

Populism Vs Growth

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Abstract

In this article we present an evolutionary model that shows the possibility of political cycles in which populist and non-populist elites alternate in power. Although, on the one hand, the platforms promoted by the political sectors are presented as valid alternatives to solve the most pressing problems of society at a given moment, they do not pay attention to the imbalances or social problems that their macroeconomic political decisions can generate in the long run. On the other hand, the policies promoted by a non-populist ruling elite, which, precisely, pay attention to macroeconomic balances, productivity, the promotion of private investment and, in general, long-term growth, many times do not consider the difficulties that these policies can generate in some sectors of society. This can give rise to the emergence of a populist alternative, thus creating a political cycle of alternation of power between populist ruling elites and non-populist ruling elites. We will also analyze the possibility that society evolves in a sustained manner towards a populist government with the growing approval of the citizens, as well as the possibility of achieving a non-populist government with the growing support of the citizens.

Keywords: populism, growth policies, political cycles.

JEL Codes: H11, H24, H30.

1 Introduction

Today we are witnessing a shift from the traditional class-based politics and left-right divide to a distinction based on cultural attitudes and education. This change is having profound effects on the political systems of democracies organized along the traditional left-right divide. The traditional economic and redistributive conflict between left and right is fading. In its place, a new conflict has arisen between nationalist and socially conservative positions versus cosmopolitan and socially progressive positions. Although it is possible to distinguish different variants between the antagonists, we will not go into more detail here on this point, we will represent this new political division as the one that exists between populists and non-populists. Undoubtedly a somewhat schematic distinction but appropriate to the purposes of this paper (for the interested reader we refer to Giampaolo Bonomi (2021) and Rodrik (2017)).

In support of our schematic classification we can resort to authors such as such as, Alan Ware in Ware (2002) define the populism as a “political strategy deployed by a wide range of politicians”, while Cas Mudde (Mudde (2004)) argue that populism is ‘ ‘as an ideology that considers society to be ultimately separated into two homogeneous and antagonistic groups, “the pure people” versus “the corrupt elite” and which argues that politics should be an expression of the *volonté générale* (general will) of the people.’’ For Robert Barr (Barr (2009)) for example populism is “a mass movement led by an outsider or maverick seeking to gain or maintain power by using anti-establishment appeals and plebiscitarian linkages.”

Although populism appears in many different forms in political life, there is a clear distinction between what we can call left-wing populism and right-wing populism Krämer (2017), the first form typical of Latin

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American countries and the second in Europe and the United States (see Rodrik (2017) for a more detailed description) in general, beyond the specifics of the problems, populism is characterized by presenting a platform of apparently simple solutions to implement to complicated problems with the supposed objective of benefiting sectors of the population that for various reasons have been affected by the factors that give rise to such problems (see for instance Derks (2006)), at the expense of the state and those considered responsible for the misfortune. An example of this is inflation, which is often generated by the expansive macroeconomic policies of populist platforms, which populist governments intend to combat with price adjustment policies or transfers to the sectors most affected by this phenomenon. These same platforms usually affirm that the problem of the lack of work that affects sectors of the autochthonous population is solved by preventing the entry of immigrants or protecting the national industry with taxes on imported goods, or by creating public employment. All these policies that may have a short-term social benefit for the most disadvantaged sectors, in the medium and long term only end up aggravating the existing problems, since implementing them implies transferring resources originally destined for infrastructure, education or research works, as well as taxing with abusive taxes on the productive sectors of society, to finance immediate social policies (see Dornbusch (1991)), thus cutting off all possibility of growth and social mobility.

At present, in different countries of the world, those who support these policies -generally governments or politicians who aspire to be elected or re-elected to their positions, Acemoglu et al. (2013) without adequate preparation, putting their ambitions for power before general interests- They are opposed by those who argue that the way to overcome the most pressing problems facing society is through investment in infrastructure, the development of competitiveness, research, the search for energy, clean and economic alternatives that allow efficient growth without neglecting the environment. However, beyond the possible effectiveness of each of these opposing proposals, the truth is that the urgent problems of the popular sectors in different countries require immediate solutions (a phenomenon that is not characteristic only of less developed economies Rodrik (2017) and Guiso et al. (2017)), which makes populist policies make sense and are supported by broad sectors of the society in different countries regardless of their level of development. This translates into an increase in the chances of winning elections for governments and/or politicians who argue in favor of this type of immediate policy without considering the fact that the economic policy promoted by populist politicians requires the transfer of resources from areas growth, with the consequent deterioration of the quality of life of citizens, which will lead to even those who support these governments with their votes, feeling disappointed by what they live or because they consider the results of the initially proposed policies insufficient. At this point, these citizens will prefer to support sectors opposed to these policies that are presented as alternatives to overcome these difficulties. In this paper we will analyze the evolution of support for one or the other policy over time, considering an evolutionary model from the field of games.

The rest of the paper is organized as follows: Section 2 introduce the model. Section 3 introduce a two population normal form game, that represents the pure strategies of the different populations immersed in the conflict of the democratic alternation of power that confronts citizens and the political class. Section 4 analyze the Nash equilibria available for the game depending on the values of the parameters expressed in the return matrix of the game. Section 5 Introduce the replicator dynamics to analyze the evolution of the behavior of the citizens and politicians, In this section we consider the possibility of cycles of alternance between populist and non populist ruler elites. Section 6 gives some conclusions and refer futures work.

