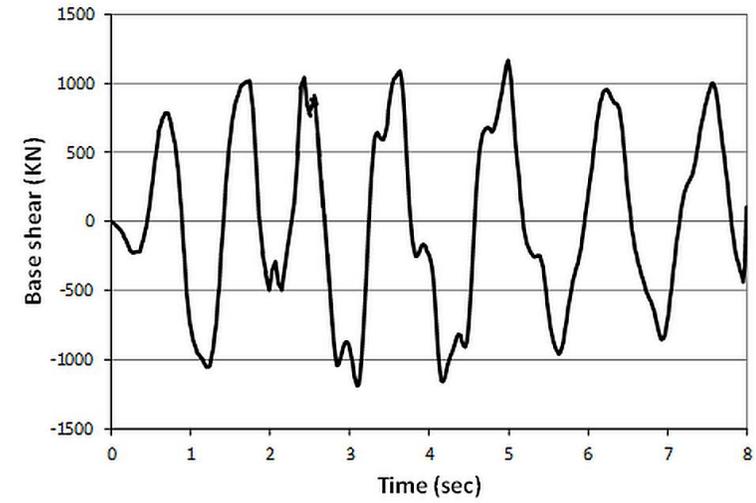
Simulación numérica



Ensayo

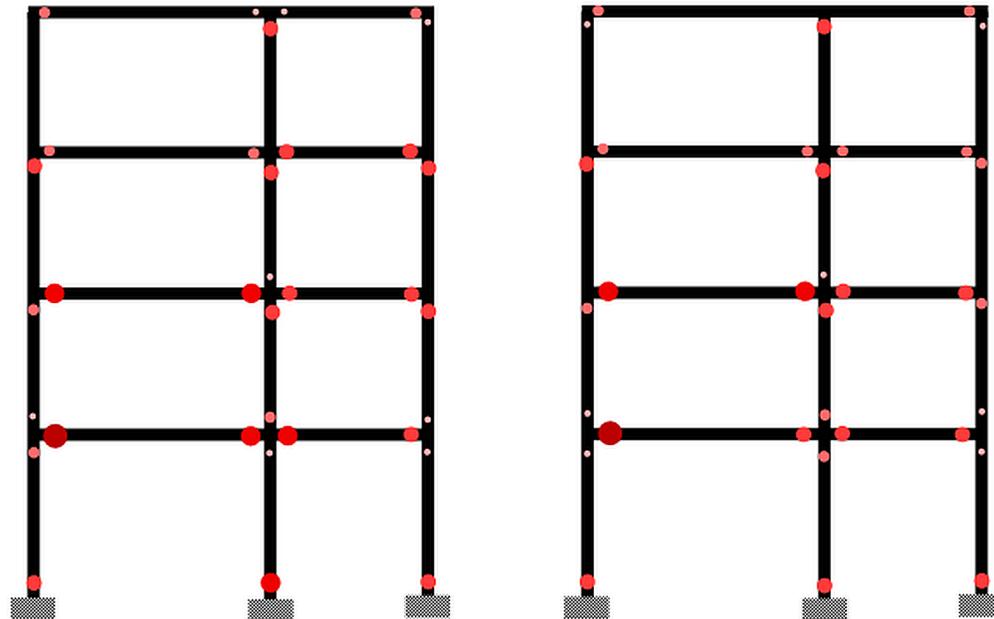


Simulación numérica



Ensayo Pseudodinámico de un pórtico de concreto armado

Pórtico interno

