Changes in the education systems after COVID-19: A game theorerical approach.*

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Abstract

In this paper we study, from a game theoretical point of view, the repercussions of COVID-19 in the education. For this purpose, we will build a dynamic evolutionary model based on replicator dynamics, to describe the evolution of the teaching-learning strategies followed by professors and students since the beginning of the COVID-19 pandemic. In particular, we focus on studying the transition from offline education to education based on the use of information technologies as a consequence of the global health crisis. In addition, we show that possibly based on the different economic realities experienced in each country, teaching-learning methodologies can evolve towards methodologies completely based on traditional offline education or, on the contrary, towards teaching-learning methodologies that favor the use of so-called technologies of the information.

1 Introduction

The COVID-19 pandemic was a period of sanitary emergency that has had a significant impact on education system, as a consequence more than 1.3 billion learners are affected by school closures [1]. Students were ordered to remain at home, forcing the education sector to develop and create new educational techniques for to migrate from face-to-face learning to online education through the use of digital technology (first stage).

Each country implemented different teaching methods according their possibilities, in Mexico, for example, the television was the medium principal for the basic education and radio was used as informative communication medium, while in the university dominated the digital platforms as Microsoft Teams, Zoom, Google Meet, among others. These were abrupt changes owing education and technology digital were not highly related in many countries. Although in some countries, such as England, where the Higher Education Funding Council for England (HEFCE) was founded in 2005 integrating e-learning (explain what it is) into higher education in some British schools, or in the Netherlands where in 1987 the European Association of Distance Teaching Universities for distance education in Europe was founded [13], the education that included the use of technology in its teaching process has been a constant for many years, this was not the case, until before the pandemic caused by COVID-19, in many developing countries ([14], [7]) and even in many developed countries, since among the main factors that prevented (and still continue to prevent in many cases) the use of ICTs in the teaching-learning process in many countries (including develop and developing countries) are the lack of investment on the part of the educational authorities to provide technological infrastructure to all schools (high-speed internet access, computer equipment in the classrooms, digitizing tablets to teach the class, projection guns, etc.), as well as the lack of training for teachers in

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handling these technologies, this on the side of the educational system. However, on the side of the students, social, economic and geographical location factors within their own country or state have made it impossible to implement education based on the use of technology over the years, this was evidenced as a result of this pandemic, since with the beginning of this the number of student dropouts also increased, as a consequence, in many cases, of the lack of computer equipment or access to a good internet, since sometimes the computer equipment had to sharing among the different members of the family at the same time, which made it impossible for some of them to take classes or complete tasks, and even if the number of computers was sufficient for the family members, connectivity problems, difficult to solve due to lack of economic resources or the fact of living in places where high-speed internet does not reach (distributed by fiber optics, for example), were present. So online teaching and learning caused an educational gap during the time of the pandemic caused by COVID-19 (see [18]).

When sanitary conditions improved, hybrid education was implemented (second stage), this is a combined face-to-face teaching with online education, some time after the start of the pandemic, some countries opted for hybrid teaching models while the rest continued with their distance education [20], [22], [24], [26]. This model consisted of in which some students, through reservations or direct assignment, had to attend classes at their school (in person), while the rest took classes remotely, for which the teacher, through the use of some digital platform, had to attend and try to impart the content of their subject to a sectioned group (those who were face-to-face and those who did it remotely), this brought with it numerous challenges that both public and private institutions had to attend to and try to solve in the best way, since they had to develop contingency plans for the safe return of students to classrooms at the same time that the classrooms for the use of technology to teach a class and the teacher was trained for this objective, See for instance [6], [17] and [30].

In a third stage, it was decided to return to classes in a completely face-to-face mode and this brought with it a new challenge for education, since not only did the possibility of a new wave of infections have to be faced, but also the teaching conditions. Learning had changed since the start of the pandemic, that is, the time had come for the so-called "new normality", now the students were accustomed to the teaching methodology that included both the use of traditional teaching techniques, the use of blackboard and markers, such as the use of digital platforms (Blackboard, Moodle, Teams, Zoom, YouTube, etc.) to solve their academic doubts, to a large extent these platforms to obtain information and review didactic materials had replaced the advice and tutoring of teachers [9], [13]. On the other hand, teachers now had the ability to carry out their teaching work both traditionally and digitally. This brings with it a great dilemma for which we are still far from seeing an answer: return to the teaching-learning techniques prior to the pandemic, use exclusively the techniques learned during this time, or use a combination of both?.