2 The model

We will consider a social conflict between two populations, that of the political class divided into populists and non-populists, and that of the citizens (the voters) also divided into two subpopulations, which we call, to simplify the notation, also as populists and non-populists. Citizens through universal suffrage

decide the policy to follow once each period of government is over.

Beyond nuances, which are surely important to consider in other areas, populist politicians are defined as those who propose immediate solutions to real problems but without considering the repercussions that these may have in the long or medium term. We will denote the subpopulation of populist politicians by p_p . We will call the subclass of citizens who support this political sector populists, whom we will represent by c_p .

The subpopulation of non-populist politicians will be represented by p_{np} , and it is made up of those politicians who see productive investment as the basis for solving the problems that the country faces for its growth and even as a solution to the most urgent problems suffered by some social sectors. Through c_{np} we represent the subpopulation of citizens who support this political sector.

This division is undoubtedly simplifying social reality and somewhat arbitrary. We understand that it largely represents the current political division in many countries, both developed and developing. The insecurity caused by free enterprise in the most vulnerable sectors, or immigration that seems to threaten the jobs of nationals or free trade are factors that encourage the offer of quick solutions and populist proposals, many of which appear loaded with a vengeful spirit against those who support the supposed causes of discontent. Undoubtedly, the easy promises to eliminate these causes of discontent attract popular sectors supposedly affected by them in their well-being. The most realistic solutions seem to be long-term and insufficient, at least from the point of view of their immediate expectations, by the least favored sectors. This situation often leads to a polarization of society, which somehow justifies our simplifying assumptions.

We will assume a fixed number of politicians defined by $\#P$, not the number of politicians in one population or another, which we understand can change over time, $\#P = P_p(t) + P_{np}(t)$, for all $t \geq 0$. Similarly for citizens, the total will remain fixed over time, but the individuals in one sector or another can change over time, this can be represented by equality $\#C = C_p(t) + C_{np}(t)$, for all $t \geq 0$.

We will consider that the activity of the political class is driven by the desire to maintain or gain power, which they will only access or remain in if they win the majority of the citizens' votes. In this sense, the appearance of populist sectors is clearly influenced by the growth of discontent among citizens, or a part of them, due to problems of poor distribution of wealth, corruption, lack of employment or opportunities, or at least due to the sense of existence of such factors. These problems in the best of cases do not have quick solutions and often survive a generalization even under the best efforts of governments and citizens. The promise of easy solutions, through tax burdens, direct transfers or the creation of public jobs seem like a solution for large sectors of the city in times of economic recession. Options that in most cases do not solve the problems in the medium or long term and end up becoming negative facts for their own drivers.

Next we will measure the level of satisfaction that the ruling elites obtain from their political actions by the number of votes that their actions determine. While the level of satisfaction of the citizens before the government of one sector or another will be measured by the individual welfare that is deduced from the current political actions or promised by these sectors

1. Citizens of with preferences for populist parties:

- Receives from populist governments, satisfying for ideological affinities with populist proclamations, increased, in some cases with direct helps direct help including bonuses or money. These citizens feel satisfied, both intellectual and materially with this type of governments, fact that can be measured in terms of utility. This satisfaction will be represented by $uc_{pp} = id_{pp} + c_{pp}$ composed of two ads, ideological affinity d_{pp} and c_{pp} that measures the degree of satisfaction with monetary transfers and government tax policies.
- These citizens faced by non-populist governments will feel affected because, even in the case of receiving direct transfers, they will be in minor quantities and generally in the short term. Not stubborn can find long-term satisfactions due to the construction of infrastructure and the possibilities of labor. The utility received by these citizens in case of facing a non-populist

government will be of the type $uc_{pnp} = c_{pnp} + \alpha^k in, k \geq 0$. It is important to keep in mind that non-populist governments of different types offer some compensations to citizens, in terms of scholarships for a personal study or overcoming, this fact is measured by c_{pnp} and it strongly related with government tax policy, and $\alpha \in [0, 1)$ is a discount factor associated with the benefits of long or medium term that infrastructure construction can contribute to the citizen, which multiplied by the amount of investment in that the non-populist government makes in this area, represents the satisfaction that the citizens with preferences by governments non-populists obtains for said investment and $k > 0$ corresponds to the time expected for the results of the investment in infrastructure.

