Mathematical models of Spikes Codes as deterministic self-adapting dynamical systems.

Eleonora Catsigeras
Instituto de Matemática. Facultad de Ingeniería.
Universidad de la República. Montevideo. URUGUAY.
eleonora@fing.edu.uy
www.fing.edu.uy/ eleonora

Abstract and slides of the talk in the
7th. International Neural Coding Workshop
in honour of Prof. José Segundo,
Montevideo, Uruguay, 2007

ABSTRACT

In a scale level of a whole system the network of several interacting neurons can be modeled as a deterministic system, whose actual state is modified by the probabilistic incidence of external excitations. It can be mathematically studied with the theoretical tools of the Dynamical Systems Theory in a qualitative description, rather than using a quantitative method. Also Ergodic Theory known results are applicable.

Those abstract mathematical tools provide rigourously proved properties of some n-neurons system models (with n as large as wanted) and the qualitative tasks of its spike trains. For instance, some systems are mathematically proved to exhibit several characteristic structural stable limit cycles while its evolution is just remaining from its internal spikes, in the meanwhile it is not modified by the external excitation spikes of the sensorial neurons of the system. But at the same time, the same system can have a response capable of processing a large amount of information while a probabilistically generated external excitation is added to the system.

It is a known mathematical theorem of the Ergodic Theory that each deterministic (not random) system does define a self characteristic probabilistic distribution (know as "measure of maximal entropy or of topological entropy of the system") such that, if external excitation were added to the system according to that theoretical probabilistic distribution, the system should optimize the information that is internally processed.

These slides can be downloaded from the web page:

http://eleonora.wikispaces.com/NeuralCoding07

MATHEMATICAL MODELS OF SPIKE CODES AS DETERMINISTIC DYNAMICAL SYSTEMS

Eleonora Catsigeras

Universidad de la República

Uruguay

eleonora@fing.edu.uy
www.fing.edu.uy/~eleonora

Neural Coding 2007. Montevideo
in honour of Prof. José Segundo

NERVOUS SYSTEMS OF n NEURONS WITH UP TO n(n-1) SYNAPSIS:

modelled as a **DETERMINISTIC DYNAMICAL SYSTEM.**

highly **COMPLEX:** n ~ 10 exp(12) cells

strongly **NON-LINEAR** → each cell has a **THRESHOLD** for producing **SPIKES**

→ System of n cells → may have **LIMIT CYCLES**, **CHAOTIC ATTRACTORS**

→ make **DECISSIONS**

EACH CELL DIFFERENTIAL EQUATION Hodgkin-Huxley model (1952)

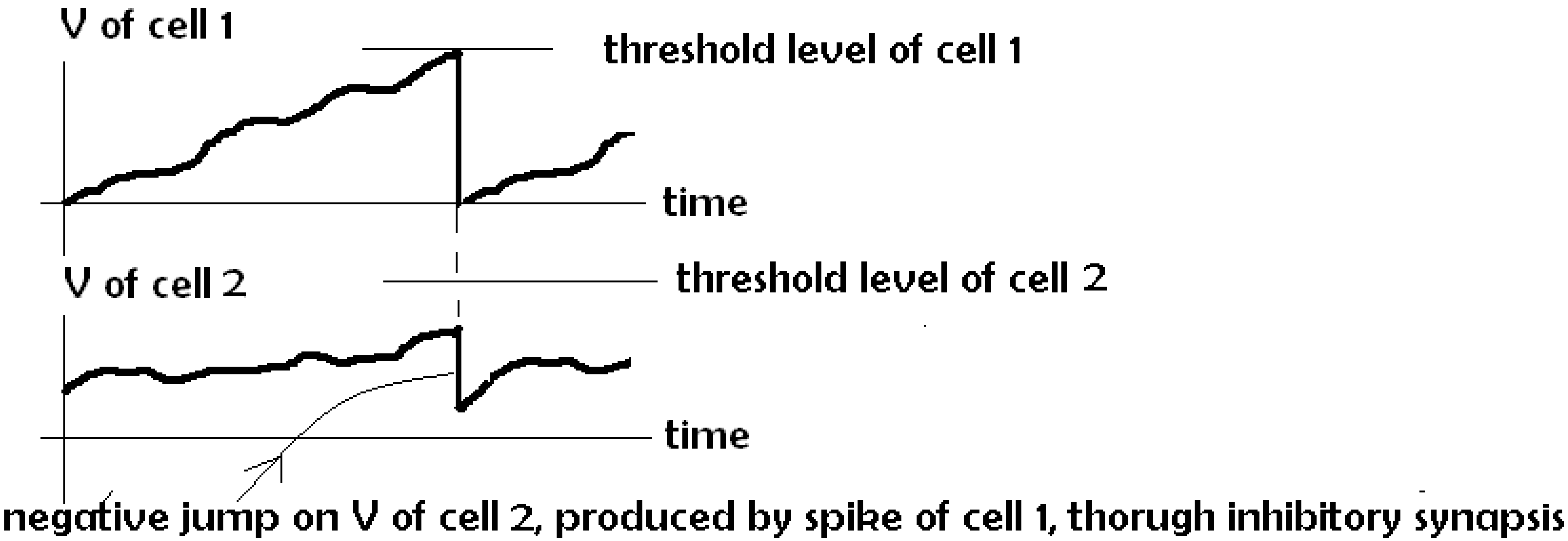
particularly Integrate and fire type (Stein-French-Holden 1972)