Although the use of ICTs (Information and Communications Technology) seems to be the point of convergence of the teaching methodology in developed countries and the natural step to be followed by developing countries [4], [23], this could be difficult to carry out in these countries because in most of these the internet does not reach all corners of their territory, so the use of these tools may not be desired by those students with unstable internet connections or without internet access, on the other hand of the teachers, perhaps with the return to their normal activities, having high-speed internet is now unnecessary, so they could decide to opt for cheaper connection packages, which favors their economy, which consequently could bring with it a decrease in the use of technology for their teaching work, thus causing a return to traditional teaching techniques.

Due to the fact that the investment made in technological infrastructure was enormous in most of the countries, it is to be expected that the educational authorities seek, on the one hand, that these be implemented in the day-to-day teaching work and, on the other hand, to measure the level of knowledge acquired, on the management of these, by their teaching staff, since in many cases it was necessary to offer, at cost 0, training for teachers in the use of ICTs, so that an indirect way of measuring the use of ICTs in class and the knowledge in their management by teachers is through student evaluations of teaching

Actually, many investigations about the advantage and disadvantages of online education have been analyze, the principal benefit of online class is the saving travel time [34], and big disadvantage is the poor verbal communication between teacher-student. Recently, due the vaccination process vs Covid 19, the students returned to school and then the interrogation was how will the adaptation process be post-covid?, This paper investigates the perception pre and post COVID 19 epidemic using an evolutionary game theory xXXXX resultadosXXX.

2 The model

Following the models, in different contexts, propose in [1], [2] and [3], in this paper we propose a model of evolutionary game theory, based on the replicator dynamics, to explain the evolution of the teaching-learning methodologies followed by teachers and students in a post-pandemic educational environment. For this purpose we consider a game theoretical model where two populations exists *Professors* and *Students* each one having two different strategies to be followed:

Professors: *Professors* who, upon returning to the classroom in person with students in the classroom, are faced with the decision to resume the traditional teaching method consisting of the use of the old blackboards to teach classes, leaving aside the use of technology (digitizing tablets and projectors instead of the blackboard, computer equipment to present programs and/or graphics instead of doing them by hand, etc.) made available to teachers by the authorities and educational establishments during the hybrid teaching period that was developed as a test measure for the return to classrooms. We will say that professors who return to traditional teaching, used before the pandemic in many countries, without the use of ICTs employs traditional pre-COVID or offline teaching techniques and we will denote it by O_T, while we will say that professors who have upgraded and now use teaching methods that include more and more the use of *information and communication technologies* (ICTs) as a method to receive and review assigned tasks or as a means to upload online review classes (use of YouTube channels for example) or cover extra class material that helps cover the content of a subject, they use post-pandemic teaching methods based on the use of ICTs, for simplicity we will call to this methodology *ICT-teaching methodology* and we will denote this strategy by E_T.

Students: On the other hand the *students*, now back in the classroom, must adapt to new educational and learning challenges after two years of studying remotely [9], these challenges range from the way they take notes in class to how they find information and turn in assignments. Before the pandemic, the use of cell phones in class, for example, was limited occasionally to looking for some kind of information necessary in the development of a class (formulas in mathematics, physics or chemistry, to mention one use). However, due to the different methodologies of teaching used throughout the world as a consequence of the COVID-19 pandemic, the use of this and other devices became essential in the development of the classes, since they were used to take notes, capture screens of the content presented by the teachers during class time, make recordings of them, etc., so that when they return to school, most of the students face the dilemma of how to take notes in class or study and/or hand in homework, because on the one hand they can do it in the traditional way, take notes on paper and sheets, carry out academic research in the school library or physical reference books and/or physically deliver their us class activities, or use their telephone devices to conduct research, take pictures of the material presented by the teacher in the classroom and digitally deliver their activities. We will say that students who follow an offline work methodology apply pre-COVID study techniques and we will denote it by O_l, while when they use techniques that include the use of electronic devices they use post-COVID learning techniques (or *E-learning* techniques) and we will denote it by E_l.

3 A two-pupulations normal form game: Players and strategies

In current educational models, the use of traditional education methods combined with the use of information technologies has been a growing trend in recent years and its use has undoubtedly increased drastically as a result of the pandemic ([30]), both in developed countries and in developing countries, although in the latter, as reported in [19],[27], [32] and [33], was where connectivity problems and access to a good internet signal occurred, for which reason the use of ICTs was unfeasible in many communities and it cause an educational backlog that will take many years to correct. However, even with these problems, the use of the internet and computers as a teaching-learning medium was implemented in these countries (*E-learning - E-teaching* methodologies). Hence by x_{O_l} we denote the portion of students using the studying techniques they have before the COVID-19 pandemic and $x_{E_l} = 1 - x_{O_l}$ denotes the portion of students who, even resuming their academic activities in person, decide to continue using the study techniques acquired during the pandemic, that is, they continue to use digital platforms (Youtube channels for example) to study instead of seeking face-to-face academic advice. By x_{O_T} denotes the portions of professor who decide to continue using the teaching methods or techniques that they used before the COVID-19 pandemic, while $x_{E_T} = 1 - x_{O_T}$ is the portion of professor that changes their teaching methods to incorporate the use of digital platforms.