2. Citizens with preferences by governments non-populists.

- Faced with populist governments, they will be affected from the ideological point of view as well as in their believers about future intellectual and financial development. We understand that these citizens are generally related to intellectual development, free initiative and without the need for external support not based on individual achievements to achieve success, which makes them feel excluded from paternalistic policies typical of populist platforms. The utility that these citizens will receive from populist governments we represent as $uc_{npp} = id_{npp} - e_{pp} + c_{npp}$, where $id_{npp} < 0$ represents the discomfort that these citizens feel before a populist government and $e_{pp} > 0$ is an index that measures the average exclusion feeling. The work carried out by the government, measured through the infrastructure carried out (communication routes, lighting modernization, road improvements, etc.), allocation of resources for low-income people or support through scholarships for students, support for mothers of family, etc., benefits the entire population regardless of the ideologies of the latter, so in our model, the vision of the non-populist population towards this kind of work by a populist government is represented by c_{npp} .
- These citizens will feel encouraged by non-populist governments, generally motivating commercial activity and independent development of the State. In this case, these citizens will feel satisfied with the creation of infrastructure that allows a development of commercial and industrial activities, and for the ideological affinity for the factors that drive the growth of the economy. We will represent this level of satisfaction for $uc_{npnp} = id_{npnp} + \alpha^h in + c_{npnp}$. Where id_{npnp} represents the degree of ideological affinity, $\alpha \in (0, 1]$ a discount on the discount factor in the creation of infrastructure and the exponent $h < k$. It is noteworthy that the infrastructure investment carried out in the government period does not necessarily imply benefits for society in the government period, but in the future, even in this situation, the work developed by the government is valued by the different members of society, this is represented by c_{npnp} .

Note that we do not make assumption about the sign of c_{npp} and c_{npnp} , nor of their relationship between it, but we claim that this are negative whenever the assessment of the government's actions (populist or non-populist) is not well perceived by different types of citizens and positive when it is well received.

3. Populist governments face:

- Populist citizens: In general, populist platforms will be favored by the vote of these citizens. Fidelity will be increased by the level of promised aid. We will measure the utility of the government to $ug_{pp} = \beta_1 c_{pp} + \beta_2 id_{pp}$ the utility will be measured by the possibility of vote, which depends proportionally on the current promises or subsidies that are offered on the populist platform and the ideological affinity that these popular sectors feel with the populist elite.

- Non-populist citizens: While these sectors are not prone to populist policies, some intellectuals concerned about the problems that affect the most vulnerable sectors of society and expect immediate solutions for them without taking into account the possible negative consequences that in the long term, for the economy, a populist policy can have, worse even in the case of consolidating, see Funke et al. (2021) We will measure the utility of the populist government for the resonance that its promises achieve in this sector of intellectuality. To achieve such a result, they must present their proposal as a valid alternative to get over the situation of those who are in the worst conditions.

4. Non-populist government face:

- Populist citizens: They must convince them that there are no easy solutions and that the deep causes of difficulties are not necessarily apparent and generally the only ones indicated by populist sectors. Notwithstanding the degree of difficulty that these sectors of the population find for their day to day can lead to no favorable response from these sectors for non -populist elites. The probability that an individual from the group who in Progress feels favored by the populist platform faced by these sectors, The utility that this political sector will obtain from its relationship with this sector of the population will correspond to the probability of convincing about its proposals to a citizen of this group, then equal to $ug_{npp} = \delta CC$ where CC measures the ability to convince.
- Non-populists citizens: The probability that the citizens of the group vote for this sector of the political spectrum will depend on their proposals for the growth and development of the country and its ideological proposal, especially the elements related to government efficiency. $ug_{npp} = \gamma pr$ γ next to 1, by pr we represent the clarity of the proposal presented by the candidates for the government or to be reelected

3 The game

It is a two-populations normal form game that is repeated. The utility associated with each possible strategy of the political class will be defined by the expected quantity of votes she presents. While the utility for citizens is measured by the expected value of the degree of well-being that the proposed political platform supposedly offers.

For this we will consider ug_{pp}, ug_{pnp} as the value of the strategy P_p , that is, the ability to win the vote of an individual of the classes p or np when a populist platform. Similarly ug_{npp}, ug_{nnp} represent the ability to win the vote of one individual from each class when presenting a non-populist platform, that is when the ruler elite follows the strategy P_{np} .

Citizens of the populist sector (C_p) expect to reach a level of welfare uc_{pp} , or uc_{pnp} depending on whether a populist or a non-populist government wins. Similarly, uc_{npp} and uc_{nnp} represent the expected welfare values for non-populist citizens, in case a populist or non-populist government wins respectively.

The return matrix of the game is given in the following table.

	C_p	C_{np}
P_p	ug_{pp}, uc_{pp}	ug_{pnp}, uc_{npp}
P_{np}	ug_{npp}, uc_{pnp}	ug_{nnp}, uc_{nnp}

Table 1: Payoff matrix.

where

$$ug_{pp} = \beta_1 c_{pp} + \beta_2 id_{pps}, \quad ug_{pnp} = \gamma Re, \quad ug_{npp} = \delta CC, \quad ug_{npnp} = \gamma Pr$$

$$uc_{pp} = id_{pp} + c_{pp}, \quad uc_{npp} = id_{npp} - e_{npp} + c_{npp}, \quad uc_{pnp} = c_{pnp} + \alpha^k in, \quad uc_{npnp} = id_{npnp} + \alpha^h in + c_{npnp}$$

According to Von Neumann's expected utility theorem, a rational player will choose the strategy that offers the highest expected value.

$$E(P_p) - E(P_{np}) = x_{C_p} (\beta_1 c_{pp} + \beta_2 id_{pps} - \delta CC + \gamma Pr - \gamma Re) + \gamma Re - \gamma Pr \quad (1)$$

$$E(C_p) - E(C_{np}) = x_{P_p} (id_{pp} + c_{pp} + e_{npp} - id_{npp} - c_{npp} + id_{npnp} + \alpha^h in + c_{npnp} - \alpha^k in - c_{pnp}) \\ + c_{pnp} + \alpha^k in - id_{npnp} - \alpha^h in - c_{npnp}. \quad (2)$$