Relaxation

in general: **INCREASING WITH TIME POTENTIAL with:**

- **a THRESHOLD LEVEL TO PRODUCE one SPIKE**
- **the (n-1) SYNAPSIS connections (for each of n cells):**

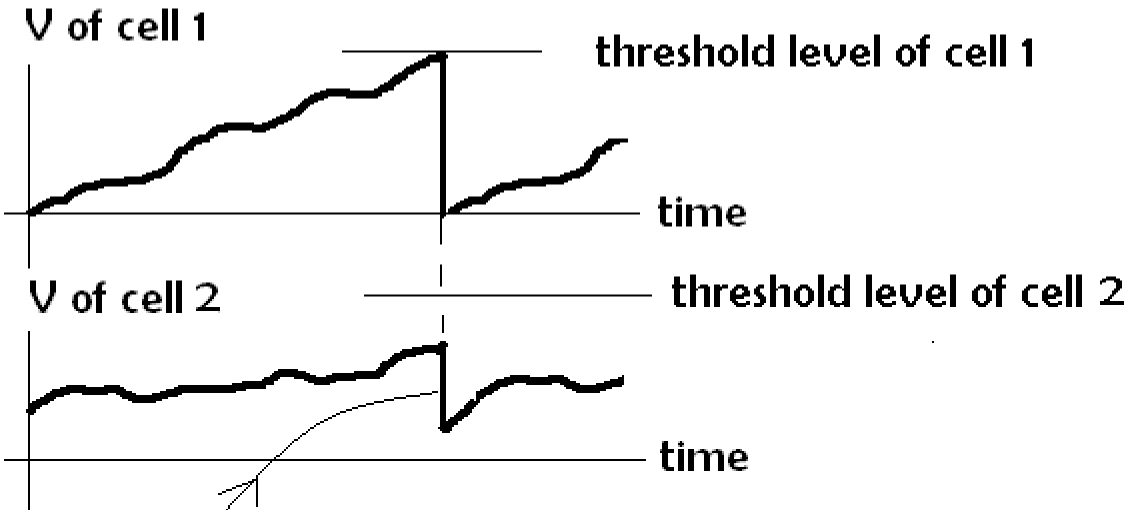
  positive (exhitatory) or negative (inhibitory) jump when other cells of the same system produce spikes (SYNAPSIS).