Although the percentage of individuals from one population or another that follow one or another learning or teaching strategy may change over time, for the purposes of facilitating the model we will assume that the total number of individuals in each population remains constant.

We will denote by S the total number of students and by T the number of teachers. By $S_{Ol}(t)$ and $S_{El}(t)$ represent the total number of students who in the moment t follow *offline-learning* and *E-learning* techniques, respectively. In the same way $T_{OT}(t)$ and $T_{ET}(t)$ will represent the teachers who at the moment t follow the traditional (*offline-teaching*) and non-traditional modality (*E-teaching*), respectively, in this way:

$$\begin{aligned}
&S = S_{Ol}(t) + S_{El}(t),\\
&T = T_{OT}(t) + T_{ET}(t),\\
&x_{O_l}(t) = S_{Ol}(t)/S,\\
&x_{E_l}(t) = 1 - x_{O_l}(t) = 1 - S_{Ol}(t)/S,\\
&x_{O_T}(t) = T_{OT}(t)/T,\\
&x_{E_T}(t) = 1 - x_{O_T}(t) = 1 - T_{OT}(t)/T.
\end{aligned}$$

To save notation along this paper we will omit the parameter t.

We will present below in the form of a matrix, the incentives that students have to choose one or another way of acting. Teachers will receive incentives directly related to the policies preferred by the school management, their computer skills and the results (evaluations) obtained during the teaching process. On the other hand, students will prefer one or another way of acting according to the availability of electronic resources that are within their reach, their computer skills, individual tastes, geographic location (it could make them have better or worse access to the Internet) and their economic resources (which eventually allow them to obtain better internet plans).

Although when grading their students, teachers will only take into account the final grade results without considering the methodology followed by the student, indirectly the final grade obtained by the student involves the processes used by the student, since a student with connectivity problems, statistically speaking and just to mention an example, usually delivers work of lower quality than the students with better opportunities to connect, which is undoubtedly reflected in their evaluations. Students will rate their teachers according to the knowledge acquired during the course and taking into account their empathy and preferences towards the teaching methodology followed by the teacher throughout the course.

3.1 Students' Payoff matrix

In this subsection we describe the students payoff matrix, C denotes the note or knowledge obtained by a student who use traditional studying techniques or the same techniques followed before the pandemics (O_l) when he/she face a professor who adopt a traditional education methodology (O_T), while c denotes the knowledge obtained by the same kind of student when he face a professor who use post COVID teaching methodology (E_T), that is, whenever their professor use *E-teaching* techniques. On the other hand, $\overline{c}$ and $\overline{C}$ denotes the level of knowledge acquired by an student applying the studying techniques learned during the pandemic (E-learning techniques (E_l)) when he/she face traditional educational methodologies (O_T) or those acquired during the pandemic by the professors (E_T), respectively. In the following table we present the payoff matrix of students:

	O_T	E_T
O_l	C	c
E_l	$\overline{c}$	$\overline{C}$

Table 1: Students' Payoff matrix.

According to the Von-Neumann-Morgenstern theorem, students will choose their study strategy according to the expected value associated with each of their possible behaviors. The expected value of the strategy to follow a traditional study method is given by

$$E\left(O_l\right) = x_{O_T} C + (1 - x_{O_T}) c$$

and the expected value of the strategy to follow an *E-learning* study method has an expected value given by the expression

$$E\left(E_l\right) = x_{O_T} \overline{c} + (1 - x_{O_T}) \overline{C}.$$

Therefore, if $E(O_l) - E(E_l) > 0$ the number of students following the studying techniques used before the COVID-19 pandemics will increase. If $E(O_l) - E(E_l) < 0$ the number of students that continues using the studying methods acquired during the pandemics will increase while if $E(O_l) - E(E_l) = 0$ the students will be indifferent between both strategies. In this case we observe that:

$$E(O_l) - E(E_l) = x_{O_T}(C - \overline{c}) + (1 - x_{O_T})(c - \overline{C}) = x_{O_T}(C - \overline{c} + \overline{C} - c) + c - \overline{C}. \tag{1}$$

In the following subsection we discuss some restrictions and interpretation about the parameter $C, \overline{C}, c$ and $\overline{c}$ which fit with the social and economic realities of the students in some (developing) countries.