Where by

$$x_{C_p}(t) = \frac{C_p(t)}{\#C}, \quad x_{C_{np}}(t) = \frac{C_{np}(t)}{\#C}, \quad x_{P_p}(t) = \frac{P_p(t)}{\#P}, \quad x_{P_{np}}(t) = \frac{P_{np}(t)}{\#P}$$

we denote the respective percentage of individuals in each sub-population in each time. Note that the expected value depends on time, however to simplify notation we omitted to write to the variable t in the previous expressions

In addition in that follows order to save notation we set

$$A_{pp} = \beta_1 c_{pp} + \beta_2 Id_{pps} - \delta CC + \gamma Pr - \gamma Re \\ B_{pp} = \gamma(Re - Pr) \\ A_{cp} = id_{pp} + c_{pp} + e_{npp} - id_{npp} - c_{npp} + id_{npnp} + \alpha^h in + c_{npnp} - \alpha^k in - c_{pnp} \\ B_{cp} = c_{pnp} + \alpha^k in - id_{npnp} - \alpha^h in - c_{npnp} \quad (3)$$

Observe that according to our initial assumptions, although the number of individuals in each population (politicians and citizens) are considered as constants of the model, this is not the case with the percentages of individuals in each subpopulation. Since these percentages can change over time, the expected values of the strategies are functions of time. Furthermore, the supposed rationality of the participants implies a growing percentage of individuals in the subpopulation whose strategy has a higher expected value. Note that this does not necessarily imply that the once winning strategy will continue to be so. For example, in the event that individuals waiting for direct transfers increase in number, the budget of the State is finite, which means that more and more resources are required to maintain the level of aid, or that the amount of each aid is reduced, all of which can result in a change in the attitude of the voters.

For each of the possible strategies there are restrictions. One of the most important constraints is the government's budget constraint, this means that the total possible aid that a populist government can offer cannot exceed revenue (we do not consider the possibility of the government borrowing abroad). Therefore, even a populist government must find some source of resources or reduce the aid provided. Many times the resources allocated to subsidies come from cuts in basic services, education and health, as well as the cut of resources destined to infrastructure, or the collection of taxes in particular to the productive sectors of society. The need for infrastructure can become a critical factor for the growth of the country, the neglect of communications and transport and in general of basic services, can make even populist governments feel obliged to allocate resources for the creation of said infrastructure. Investment that will often be made at

the expense previously granted aid, inflation or indebtedness, factors that can lead the country to financial crises that end up harming society as a whole. This situation will once again put citizens into consideration the need for governments that address the problems of growth and economic balances. On the other hand, a heavy tax burden can harm the productive sectors, causing shortages even of basic necessities, all of which will lead citizens, including those who originally transferred the government, to reconsider their vote for the next period. Argentina during different Peronist governments is a clear example of this situation. see for example OECD (2019).

Similarly, non-populist governments will face budget constraints. Inefficient investment in public services and infrastructure in general, can generate financial difficulties, which in turn cause difficulties in the most vulnerable sectors of the population, unemployment and loss of quality of life, as well as discontent in sectors of the population linked to the industry and commerce that require efficient investments in services and infrastructure for the normal development of their activity. On the other hand, economic growth as a result of investment in science and technology can generate unemployment and unexpected changes with negative repercussions for some sectors of the population (see Accinelli and Muñiz (2021)), which in the absence of a policy Adequate public policy can generate discontent with the government's economic policy, even when it is aimed at improving the performance of the economy and economic growth. This situation can lead to changes in the electoral decisions of a large part of the citizens who until now felt related to the government, opening the door to the presentation of populist platforms.

4 The Nash Equilibrium of the political game

While a Nash equilibrium in pure strategies assumes that the entire political class behaves in a single way, while the same occurs for the population of citizens, a mixed strategy corresponds to a percentage distribution of the individuals of each population among its possible subpopulations (this equilibrium is said strictly mixed if both distributions are not degenerated). The latter corresponds to a certain situation of uncertainty, where it is not clear to the political class what the preferences of citizens are, which means that not all politicians act in the same way. As we will see, for this type of equilibrium the model presents two alternatives. The first one corresponds to a stable equilibrium, in the sense that the distribution over time between the subclasses can be modified, if we start close to the equilibrium distribution, society will It will change a lot, cycles will result in which political elites of one ideology and another alternate in power, with the support of the majority of citizens. Or the second possibility corresponds to the case where the strict mixed Nash equilibrium is unstable, in the sense that it will be difficult for society to reach it, but even if the respective distributions are at some point close to the equilibrium distributions, they will move away from them forever.

We will consider these possible scenarios in the next section, however the evolution can only be considered once we introduce a dynamic for the game, which will be done later in the section 5.