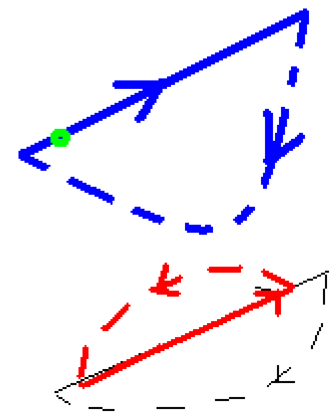


EACH CELL DIFFERENTIAL EQUATION Hodgkin-Huxley model (1952)
particularly Integrate and fire type (Stein-French-Holden 1972)
Relaxation

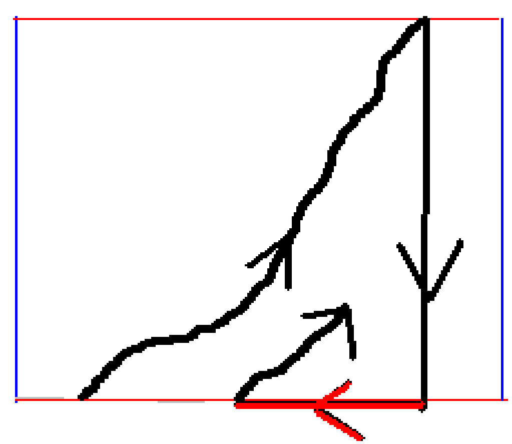
in general: **INCREASING WITH TIME POTENTIAL with:**

- **a THRESHOLD LEVEL TO PRODUCE one SPIKE**
- **the (n-1) SYNAPSIS connections (for each of n cells):**
  positive (exhitatory) or negative (inhibitory) jump when other cells of the same system produce spikes (SYNAPSIS).

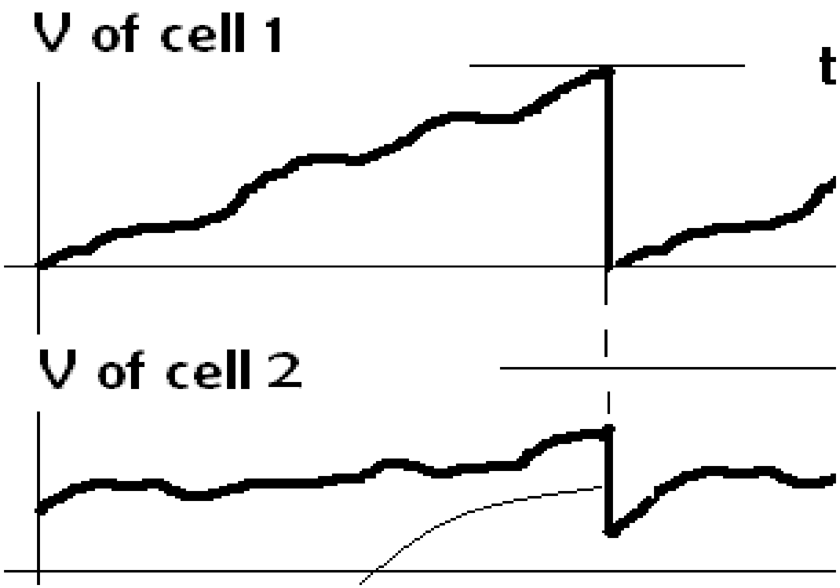


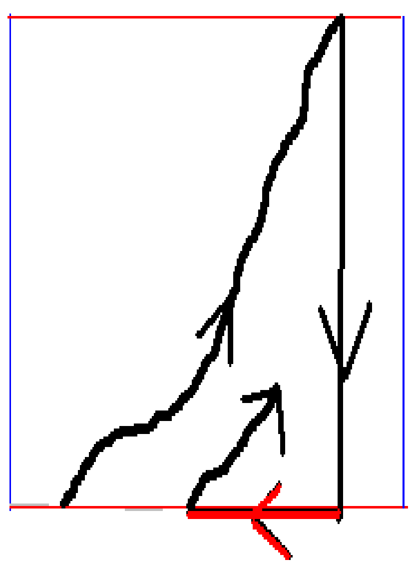


negative jump on V of cell 2, produced by spike of cell 1, thorugh inhibitory synapsis



Dynamical system of 2 cells: flux in a 2-dimensional TORUS:
Discontinuity in the V level of cell 2 (red) when cell 1's spike is produced.