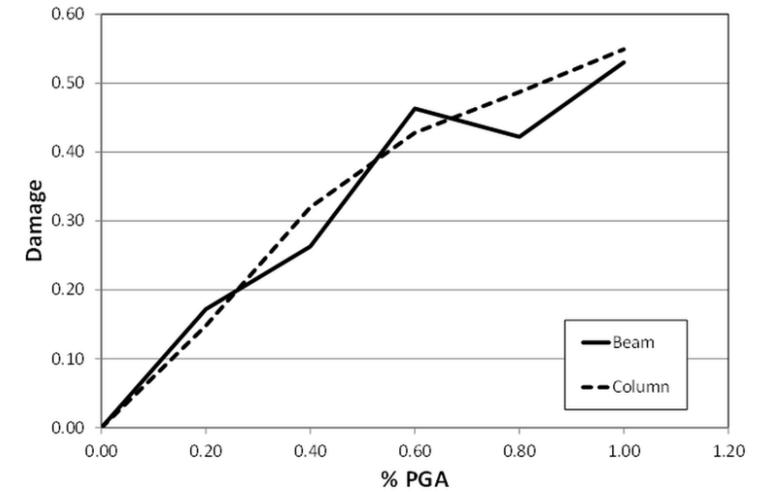
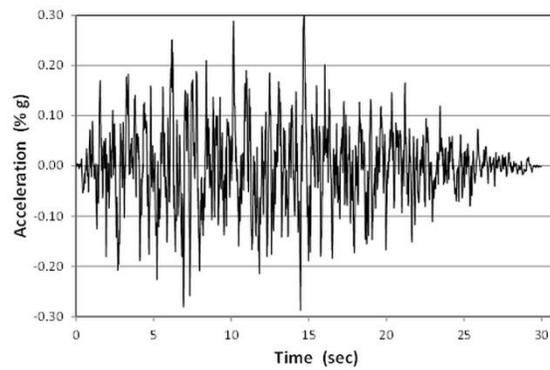
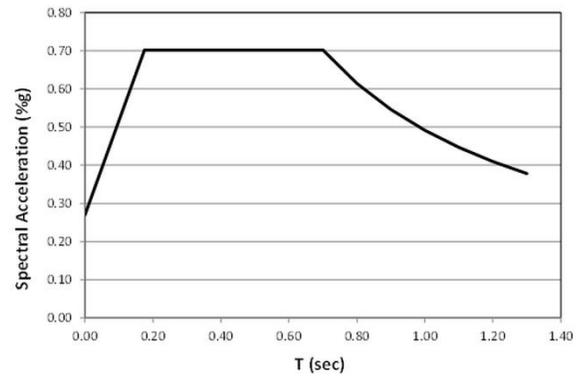
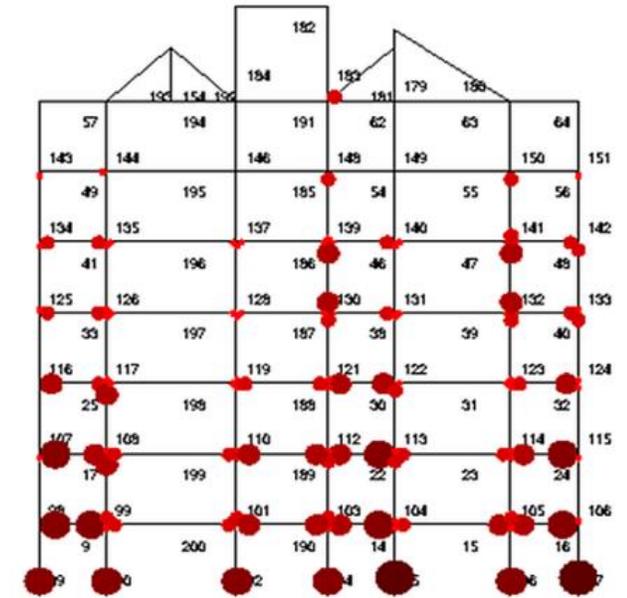
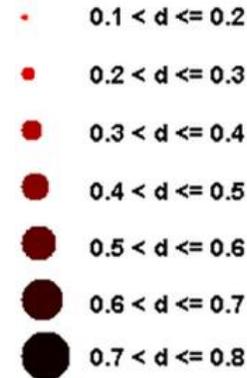
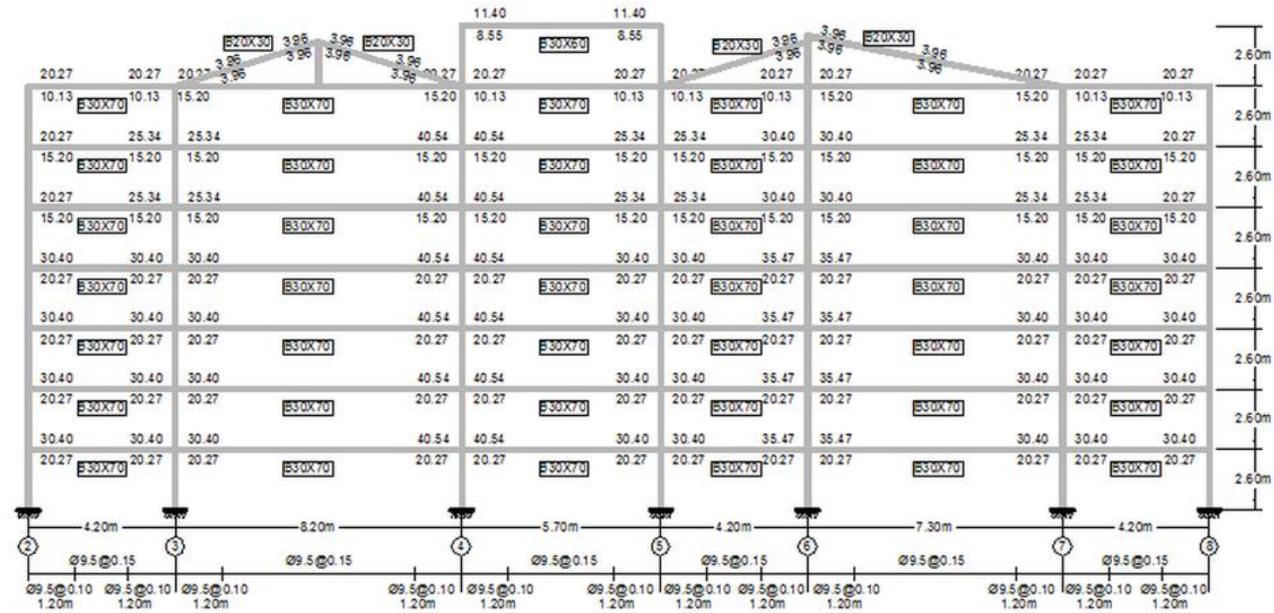


