Contractive piecewise continuous maps modelling networks of inhibitory neurons.

Ruben Budelli[*], Eleonora Catsigeras[†] and Álvaro Rovella[‡]

15th. August, 2005

Abstract and slides of the talk in the
9th. Tamagawa Dynamic Brain Forum,
Auckland, New Zealand, 2005

Abstract

We consider piecewise continuous maps on a compact region of a finite dimensional manifold, that separates the distances between the different continuity pieces but are locally contractive. We prove that generically those maps are assympthotically periodic, having a finite number of persistent limit cycles. We apply this result to prove that a generic network of more than two inhibitory neurons phase lock to a periodic behaviour that persists under small perturbations of the set of parameter values.

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CONTRACTIVE PIECEWISE CONTINUOUS MAPS

MODELLING NETWORKS OF INHIBITORY NEURONS

Joint work of

Eleonora Catsigeras

Álvaro Rovella

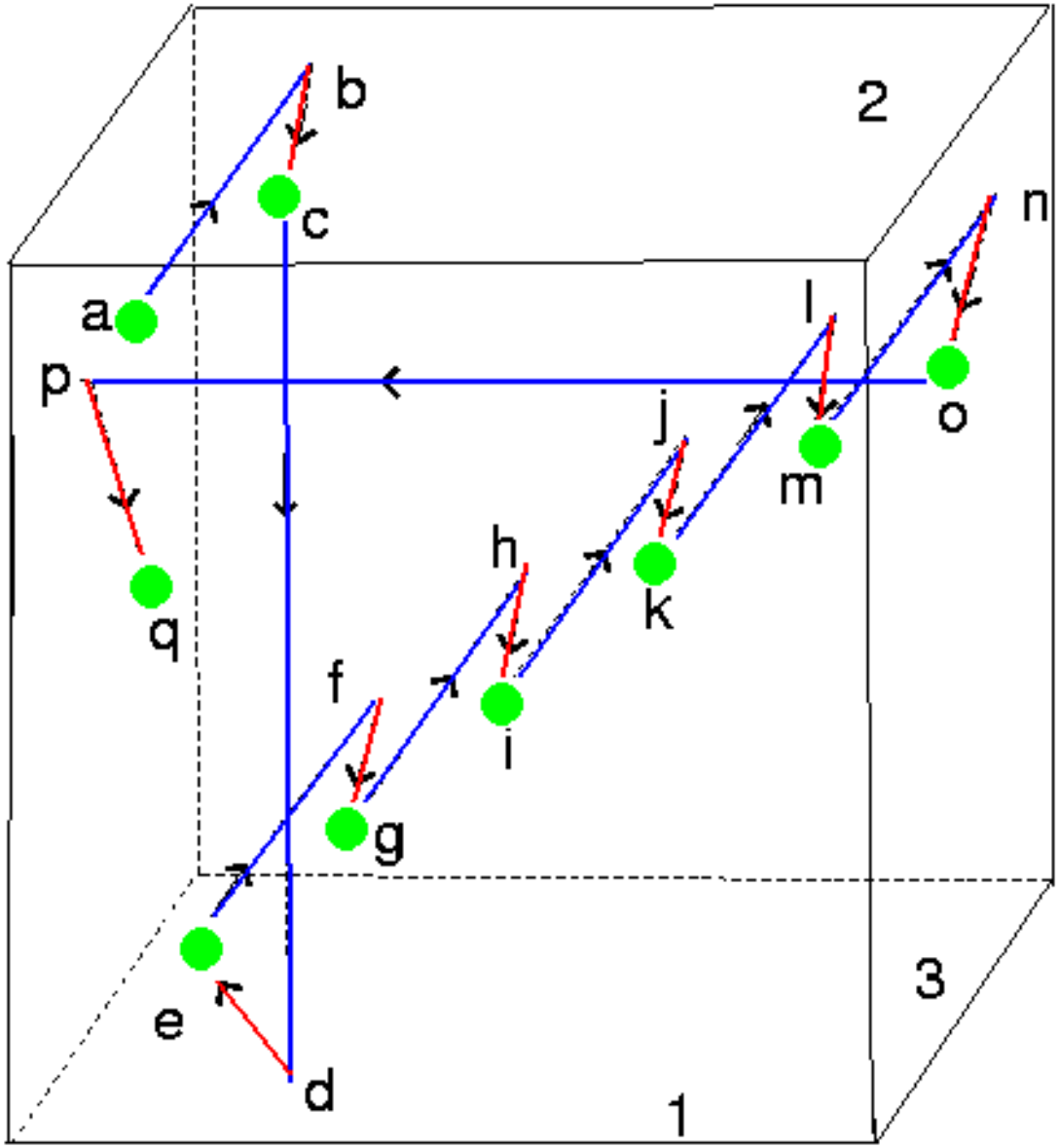
Ruben Budelli

Universidad de la República

Uruguay

9º Tamagawa Dynamic Brain Forum, Auckland 2005

Front faces 1, 2 and 3: Threshold levels of neurons.
Backward faces: Zero levels of voltage of the neurons.