3.1.1 Students in developing countries

The different realities that governments and citizens faced from the pandemic that emerged at the end of 2019 brought with them enormous challenges to overcome, from the supply of essential goods to educational challenges that triggered the emergence or adaptation of methodologies of study-teaching have been problems that each country has faced from its different realities [5], [25]. In the educational field there were countries better prepared for the distance learning environment than others, in developed countries the use of digital educational tools to record and transmit classes and/or or didactic material has been a constant for several years now, so that in the school environment, adapting its technology to the new reality faced implied a lower investment than that made by developing countries, where the use of digital platforms for these purposes were just beginning to emerge in the years before the pandemic [18]. Although many schools in these countries had already introduced, through training, their teachers to the use of technology to carry out their teaching work, this was not a constant in all schools, since many of them lacked infrastructure and

monetary resources to provide its facilities and its teachers with the necessary technologies for this purpose, so its use did not reach practice before the mandatory confinement of 2020, causing that professors learned by trial and error to handle the different technological tools that they had to use to to teach.

On the other hand students who attended school in a traditional way, now had to start their education in distance mode, that is, they had to take classes through the computer using platforms such as Teams and Zoom. This brought with it a high number of dropouts and a drop in the academic performance of some students, as a result of connectivity problems and/or economic resources to access internet plans or computer equipment that would allow them to attend their classes on a regular basis ([15], [16]). Even though many students were able to continue their studies under these conditions, their grades decreased as a result of the confinement, since they did not have the necessary infrastructure to function in the best way in the distance education environment, so it could be expected that when returning from the distance modality to face-to-face education with professors teaching classes in a traditional way (use of the blackboard and physical delivery of tasks), their grades will increase and even exceed those of a student who must attend school and work under the guidelines of a teacher who implements traditional teaching methods, but who prefers online education, this fact is reflected in the first inequality of the following pair.

$$C \geq \overline{c} \quad \text{and} \quad \overline{C} \geq c. \tag{2}$$

The second inequality implies that when students must go to school in person, but teachers continue to use digital platforms to assign homework or carry out activities, those students who faced technological difficulties during the pandemic will continue to obtain lower grades than those who did not. This inequality reflects the reality of millions of students around the world, especially those who live in developing countries and who do not have the economic resources to access information technologies to develop in a school environment based on the use of new digital technologies and that consequently are left behind in the use of this type of applications in comparison with those who have better economic possibilities.

In conclusion, inequality (2) refers to the fact that students that feels more comfortable with the traditional teaching methodology, the one used before the COVID-19 pandemic, obtain a higher grades (measured through the result in an exam for example) than the one obtained by an student that prefer to follow the study methodology and homework delivery used during the pandemic. This inequality reflects the preferences and realities towards the use of technology of today's youth.

3.2 Professors

Professors who return to their usual face-to-face teaching activities must face the decision of whether or not to use the technology to carry out their teaching work, always seeking to obtain the best teacher evaluation from their students, since a good evaluation could bring recognition that benefits them to obtain a promotion [21]. By N we denote the evaluation note given to a professor using traditional teaching methods (*offline-teaching* (O_T)) when he/she face an student that prefer to study using his/her pre-COVID studying techniques (*offline-learning methods* (O_l)), while n is the evaluation note of a professor following the same kind of teaching techniques when he face an student who use studying techniques developed during the COVID-19 pandemic (*E-learning techniques* (E_l)). For $\overline{n}$ and $\overline{N}$ we denote respectively, the evaluation given to a professor when he follows the usage of technology in his/her teaching work (*E-teaching* methods (E_T)) and he/she faces to students that prefers the use of technology or not, respectively.

	O_l	E_l
O_T	N	n
E_T	$\overline{n}$	$\overline{N}$

Table 2: Students' Payoff matrix.

Once again according to the Von-Neumann-Morgenstern theorem, professors will choose their study strategy according to the expected value associated with each of their possible behaviors. Hence from a direct computation the expected value of the *offline-teaching* strategy is given by

$$E(O_T) = x_{O_l} N + (1 - x_{O_l}) n$$

while the expected value of the *E-teaching* methodology is

$$E(E_T) = x_{O_l} \overline{n} + (1 - x_{O_l}) \overline{N}$$

Then, if $E(O_T) - E(E_T) > 0$ the number of professor using the same teaching techniques that they used to use before the COVID-19 pandemic (*offline-teaching techniques*) will increase. If $E(O_T) - E(E_T) < 0$ the number of professors that continues using the teaching methods acquired during the pandemics (*E-teaching methods*) will increase, while if $E(O_T) - E(E_T) = 0$ the professors will be indifferent between both strategies. In this case we observe that

$$E(O_T) - E(E_T) = x_{O_l}(N - \overline{n}) + (1 - x_{O_l})(n - \overline{N}) = x_{O_l}(N - \overline{n} + \overline{N} - n) + n - \overline{N}. \tag{3}$$

In the next section we address and interpret some constraints on teacher evaluations, as results of their teaching methodologies, in different contexts.