Pórtico externo

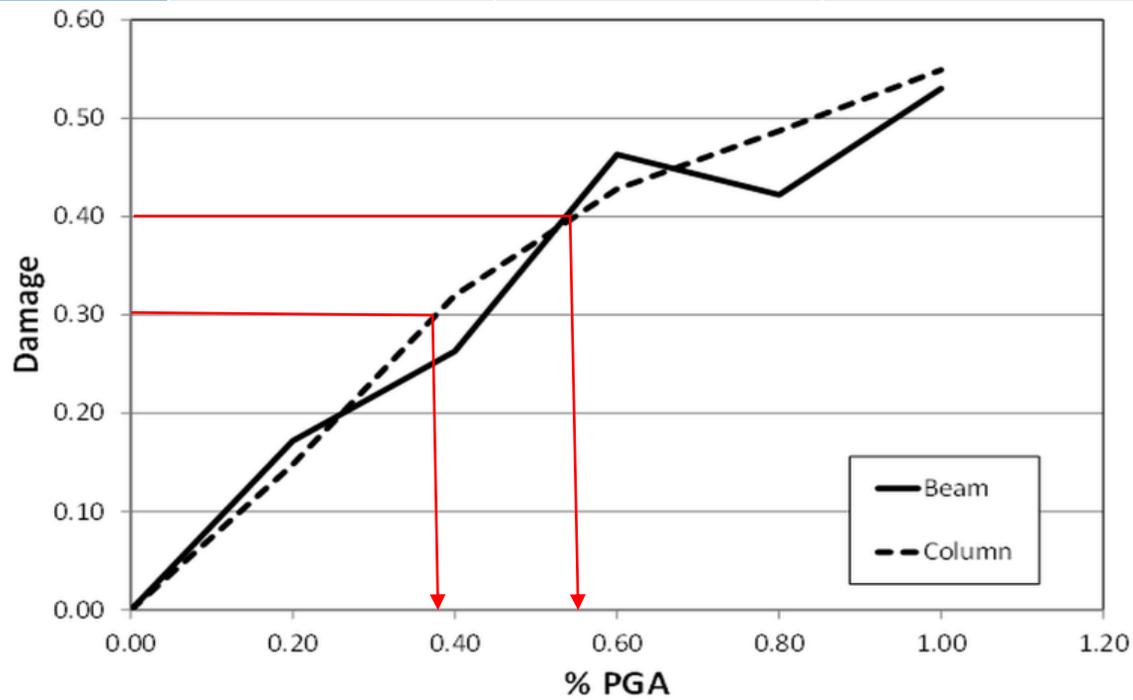
- $0.0 < d \leq 0.2$
- $0.2 < d \leq 0.3$
- $0.3 < d \leq 0.4$
- $0.4 < d \leq 0.5$
- $0.5 < d \leq 0.6$
- $0.6 < d \leq 0.7$
- $d > 0.7$

Mapa de daño

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Performance level	Max expected damage in Beams	Max expected damage in Columns	Description
1	$0.30 \approx d_p$	0.10	The elements do not require any intervention after the event.
2	0.40	$0.30 \approx d_p$	Some minor repairs may be needed.
3	0.50	0.40	The element requires reparation at reasonably costs.
4	$0.6 \approx d_u$	0.5	The structure requires a major rehabilitation process.
5	> 0.7	$> 0.6 \approx d_u$	Inacceptable structural behavior.



Dynamic Capacity Curve

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