In green: Evolution of the system while not firing. A line from backwards to front, seen like a point due to perspective.

In blue: Firing of a neuron. A line from the front faces to the paralell backward faces.

In red: Negative synapses. Reduction of the voltage of other neurons not fired.

- For n neurons: dynamics in a n-dimensional cube.
- G = First return map to the backward faces of the cube.
- F= G o G = Second return map.
- F^k = 2k-th. return map.
- Atoms of generation k = continuity pieces of F^k.

- F is piecewise continuous (discontinuity (n-2)-dimensional lines).

- F is contractive in each continuous piece (with a well chosen metric in the (n-1) dimensional dominium of F).

- F has the separation property:

  Atoms of the same generation are pairwise disjoint, so there is a positive minimum distance between them (if the neurons are not very different).That is why we choose the Second return map.

THEOREM 1:

Contractive piecewise continuous maps

F in $n - 1 \geq 2$ dimensions

that have the separation property

generically exhibit a persistent limit cycle

(i.e. a periodic attractor that persists under small perturbations of the map F).

COROLLARY:

Generic networks of $n \geq 3$ inhibitory neurons

(that are not very different)

phase lock to a persistent periodic

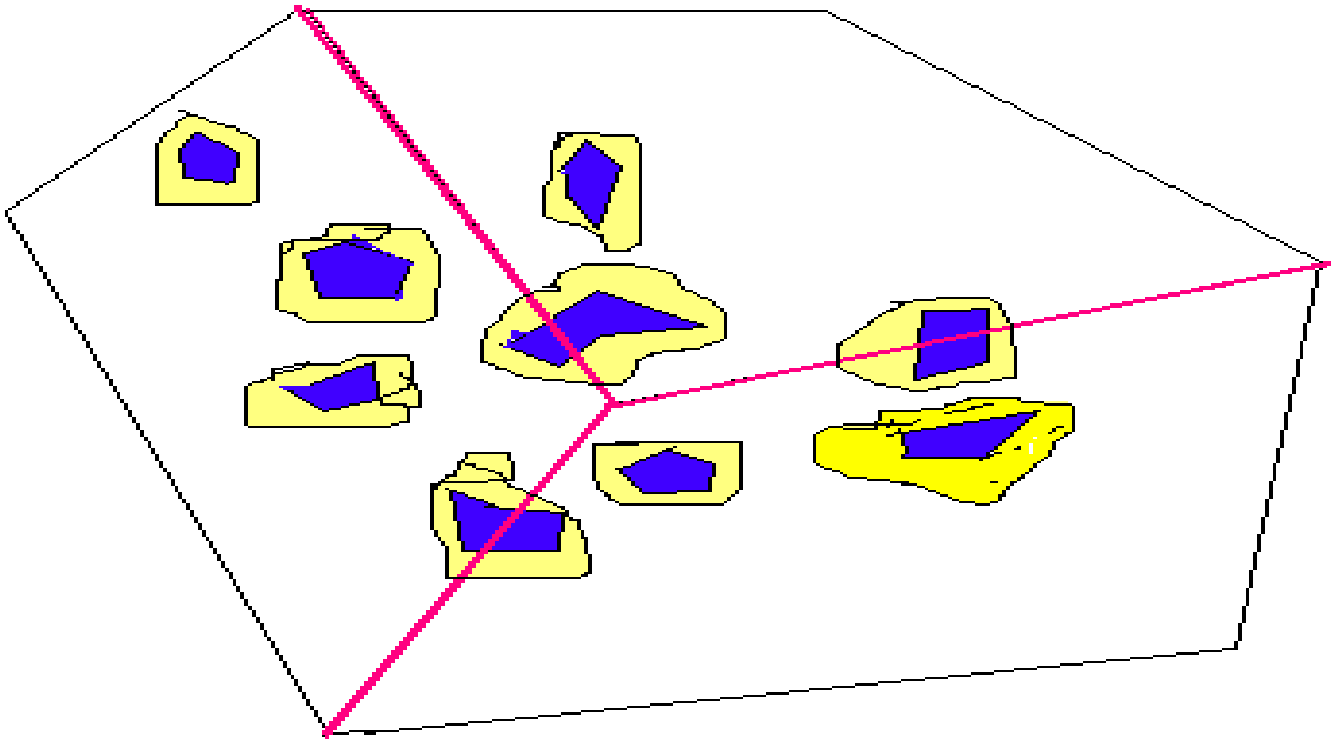
behaviour.