3.2.1 Professor methodologies

Evaluations are often used by the educational authorities as a measure of student satisfaction with the teachers' work, as well as a way to quantify and identify the strengths and weaknesses of their teachers and consequently offer them adequate training (in most cases at 0 cost) that help to improve their teaching performance and thus can better achieve the objectives of the academic entity in which they work (see [28]). In this line and as a result of the pandemic, the educational authorities have made large investments in technological infrastructure, and in training and/or certifying the knowledge, at different levels, of their teachers in the use of digital platforms for teaching classes (teams, zoom, google meet, moodle, etc., see for reference [8], [10] and [12]). Therefore, a way to measure the implementation of the acquired infrastructure (use of ICTs in class) and the level of use of the training offered to its teaching staff, could be through what is reflected in the evaluation of teachers by the students.

As is mention in [29] the students evaluation of teaching not only is used to provide feedback to professor for improve their teaching effectiveness, but also it is used as a method to determine the academic merit of its professors, as well as for their possible promotion or permanence as part of their faculty, so the importance of a teacher obtaining a better evaluation of his work by his students lies in the fact that these notes could eventually favor teachers to obtain better academic positions and in consequence better salaries, especially those teachers hired by class time (pay by hour, more hours of work imply a higher salary for them), since the best evaluated teachers, most of the time, are rewarded by offering a greater number of hours of work, special courses, tutorials and/or academic advisory services, for which they receive an extra economic remuneration.

Once we have addressed the importance of teacher evaluation, we make the following assumptions about the parameters of the model and later discuss its implications.

$$N \geq \overline{n} \quad \text{and} \quad \overline{N} \geq n. \tag{4}$$

This inequalities are related to the students evaluation of professors activities in the classroom. The first inequality implies that an student who prefers traditional teaching methods give a higher evaluation note to a professor who teaches without the use of technological applications than the note assigned to the same kind of professor by an student who feels more comfortable in a teaching environment based on the use

of technological tools. In developing countries, there are many regions without access to a good internet connection or even where high levels of poverty prevent parents from providing a computer to their children, which makes it difficult for students to fulfill activities that involve the use of digital platforms either to perform tasks or as a means of delivering them, so this could be reflected in the best evaluation of a teacher who favors the least use of technology, however those students who have the sufficient means to fulfill the activities remotely could feel that the traditional methodology for the delivery of tasks is inefficient and expensive (delivering tasks in physical form implies a greater expense for them than sending them by email or uploading them to platforms designed for this end) so they could give a lower grade to a teacher who insists on the traditional methodology.

Second inequality means that professors using the teaching techniques acquired during the pandemic gets a higher evaluation note when his/her face an student using studying methods acquired during the pandemic than the one obtained by a professor who use the teaching techniques used before the pandemic when his/her face the same kind of students. Once again these inequalities may be a reflection of the social and economic conditions experienced by students during the pandemic. Students with a better economic position or who live in areas where better internet is received (in countries like Mexico, fiber optic internet, which in theory is faster, does not reach some areas including the main cities of the country) have better possibilities of taking advantage of the use of digital technologies in their educational development, so without a doubt, being in a school environment in which the use of ICTs is encouraged, they would be willing to better evaluate those teachers who work with them.

The use of so-called new technologies could be related to the education policies implemented by their workplaces, since many schools, in order to face the problem of distance education, decided to acquire technological equipment that would allow their workers to carry out their work in the classroom in the best and most efficient way possible, so it is to be expected that this equipment will be used even in face-to-face mode, otherwise the investment made in this classroom equipment would be a useless expense with no possible return on investment.

4 The existence of strictly mixed Nash equilibrium

As it is well the existences of strictly mixed Nash equilibria occurs whenever, for each participant in the game, the expected value associated to each one of its strategies, given the strategies of the others players in the game, is the same. Or put another way, a Nash equilibrium in mixed strategies appears in a game when the two players in the game are indifferent to the choice of each of their pure strategies. In symbols this means that an strictly mixed Nash equilibrium there exists if and only if there are positive numbers $x^*_{O_T},\ x^*_{O_l} \in (0,1)$, for which

$$E(O_l) - E(E_l) = E(O_T) - E(E_T) = 0,$$

or explicitly

$$x_{O_l}(C - \overline{c} + \overline{C} - c) + c - \overline{C} = x_{O_l}(E - \overline{e} + \overline{E} - e) + e - \overline{E} = 0 \tag{5}$$