Dynamical system of 2 cells: flux in a 2-dimensional TORUS:

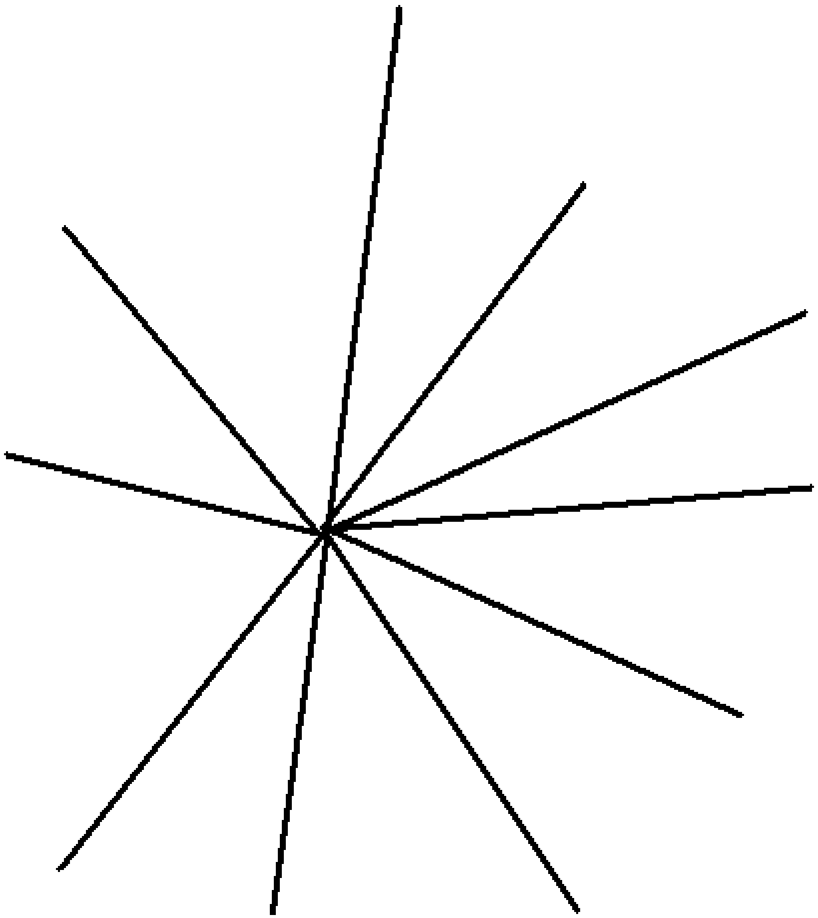
IT IS IMPORTANT
because it changes
•the skype train
•its code
•its information amount

THE TIME ELAPSED
between

A spike of cell 1
next spike of cell 2
and reciprocally

J.SEGUNDO FIRST POSSED THE QUESTION:

INFORMATION QUANTITY IS NOT ONLY DEPENDENT OF NUMBER OF SPIKES OF EACH CELL, BUT ALSO OF TIME ELAPSED BETWEEN SPIKES.



n neurons:

n independent axis = n dimensions.