4.0.1 Nash equilibria in possible political-ideological scenarios

In this section we discuss about some possible scenarios to which the different political options can give rise considering the declared objectives and the perception of the citizens on the results of the same. Although it is possible in principle to characterize economic policies in two large categories as we have done up to now, the results obtained by governments that fall into one or the other are not necessarily the same or affect citizens in the same way. This means that citizens do not appear as a homogeneous whole, and even under the same government, the citizens' perception of the results of government policy may be different, which in principle means that we can place them in two large groups, the sympathetic to the government, those who see in its politics a way to improve their standard of living and those who feel harmed by the same policy. Since the ruling elite intends in principle to remain in power, it will require the majority vote

in democratic countries, so it will be attentive to citizen opinion, and it may turn out that it does not always act as a populist even if it is, or even when it is not to make decisions that can be characterized as populist. This causes the percentages of citizens in one or the other group to change over time, which in turn leads to possible variance in government actions. The result is that the political sector with which the majority of citizens feel identified ends up triumphing. In other words, if populist policies are perceived by the majority as causing bad results, the choice of the citizens will fall on the non-populist elite, if this policy is perceived as successful by the majority, the populists will be re-elected. In the same way for non-populist governments.

We will show different scenarios, to which correspond different game structures that characterize our model, structures that give rise to different return matrices and, consequently, to different sets of Nash equilibria. In other words, each of these scenarios will ultimately be characterized by a set of Nash equilibria.

4.1 Scenario with several Nash equilibria

Under these conditions it is not possible, at least in principle, to decide which of the possible Nash equilibria will end or be enhanced in society, at least not, as long as we do not incorporate a dynamic into the model that allows us to decide on the stability of these equilibria. In short, only stable equilibria are observable.

Theorem 1. *If $u_{c_{npp}} > u_{cnp}$ and $u_{c_{pp}} > u_{cnp}$ together with $u_{g_{nnp}} > u_{g_{np}}$ and $u_{g_{pp}} > u_{g_{np}}$ holds then the game has three Nash equilibria*

- i) (pure Nash equilibrium) populism-populism*
- ii) (pure Nash equilibrium) non-populism-non-populism*
- iii) (strictly mixed Nash equilibrium)*

$$x_{cp} = -\frac{B_{pp}}{A_{pp}}, \quad x_{pp} = -\frac{B_{cp}}{A_{cp}} \quad (4)$$

Before to present the proof of the theorem we remark that first equilibrium can be identified with the probability distribution $((x_{cp}, 1 - x_{cp}), (x_{pp}, 1 - x_{pp})) = ((1, 0), (1, 0))$ which for short we write $(x_{cp}, x_{pp}) = (1, 1)$, similar for non-populism-non-populism which corresponds with $(x_{cp}, x_{pp}) = (0, 0)$ which means that no player are taking populist behaviour, hence they act in a non populist way.

Proof. See figure 1, since the dynamics of the edges of the unit cube correspond with the best response of each player against the alternative that the other is playing, then the vertices with two arrows pointing in their direction correspond to Nash equilibria in pure strategies, so $(0, 0)$ and $(1, 1)$ are the only pure Nash equilibria of the game. Certainly the expression 4 satisfies $E(P_p) - E(P_{np}) = 0 = E(C_p) - E(C_{np})$, so to guarantee that this is a Nash equilibrium in mixed strategies, we must prove that $0 < x_{cp}, x_{pp} < 1$. To see this note that the assumptions of the theorem implies that $u_{g_{nnp}} - u_{g_{np}}$ and $u_{g_{pp}} - u_{g_{np}}$ are positive, hence the result follows from the fact that $-B_{pp} = u_{g_{nnp}} - u_{g_{np}}$ and $A_{pp} = u_{g_{pp}} - u_{g_{np}} - B_{pp} > -B_{pp}$, then $0 < x_{cp} < 1$. A completely analogous argument shows that the assumptions of the theorem together with the definition of B_{cp} and A_{cp} implies that $0 < x_{pp} < 1$. \square

The theorem describes the general conditions for which populist citizens have greater satisfaction when political leaders have the same ideologies as them, and vice versa, that is, populist leaders have a greater utility when backed by citizens with their same interest. In the same way it reflects the conditions under which non-populistic leaders if non-populist citizens have no interest in changing their political ideologies unilaterally.

4.2 Scenario with only one Nash equilibrium

Theorem 2. *Assume that $uc_{pnp} > uc_{nnp}$, $ug_{pp} > ug_{npp}$, $uc_{npp} > uc_{pp}$ and $ug_{nnp} > ug_{pnp}$ then the only Nash equilibria of the game is strictly mixed and is given by the expression:*

$$x_{cp} = -\frac{B_{pp}}{A_{pp}}, \quad x_{pp} = -\frac{B_{cp}}{A_{cp}} \quad (5)$$

The same result holds whenever all the inequalities in the hypothesis of the theorem are in the opposite sense.