Theorem 1 (Existences of a unique strictly Nash Equilibrium)**.** *If strict inequalities in* (2)*,* (4)*, then the game has a unique strictly mixed Nash equilibria* [1] *and this is given by*

$$x^*_{O_T} = \frac{\overline{C} - c}{\overline{C} - c + C - \overline{c}} \quad \textit{and} \quad x^*_{O_l} = \frac{\overline{N} - n}{\overline{N} - n + N - \overline{n}}. \tag{6}$$

[1] Note that by complementary we know that $x^*_{E_T} = 1 - x^*_{O_T}$ and $x^*_{E_l} = 1 - x^*_{O_l}$, hence it is enough to write $(x^*_{O_T}, x^*_{O_l})$ to denote the Nash equilibria in strictly mixed strategies. Note that by complementary an increase in the percentage of teachers using offline-teaching methods implies a decrease in the percentage of teachers using E-teaching methods and vice-versa and analogous relation holds for por percentage of students using each of the strategie.

Proof. The conclusion of the theorem follows from a direct computation. □

Strict inequalities in (2) have the following interpretation:

i) ***Offline-learning - Offline-teaching*** *vs* ***E-learning - Offline-teaching*****:** Inequality $C > \overline{c}$ means that students using studying methods that do not involve the use of technology (*Offline-learning*) get better grades when they face a teacher who uses (offline) traditional teaching methods than when they use learning methods based on the use of technology (E-learning) against this same class of teachers.

ii) ***E-learning - E-teaching*** *vs* ***offline-learning - E-teaching*****:** Inequality $\overline{C} > c$ in last theorem means that students using the same studying techniques that they use during the pandemic (*E-learning*) gets a higher grades when professor use teaching techniques used during the COVID-19 pandemic (*E-teaching* methodologies) than a student using traditional studying methods (*offline-learning*) when he/she face the same kind of professor (*E-teaching*).

While strict inequalities in (4) means $N > \overline{n}$ and $\overline{N} > n$, which can be interpreted as follows:

iii) ***Offline-teaching - Offline-learning*** *vs* ***E-teaching - Offline-learning*****:** First inequality implies that students who prefer traditional learning methods (*offline-learning*) rate higher teachers who do not involve technology in their teaching (*offline-teaching*) than those who favor its use in class (*E-teaching*).

iv) ***Offline-teaching - E-learning*** *vs* ***E-teaching - E-learning*****:** Second inequality can be interpreted as the fact that students who prefer to use technology to study (*E-learning*) rate better teachers who favor its use (*E-teaching*) than teachers who prefer traditional teaching methods (*offline methods*).

Remark 1. *Combining item i) and iii), we can conclude that the scenario offline-teaching offline-learning turns out to be a Nash equilibrium in pure strategies for this game. Similarly using ii) and iv) we can conclude that E-teaching - E-learning turns out to be another Nash equilibrium in pure strategies. We will recover this discussion at the end of section 5.*

5 Evolutionary dynamics

To explain the evolution of the decisions of professors and students about their teaching and learning process, we will introduce the dynamics of the replicator, originally this dynamic was introduced in the biology field as a way of explaining how a type of behavior is reproduced in a given species [31]. The natural pressure causes that behavior that best adapts to the environment tends to reproduce with bigger probability. The concept assumes a large population of replicators, in which the different types are found in proportion to their participation in the population. Then introduced into economic theory, where rationality replaces natural pressure, as a dynamic for game theory.

In a conflict situation, different populations with antagonistic behaviors are subdivided into subpopulations according to the strategy, or conduct followed by each of its members, that is, individuals are divided by the behavior followed by each of them. The dynamics of the replicator tries to explain the evolution of the population behavior, when the strategic choice that each individual must make is made within a framework of limited rationality (see for example [11]), since this our case (students and professors does not the strategy that each and other is going to follow), then we can use the evolutionary dynamics tools to explain the trajectories that the teaching-learning strategies will follow along the time, that is, to explain the evolution of the election of students and professors about the teaching methodology and studying techniques that each one can take along the time after the COVID-19 pandemic.

In the long term, professors and students will learn and will follow, the behavior associated with better returns, that is, they will behave as replicants of the most successful strategy. They will choose at the end of each period between maintaining or changing their strategy in a set of possible strategies or behaviors. We understand that the learning process perhaps, by "trial and error", leads to the most successful strategy, in our case the one with the highest expected return. Hence an increased or decreased on the use of one or other strategy in the population will depend on the fact that the payoff that is obtained with one strategy is better than or a lower than average performance.