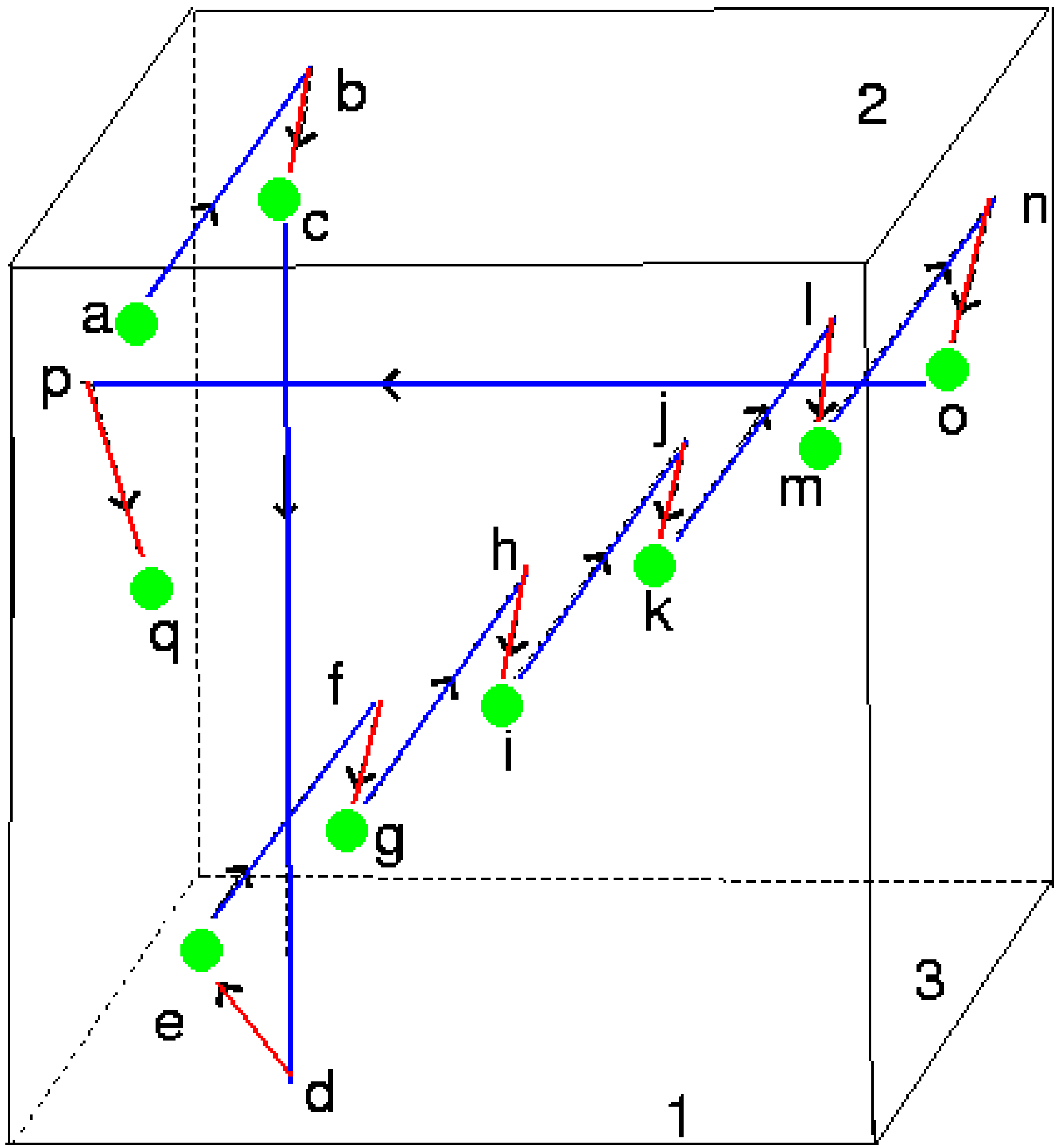
flux in a n-dimensional TORUS
(Budelli 1998)

POINCARÉ SECTION: FIRST
RETURN MAP:

it is a descrete dynamical system
in a n-1 dimensional ball with
discontinuities over a n-2
dimensional surface, of finite jumps.

A PIECEWISE CONTINUOUS
DISCRETE (n-1) dimensional
DYNAMICAL SYSTEM
in a topological ball.

Front faces 1, 2 and 3: Threshold levels of neurons.
Backward faces: Zero levels of voltage of the neurons.



In green: Evolution of the system while not firing. A line from backwards to front, **seen like a point** due to perspective.

In blue: Firing of a neuron. A line from the front faces to the paralell backward faces.

In red: Negative **synapses**. Reduction of the voltage of other neurons not fired.

- F is piecewise continuous (discontinuity (n-2)-dimensional surfaces).

- F is contractive in each continuous piece (with a well chosen metric in the (n-1) dimensional dominium of F). This is due to inhibitory synapsis hypothesis.

- F has the separation property:

  Atoms of the same generation are pairwise disjoint, so there is a positive minimum distance between them (if the neurons are not very different).