- **GENERIC** SYSTEMS: Not all systems but a dense class of them. If F does not exhibit a limit cycle, SOME arbitrarily small perturbation of F does.

- LIMIT CYCLE IS **PERSISTENT**: If F does exhibit a limit cycle ANY sufficiently small perturbation of F also does.

Proof of Theorem 1:

- (A) Atoms of generation k **have diameter** $< \lambda^k$ where $0<\lambda<1$ is a constant, that exists due to contractiveness of F.
- (B) If **discontinuity lines of F do not intercept atoms** of generation k then some iterate p of F transforms a atom inside itself. Due to contractiveness of F (and the Fixed Point Theorem) there exists a fixed point of F^p; so a **periodic orbit of F is the limit set** of all other orbits.
- (C) **Extend F** to small neighborhoods of its continuity pieces maintaining contractiveness and the separation property.
- (D) Perturb F using (A), so that the **discontinuity lines of new F does not intercept the extended atoms.** Finally use (B).



In blue: Atoms of generation k of the return map F

In yellow: Atoms of an extension of F. Any map perturbed from F has its atoms in the blue and yellow regions.

In red: the discontinuity lines of F.

THEOREM 2:

Contractive piecewise continuous maps F in $n-1 \geq 2$ dimensions that have the separation property

and do not have a periodic limit cycle

exhibit a Cantor set attractor

(i.e.non numerable infinite set, inside which dynamics is chaotic).

COROLLARY:

Non generic networks of $n \geq 3$ inhibitory neurons do **not** phase lock to a periodic behavior and **exhibit a Cantor set chaotic attractor**.

Proof of Theorem 2:

- (A) Atoms of generation k **do intercept the discontinuity lines of F for all k.**

- (B): (A) implies that atoms of all generations divide in the next generation.

- (C): Contractiveness and (B) imply that **the attractor is a Cantor set.**

- (D) The separation property implies that dynamics inside **the attractor is chaotic.**

CATSIGERAS, Eleonora.

9th TAMAGAWA DYNAMIC BRAIN FORUM – DBF 2005

CONTINUING A SEARCH FOR PARADIGMS TO EXPLORE THE BRAIN AS A COMPLEX SYSTEM

The 9th Tamagawa BDF will this year be held in Auckland, New Zealand, on November 7th to 9th, and will bring together theoretical and experimental neuroscientists from the Pacific Rim and from Europe, to discuss dynamic brain function at levels from abstract attractor theory, through biophysical and computational models, to experimental data suitable for the testing of theory.

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Department of Psychological Medicine
The University of Auckland
Auckland, NZ
7-9 November, 2005

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SCIENTIFIC PROGRAMME

MONDAY 7 NOVEMBER

Session 1
Chairperson Jim Wright

09:30	Christopher Rennie
10:10	Peter Robinson
10:50	**MORNING TEA**
11:20	Osame Kinouchi
12:00	Ichiro Tsuda

12:40	**LUNCH**

Session 2
Chairperson Walter Freeman

13:40	Minoru Tsukada
14:10	Yoshiyuki Yamazaki
14:40	Shigetoshi Nara
15:20	**AFTERNOON TEA**
15:40	Antonio Roque
16:20	James Wright
17:00	**FINISH**

TUESDAY 8 NOVEMBER

Session 3
Chairperson Rob Kydd

09:00	Peter Hunter
09:40	Moira Steyn-Ross
10:00	Marcus Wilson
10:20	Jamie Sleigh
10:40	**MORNING TEA**
11:00	Alistair Steyn-Ross
11:20	Bruce MacDonald
12:00	Peter Davis

12:40	LUNCH

Session 4
Chairperson Christopher Rennie

13:40	Walter Freeman
14:20	Eduardo Mizraji
15:00	Eleonora Catsigeras
15:40	**AFTERNOON TEA**
16:00	Fanji Gu
16:40	Gert Hauske
17:20	**FINISH**

WEDNESDAY 9 NOVEMBER

Session 5
Chairperson Minoru Tsukada

08:40	Xiaochuan Pan
09:20	Bernard Balleine
10:00	Jan Lauwereyns
10:40	**MORNING TEA**
11:00	Guy Sandner
11:40	Jeff Wickens

12:10	**LUNCH**

Session 6
Chairperson Antonio Roque

13:10	Robert Froemke
13:50	Stefan Rotter
14:30	Yutaka Sakai
15:10	**AFTERNOON TEA**
15:30	Muneyoshi Takahashi
16:10	Pei-ji Liang
16:50	**FINISH**

ABSTRACTS

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