In our model this dynamics is described by the following system of differential equations:

$$\begin{aligned} \dot{x}_{O_l} &= x_{O_l}(1-x_{O_l})\left(E(O_l)-E(E_l)\right) \\ \dot{x}_{O_T} &= x_{O_T}(1-x_{O_T})\left(E(O_T)-E(E_T)\right) \end{aligned} \tag{7}$$

Here $E(O_l)$ and $E(E_l)$ represents respectively, the expect value of the students' strategies *offline-learning* and *E-learning*, while $E(O_T)$ and $E(E_T)$ denotes the expected value of the professors' strategies *offline-teaching* and *E-teaching* respectively. Then using (1) and (3) system (7) becomes in

$$\begin{aligned} \dot{x}_{O_T} &= x_{O_T}(1-x_{O_T})\left(x_{O_l}(C-\overline{c}+\overline{C}-c)+c-\overline{C}\right) \\ \dot{x}_{O_T} &= x_{O_T}(1-x_{O_T})\left(x_{O_l}(E-\overline{e}+\overline{E}-e)+e-\overline{E}\right) \end{aligned} \tag{8}$$

The dynamical equilibria of this game are given by:

$$\begin{aligned} x^*_{O_T}=0 \quad \text{or} \quad x^*_{O_T}=1 \quad \text{or} \quad x^*_{O_l}=\frac{\overline{C}-c}{\overline{C}-c+C-\overline{c}}. \\ x^*_{O_l}=0 \quad \text{or} \quad x^*_{O_l}=1 \quad \text{or} \quad x^*_{O_l}=\frac{\overline{N}-n}{\overline{N}-n+N-\overline{n}}. \end{aligned} \tag{9}$$

Note that under the conditions of Theorem 1 the dynamical equilibrium

$$\left(x^*_{O_T},x^*_{O_l}\right)=\left(\frac{\overline{C}-c}{\overline{C}-c+C-\overline{c}},\frac{\overline{N}-n}{\overline{N}-n+N-\overline{n}}\right)$$

is also a strictly mixed Nash equilibria of the game. Moreover, the dynamical equilibria $(x^*_{O_T},x^*_{O_l})=(0,0)$ and $(x^*_{O_T},x^*_{O_l})=(1,1)$ are Nash equilibria in pure strategies, which corresponds, respectively, with the situation where both professors and students choose technological strategies of teaching and learning (*E-teaching - E-learning* (E_T,E_l)=$(0,0)$), and with the scenario where both players choose offline teaching and studying methods (*offline-teaching - offline-learning* (O_T,O_l)=$(1,1)$), see remark 1. A discussion on the interpretation of these will be taken up in section **??** below when we interpret some extreme scenarios of the model.

6 Stability analysis

In this section we use the Hartman-Grobman theorem to analyze the stability of the strictly mixed Nash equilibrium (6). To do this, we must first prove that none of the eigenvalues of the linearization of the system (8) have zero real part, this is justified in Theorem 2 below.

The Jacobian matrix of system (8) is given by

$$J(x_{O_T}, x_{O_l}) = \begin{bmatrix} (1-2x_{O_l})\left(x_{O_T}(C-\overline{c}+\overline{C}-c)+c-\overline{C}\right) & x_{O_l}(1-x_{O_l})\left(C-\overline{c}+\overline{C}-c\right) \\ x_{O_T}(1-x_{O_T})\left(N-\overline{n}+\overline{N}-n\right) & (1-2x_{O_T})\left(x_{O_l}(N-\overline{n}+\overline{N}-n)+n-\overline{N}\right) \end{bmatrix} \tag{10}$$

Evaluating the Jacobian matrix (10) of system (7) at the strictly Nash equilibria $(x^*_{O_T}, x^*_{O_l})$ given by (6), we obtain

$$J = \begin{bmatrix} 0 & \dfrac{(\overline{N}-n)(N-\overline{n})(C-\overline{c}+\overline{C}-c)}{\left(\overline{N}-n+N-\overline{n}\right)^2} \\ \dfrac{(\overline{C}-c)(C-\overline{c})(N-\overline{n}+\overline{N}-n)}{\left(\overline{C}-c+C-\overline{c}\right)^2} & 0 \end{bmatrix} \tag{11}$$

Theorem 2. *Under assumption*(2) *and* (4) *the strictly mixed Nash equilibrium*

$$\left(x^*_{O_T}, x^*_{O_l}\right) = \left(\frac{\overline{C}-c}{\overline{C}-c+C-\overline{c}}, \frac{\overline{N}-n}{\overline{N}-n+N-\overline{n}}\right)$$

is an hyperbolic equilibrium point of system (8)*. Moreover at this equilibrium point J has two real eigenvalues of opposite sing and same module and the strictly mixed Nash equilibrium* $\left(x^*_{O_T}, x^*_{O_l}\right)$ *is a saddle point for the replicator dynamics.*