THEOREM 1 (Budelli-Catsigeras-Rovella 2005):

Contractive piecewise continuous maps

F in n -1 ≥ 2 dimensions

that have the separation property

generically exhibit persistent limits cycles

(i.e. a periodic attractors that persists under small perturbations of the map F).

COROLLARY:

Generic networks of n ≥ 3 inhibitory neurons

phase lock to <u>persistent</u> periodic behaviours

(up to many periodic cycles, and of finite but unbounded!!! period).

PERSISTENCE OF LIMIT CYCLES:

They are structurable stable:

- Persist under (small) perturbations of the parameters of the system.
- Do not persist (but change) under significative perturbations of the parameters
- Jump from a cycle to other under external excitations (for instance spike train of cells out of the system of n neurons, but synaptically connected to it.)

THEOREM 2 (Budelli-Catsigeras-Rovella 2005):

Contractive piecewise continuous maps F in $n-1 \geq 2$ dimensions that have the separation property and do not have a periodic limit cycle **exhibit a Cantor set attractor** (i.e.non numerable infinite set, inside which dynamics is chaotic).

COROLLARY:

Non generic networks of $n \geq 3$ inhibitory neurons do **not** phase lock to a periodic behavior and **exhibit a Cantor set chaotic attractor**.

ENTROPY

As defined in the Ergodic Theory of Math. Dynamical Systems:

ENTROPY is the limit (assympthotic to the future) of the increasing RATE of the amount of information per unit of time.

It is defined in this way to measure the information INCREASING RATE for CHAOTIC SYSTEMS (sensible to arbitrarily SMALL perturbations of spatial state conditions).

It is not adequate to measure assympthotic rater of information amount for systems exhibiting LIMIT CYCLES, because it would be ZERO.

P ={ A1, A2, A3,......, Am} spatial partition.

Amount of information of the partition P given a espacial probability distribution p, is

$$\boxed{H(P) = - \Sigma\, p(Ai) \log (p(Ai))}$$

Entropy of the system F with the partition P, given de espacial distribution p is:

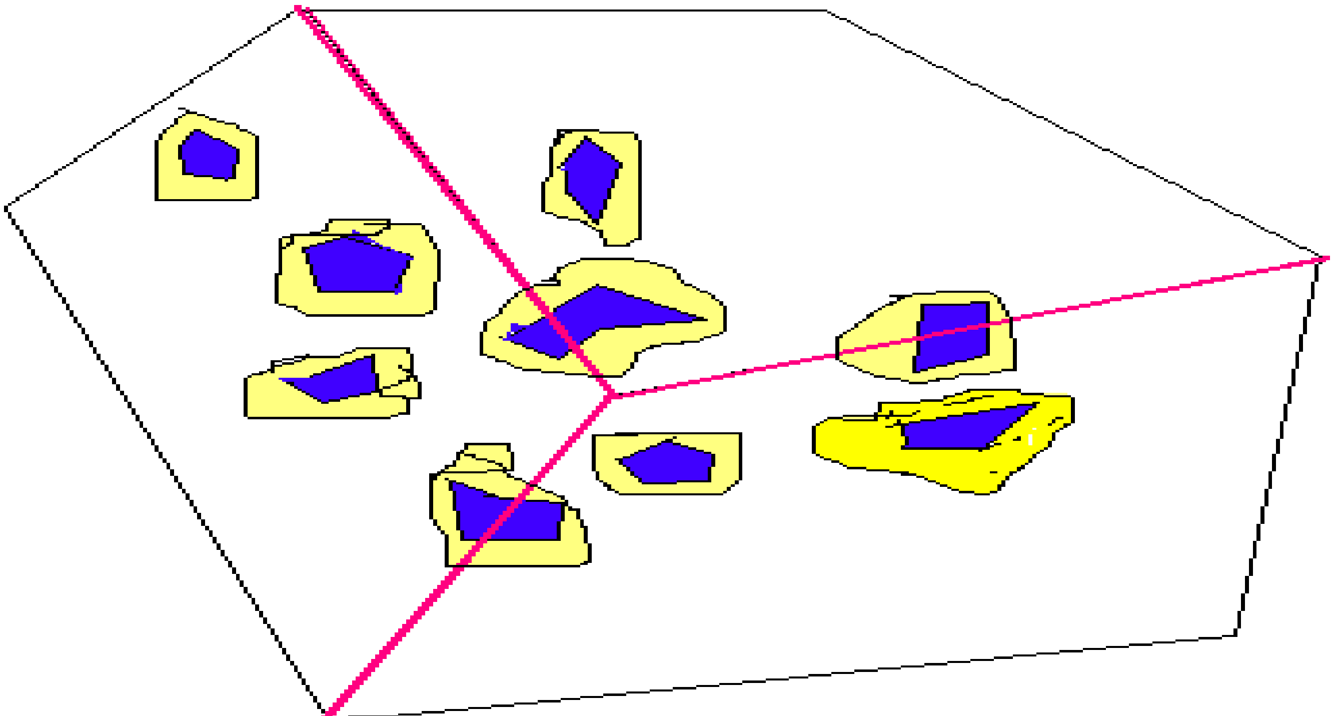
$$\boxed{H(P,F) = \lim_{n \to \infty} H(F^{-n} (P)) \,/\, n}$$