Proof. Each vertex of the figure 2 have an arrow pointing towards it and another coming out of it, since the dynamics of the edges of the unit square correspond to the best responses of each player to the alternative played by the other, we can conclude that the best response for each of the strategies played by the other player is contrary to the alternative chosen by the first, so the game does not have a Nash equilibrium in pure strategies, then by the Nash theorem for finite strategy games with finite players, we can conclude that there must be a Nash equilibrium in mixed strategies. Since for $x_{cp} = -\frac{B_{pp}}{A_{pp}}$, $x_{pp} = -\frac{B_{cp}}{A_{cp}}$ we have that $E(P_p) - E(P_{np}) = 0 = E(C_p) - E(C_{np})$, then the conclusion of the theorem follows from a the definition of $A_{pp}, B_{pp}, A_{cp}, B_{cp}$ and the hypothesis of the theorem that guarantee that $x_{cp}, x_{pp} \in (0, 1)$. \square

$uc_{pnp} > uc_{nnp}$ is equivalent to $\alpha^k in + c_{pnp} > id_{nnp} - \alpha^h in + c_{nnp}$ this situation can take place in situation where even with a non-populist government citizens with preference to populists politics has a higher utility than the non-populist citizens with a non populist government. This may be because the acts or policies followed by non-populist rulers have not been successful enough that they have not met the expectations that non-populist citizens had. It is possible that the maintenance of some subsidy policy has kept the utility of populist citizens at a higher level, even when they prefer a populist government. In this situation, even when the ideology of non-populist citizens is the same as that of the government, that is, even when $id_{nnp} - \alpha^h in > \alpha^k in$, the valuation of non-populist government actions by non-populist citizens is contrary to their political ideals, while the valuation of populist citizens to the performance of no populist government is positive, in the sense that these citizens consider that the work of the non-populist government has been adequate. In this case $c_{pnp} > c_{nnp}$ inequality that can be interpreted as the populist government is carrying out a tax policy more akin to the non-populist sectors than to the populists. The reasons for this can be several, among them, the need to encourage the country's production or exports, without which it is not possible to sustain long-term populist policies.

$uc_{npp} > uc_{pp}$ or equivalently $id_{npp} - e_{npp} + c_{npp} > id_{pp} + c_{pp}$, is a reflection of the work of the populist government to solve the problem of the population, remaining indebted to sectors related to their ideology and carrying out acts that value those citizens with ideologies contrary to theirs, that is, $c_{npp} > c_{pp}$. What can be a reflection that the programs created for infrastructure, creation of public services, allocation of scholarships, etc., have been more beneficial for non-populist citizens than for populists. This may be due to the inability of the populist government to deliver on all its promises in the short term, which is reflected in the disappointment of like-minded citizens of the government. While the perception of the rest of the citizens is that of a government that works to solve the underlying and long-term problems. This does not mean that the government has changed its populist thinking, rather it is a reflection of the reality it faces, where budgets may be insufficient to solve problems overnight and strategies to do so will take longer than expected the populists thought.

$ug_{nnp} > ug_{pp}$ and $ug_{pp} > ug_{npp}$, these inequalities capture the actions of the government, who, seeing the preference of the citizens, measured for example through surveys, takes the position that ensures it wins over the electorate, making populist campaigns when it warrants it (when facing populist citizens) and non-populist when the summons requires it (facing non-populist citizens).

5 Replicator dynamics

Our model intends to analyze the alternation in power of the elites that are defined by one or another policy. Until now the models were static, however they tried to show the possibility of alternate cycles. The replicator dynamics models the learning of rational individuals who consider the expected value of their possible strategic choices. The analysis of the stability of their equilibria makes it possible to define, in the event of the existence of multiple Nsh equilibria, which one ends up prevailing, since in reality only stable equilibria are foreseeable. It also makes it possible to analyze the appearance of cyclical trajectories centered on one of its equilibria. It is precisely this analysis that this section will deal with.

In our model the replicator dynamics is described by the following system of differential equations:

$$\begin{aligned}\dot{x}_{cp} &= x_{cp}(1 - x_{cp})(E(C_p) - E(C_{np})) \\ \dot{x}_{pp} &= x_{pp}(1 - x_{pp})(E(P_p) - E(P_{np}))\end{aligned}\tag{6}$$

Here $E(C_p)$ and $E(C_{np})$ represents respectively the expected value of playing C_p and C_{np} by citizens, while $E(P_p)$ and $E(P_{np})$ denotes the expected value of the politics playing P_p and P_{np} . Then using (2) and (1) system (6) becomes in

$$\begin{aligned}\dot{x}_{cp} &= x_{cp}(1 - x_{cp})(x_{pp}A_{cp} + B_{cp}) \\ \dot{x}_{pp} &= x_{pp}(1 - x_{pp})(x_{cp}A_{pp} + B_{pp})\end{aligned}\tag{7}$$

Note that the Nash equilibria corresponds with stationary states of the system, but the converse it not necessarily true. In section 4 in theorem 2 and theorem 1 we present some general conditions and its explanation about the political realities of the society that implies the existence of one or multiple Nash equilibria.

For the multiple Nash equilibria case using classical game theory, it is not possible, at least in principle, to indicate which of the Nash equilibria of a game ends up prevailing. The study of the stability of the equilibria of replicating dynamics, considered in the modern field of evolutionary games, will allow us to define which of them will end up prevailing. In short, both in society and in nature, only stable equilibria are perceptible. We will make the study of the stability of the multiple equilibria case in subsection 4.1.