Proof. From a direct computation using (11), we obtain that the eigenvalues of the linearization of system (7) are given by the real numbers

$$\lambda = \pm\sqrt{\frac{(\overline{N}-n)(N-\overline{n})(C-\overline{c})(\overline{C}-c)}{\left(\overline{N}-n+N-\overline{n}\right)\left(\overline{C}-c+C-\overline{c}\right)}}$$

from where the theorem follows. □

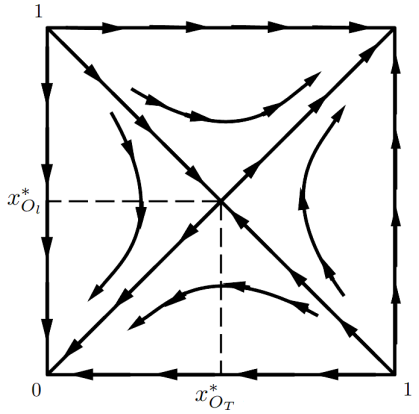

Figure 1: The phases diagram for the dynamics of education techniques.

Analyzing the the diagram above we can conclude that the dynamical equilibria $(0,0),(1,1)$ are attractors, while the steady states $(1,0)$ and $(0,1)$ are saddle points, each of which correspond to the strategic choices of the game $(E_T, E_l),(O_T, O_l),(O_T, E_l)$ and (E_T, O_l) respectively. Once we fix the initial condition

of the system, the way in which the teaching-learning techniques evolve are determined, hence we can see that the probability of convergence to the equilibrium in the interior is equal to zero, then teaching-studying techniques eventually converges to one of the attractors equilibria of the replicator dynamics, which means, that the teaching methods by teachers and the ways of studying by students will evolve together towards the implementation of *traditional (offline) teaching-learning methods* or towards the opposite scenario in which the implementation of so-called information technologies will play a key role in the educational field (towards the *E-teaching - E-learning methods*).

7 The extreme equilibrium scenarios

Now we analyze the implications of the stability of the extreme equilibria given by the vertex of the unit square $\mathcal{C} = [0,1] \times [0,1]$, that is, the steady states corresponding with the pure Nash equilibria of the game:

- **Implementation and use of technology as a studying and teaching method:** The Nash equilibria $(0,0)$ corresponds with the scenario where professors and students base its teaching-studying methods in the use of technology (*E-teaching - E-learning*). In this case, the teachers use the technology, now made available to them, to present and explain their classes, in addition they favor the delivery of assignments through digital platforms over the physical delivery and teaching using the class blackboard, at the same time that students base their study and research methods on the use of electronic devices.

  This panorama seems to fit with the reality of first world countries, where the infrastructure, in most cases, is given for a convergence towards the technological world, where teaching work can be carried out assonchronously using digital platforms and where Students have easier access to information technologies as well as better telecommunication infrastructures that will eventually allow them to study from home without the need to attend classrooms in person.

- **Classical methodology of teaching-learning** $(1,1)$ this scenario take place whenever the professors decide to continue implementing the same teaching methods they used before the pandemic caused by COVID-19 and the students for their part feel more comfortable with this method and prefer the traditional methods of delivering homework and papers over deliveries online through digital platforms. In this situation $N > \overline{n}$ and $C > \overline{c}$, this pair of inequalities means that students in favor of traditional teaching methods give a lower note to professors who use E-teaching methods than the one given to professor using offline methodologies (traditional teaching methods) and the note obtained by an student that follows offline-learning methods is higher than what he/she would get if he/she followed E-learning methods whenever the professor applies traditional teaching methods.

  This scenario is undoubtedly compatible with the reality of many developing countries, because although information technologies are currently available for implementation, the lack of investment, before the pandemic, in this sector could eventually return. to become present leading to the lack of maintenance of the acquired infrastructure and causing a recession in the way of teaching-learning, thus converging to an offline educational system.

8 Conclusions

In this work we have presented an evolutionary model of game theory, where we show the possible trajectories that teaching-learning strategies will follow in a post-pandemic world, depending on the different social and economic realities experienced in each country. We also show that although, as a consequence of the investment made by the different educational and government authorities of the different countries

worldwide, teachers and students can choose to follow teaching-learning strategies that involve the use of ICTs, these strategies will eventually converge to situations where their use is completely privileged by both students and teachers or towards scenarios where these technologies are no longer applied and the classical teaching methodology is returned, although this scenario could not being desirable may be a consequence of the lack of investment in teacher training, incentives for the use of technology, or the lack of maintenance of the technological infrastructure acquired as a result of the pandemic.

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