- **FOR PERIODIC (LARGE PERIOD) SYSTEMS: AVOID TAKE LIMIT AND DIVIDE USING n = PERIOD.**
- **WHAT IS THE ADEQUATE PROBABILITY DISTRIBUTION (?????)**

THERE EXISTS ONE THAT IS INVARIANT UNDER TIME, AND MAXIMIZES THE ENTROPY OF THE SYSTEM: TOPOLGICAL ENTROPY OF THE SYSTEM.

MAXIMAL ENTROPY OF THE SYSTEM: COUNTING THE RATE OF THE INCREASING NUMBER OF ATHOMS OVER TIME.

The espacial probability distribution thus obtained is inherent to the system.



OPEN QUESTION:

¿Is the maximal entropy probability distribution (which can be computed knowing only the dynamics of the system and not its external inputs) THE SAME (or near) the probability distribution of EXTERNAL EXCITATIONS that change the state of the neural system?

If YES: The neural system would be optimally adapted to process information that receives from outside.

CONCLUSIONS:

1) **EXTERIOR SIGNALS DO CHANGE THE BEHAVIOR** (spike train codes) OF THE SYSTEM of n neurons, and allow:

categorize the inputs
make decissions

between large number of possibilities if the system has n neurons (n ~ 10 exp 12),

---

2) **Period of spike train codes are theoretically as large as needed (finite but UNBOUNDED)** so, in practice (not assympthotically) they can be considered as "almost" chaotic.

Long periods of LIMIT CYCLES OF SPIKE TRAINS allow the system:

long term memory functions
categorize many different inputs
have a scenario of many (finitely many but a large number) of decissions

---

CONCLUSIONS:

3) INSIGNIFICATIVE PERTURBATIONS OF THE SYSTEM DO NOT CHANGE THE NUMBER NOR THE CHARACTERISTICS OF SPIKE TRAIN PERIODIC LIMIT CYCLES OF THE SYSTEM. **(THE SYSTEM IS STRUCTURABLY STABLE)**

---

4) SIGNIFICATIVE CHANGES IN THE PARAMETERS OF THE SYSTEMS for instance: some decreasing in the number of neurons,
or in the number of synapsis, (as those occuring during childhood),
also some changes of the numerical coefficients modelling the chemical reactions in the membranes of the cells, etc

DO CHANGE

the quantity, characteristics, size of its basins, and periods of the limit cycles of spike trains, and thus also CHANGE