It is important to take into account the equations of the dynamical system corresponding to those that guarantee the existence of the solution. Its uniqueness will be determined once we know the initial conditions. that is, a distribution over the possible subpopulations for each population at a given moment. Once these distributions are known, they find the only possible trajectory that the evolution of the system that corresponds to such initial condition will follow. We will call these initial conditions the initial state of the system, while the state of the systems at each moment corresponds to the pair of distributions existing at each moment, which certainly mark the political evolution of society.

5.1 The stability analysis of the multiple equilibria scenario

Using the Hartman-Grobman theorem in this section we analyze the stability of the strictly mixed Nash equilibrium (4). For this propose, we estimate the eigenvalues and its sings of the linearization of the system (7). From a direct computation the Jacobian matrix of system (7) is given by

$$J(x_{cp}, x_{pp}) = \begin{bmatrix} (1 - 2x_{cp})(x_{pp}A_{cp} + B_{cp}) & x_{cp}(1 - x_{cp})A_{cp} \\ x_{pp}(1 - x_{pp})A_{pp} & (1 - 2x_{pp})(x_{cp}A_{pp} + B_{pp}) \end{bmatrix}\tag{8}$$

Evaluating the Jacobian matrix (8) of system (6) at the strictly Nash equilibria (x_{cp}^*, x_{pp}^*) given by (5) or (4) we obtain

$$J(x_{cp}^*, x_{pp}^*) = \begin{bmatrix} 0 & x_{cp}^*(1-x_{cp}^*)A_{cp} \\ x_{pp}^*(1-x_{pp}^*)A_{pp} & 0 \end{bmatrix} \quad (9)$$

even when the same Jacobian matrix is gotten for the Nash equilibria describe by (5) and (4) the dynamical behaviour of both is completely opposite. In next theorem we present the stability analysis of the strictly mixed Nash equilibrium for the scenario of multiple equilibria. We leave the analysis of analysis of the scenario of unique strictly Mixed Nash equilibrium to the end of this section.

Theorem 3. *Under assumption of theorem 1 the strictly mixed Nash equilibrium*

$$(x_{cp}^*, x_{pp}^*) = \left(-\frac{B_{pp}}{A_{pp}}, -\frac{B_{cp}}{A_{cp}} \right)$$

is an hyperbolic equilibrium point of system (7).

Proof. Finding the eigenvalues of (9), we obtain

$$\lambda = \pm \sqrt{x_{pp}^*(1-x_{pp}^*)x_{cp}^*(1-x_{cp}^*)A_{pp}A_{cp}} \quad (10)$$

since $x_{pp}^*, x_{cp}^* \in (0,1)$ and the hypothesis of the theorem implies that $A_{pp} > 0$ and $A_{cp} > 0$ (see proof of theorem 1) the theorem follows. \square

Observation 1. *Note that under the conditions of last theorem $J(x_{cp}^*, x_{pp}^*)$ has two real eigenvalues of opposite sing and same module. Then the strictly mixed Nash equilibrium (x_{cp}^*, x_{pp}^*) is a saddle point for the replicator dynamics with a 1-dimensional unstable manifold and a 1-dimensional stable manifold passing through it. Hence the probability of convergence to the equilibrium in the interior of the unite square is equal to zero.*

One can easily conclude that the Nash equilibria $(0,0), (1,1)$ are attractors of the dynamical system, while the equilibria states $(1,0)$ and $(0,1)$ are saddle points. Before to continue we recall that these equilibria correspond respectively with the strategic profiles $(C_{np}, P_{np}), (C_p, P_p), (C_p, P_{np}), (C_{np}, P_p)$.

Once we fix the initial condition of the system, the way in which the populism evolve is determined, hence from the observation above the populism behaviour of politics and citizens eventually converges to one of the attractors equilibria, that is, the populist behaviour of both politics and citizens eventually will converge to high level of populism $(1,1)$ or to a very low level of populist acts $(0,0)$.

As we point out above, the Jacobian matrix at the strictly mixed Nash equilibria describe in theorem 1 and theorem 2 is the same for both cases. Unfortunately under the condition of theorem 2 the sign of A_{pp} and A_{cp} is opposite being positive for the first constant and negative for the second. Hence in this case we can not apply the Hartman-Grobman theorem, since the eigenvalues of the Jacobian matrix are imaginary pure. Even though the equations defining the replicator dynamics allows us to study the behaviour of the dynamica for this case, allowing to conclude the existence of a cycle as is described in the next section.

5.2 Populist, non-populistc Cycles

Most of the specialized literature, see for example Dornbusch (1991), indicates that populist policies are based on expansive monetary policies that cause inflation and economic imbalances as well as debt to face non-productive transfers, lack of investment in the productive sectors and in public services, loss of competitiveness of the economy and lack of interest in improving by large sectors of the population. These, among other evils, would give rise to an economic crisis, which in the end causes populist policies to end

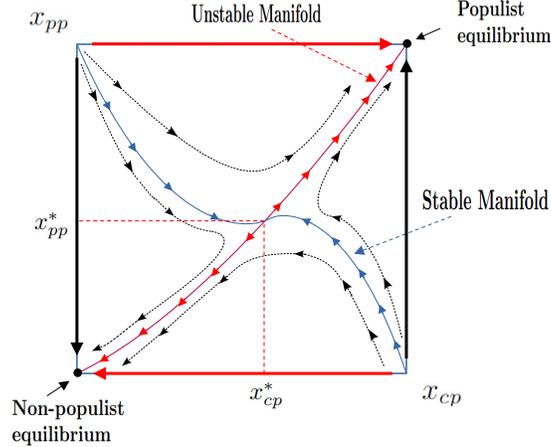


Figure 1: The phases diagram for the dynamics of populism. Arrows in the edges of the unit square represents the evolution of the pure strategies of the players: Red for citizens and black for politics.

up harming, even those who they initially claimed to represent. This makes the majority of citizens look for an alternative to populist politics in the medium term..

Although non-populist policies can give rise to growth and development, attention is due to the fact that many times these factors create inequalities which can give rise to feelings contrary to these policies in large sectors of the population. This can lead to the search for alternatives, which would be populist politics.

A populism cycle is characterized by the following four equations, which holds under the conditions of theorem 2

Citizens become populist when the political parties have stopped seeing it:

$$\dot{x}_{cp} = x_{cp}(1 - x_{cp})B_{cp} > 0. \quad (11)$$

This can happen at election time when the results of the traditional parties that are not populist have ceased to convince the people, and in the face of the appearance of politicians with proposals for short-term solutions, although these may be unrealistic or difficult to achieve, these are the solutions that voters expect.

The behavior of politicians becomes more and more populist when they observe that citizens prefer electoral populism. This scenario is summarized in the equation

$$\dot{x}_{pp} = x_{pp}(1 - x_{pp})(A_{pp} + B_{pp}) > 0 \quad (12)$$

A more detailed explanation is as follows: To ensure the preferences of the electorate, politicians carry out campaigns based on what they understand are the aspirations of the majority of citizens. To learn about them, they invest in political advisors who advise on the political platform and electoral discourse, so it is not surprising that in a scenario where society is accused of problems such as inequality, immigration or the lack of decent jobs, the The electorate prefers those platforms that promise short-term solutions, instead of those that propose long-term plans, more akin to productive investment than to social subsidies. This scenario gives rise to populist platforms with easy slogans and supposed immediate solutions to the difficult problems that society faces. those that can secure a greater number of votes. The populist alternative then appears as a real possibility.

Faced with a panorama where populist politicians predominate, people change their way of seeing politics by taking a less populist behavior:

$$\dot{x}_{cp} = x_{cp}(1 - x_{cp})(A_{cp} + B_{cp}) < 0 \quad (13)$$

Faced with a panorama where populist politicians predominate, citizens can change their view of politics by adopting less populist behavior. The economic crises that harm non-populist sectors and the inability of populist policies, based on the collection and use of public resources to make transfers with the alleged intention of solving the problems of the most vulnerable sectors of society, make that the citizens look for alternatives to the populist government. The lack of incentives for investment, the inadequate use of resources allocated to public services, the lack of medicines and quality jobs and, in general, the lack of incentives for production make social sectors that previously supported populist policies now feel the need of a paradigm shift. For a more detailed explanation of this situation see for example Rodrik (2017). This trend is reflected by the following equation

$$\dot{x}_{pp} = x_{pp}(1 - x_{pp})B_{pp} < 0 \quad (14)$$

Once in this scenario the cycle restarts.

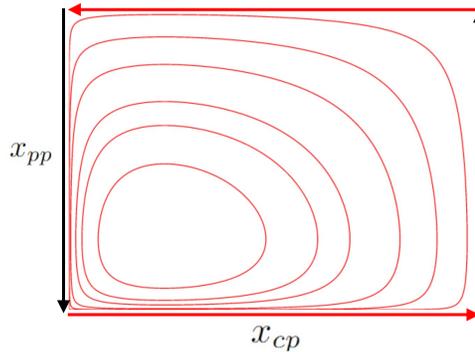


Figure 2: The phases diagram for the dynamics of populism.

6 Conclusions

In this work we have presented a model based on the theory of evolutionary games that allows us to analyze the trajectories that the populist behavior of citizens and politicians will follow over time with respect to different real scenarios.

In theorem 1 we show and interpret the general conditions that imply the appearance of cycles of populism-non-populism. Although in this case the Hartman-Grobman theorem does not allow us to conclude anything about the stability of the equilibrium in mixed strategies, the system of differential equations describing the dynamics of the replicator is the sufficient tool to show the evolution of the behavior of citizens and politicians towards populism. In this case, the analysis allows us to conclude the appearance of cycles central to the mixed Nash equilibria of periods of low level of populism on the part of both citizens and politicians, followed by the appearance of populist behavior on the part of one of the two, achieved after this. periods of high populism of both, and so on.

On the other hand, theorem 1 allows us to find conditions that guarantee the existence of multiple Nash equilibria in the model, for this scenario the Hartman-Grobman theorem allows us to carry out the analysis of the stability of the mixed equilibrium, allowing us to conclude that under these conditions there is the

possibility that both types of populist and non-populist behaviors coexist in periods of time. However, depending on the location of the initial conditions (percentages of citizens and populist politicians), the individual behavior of both actors in political life will converge to only one of these behaviors, that is, eventually the percentage of populist individuals will be extremely low or will converge to totally populist behaviors.

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