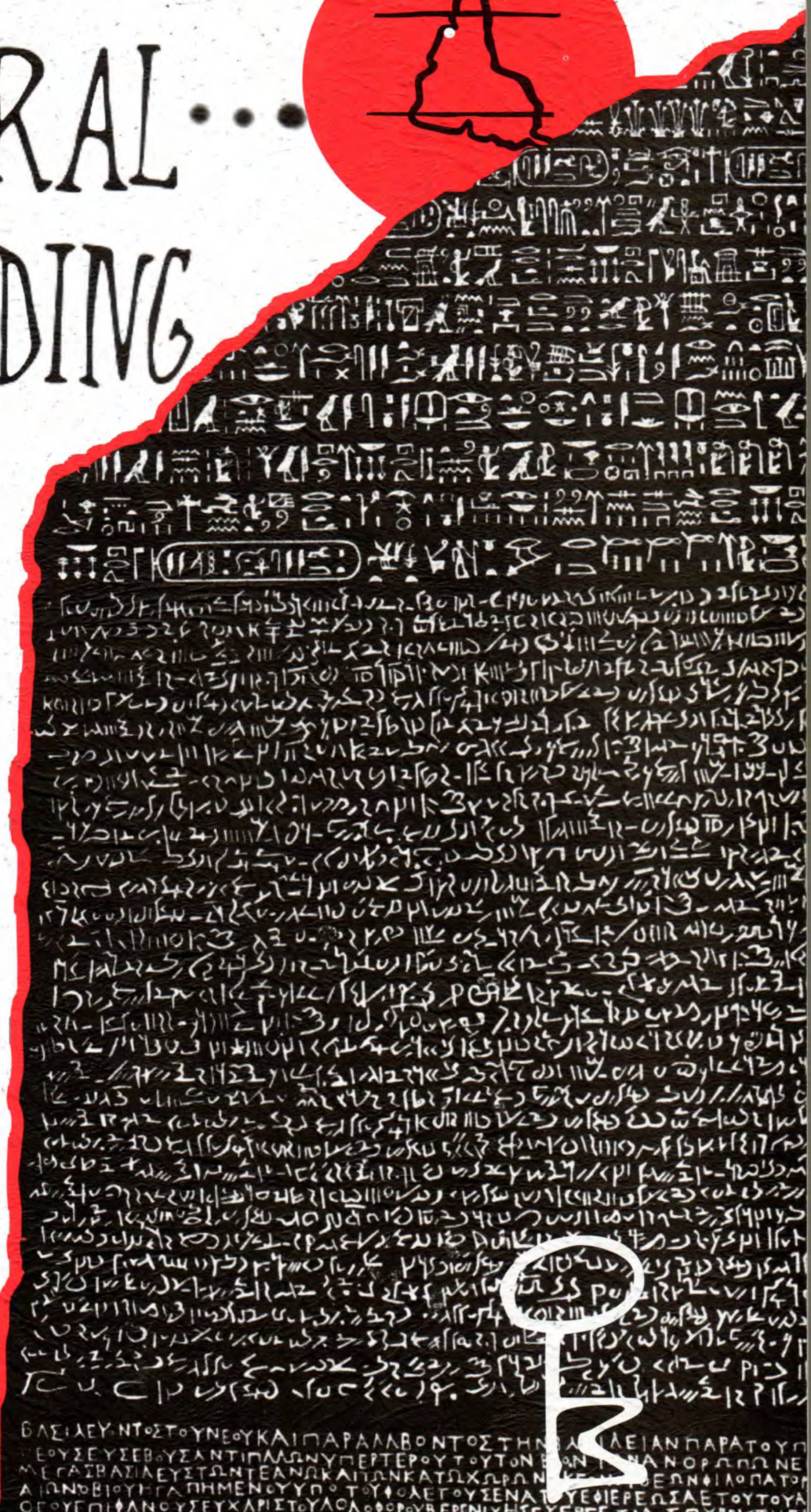
the **maximal entropy** capability of the system, and
the INTERNAL PROBABILITY DISTRIBUTION and the THEORETICAL INPUT SIGNALS that the system can process optimally.

ELEONORA CATSIGERAS

NEURAL... CODING

7th International Neural Coding Workshop

Montevideo Uruguay



7th International Workshop

NEURAL CODING 2007

7-12 November Montevideo, Uruguay

http://neuralcoding2007.edu.uy

LOCAL ORGANIZING COMMITTEE (Uruguay)

Budelli, Ruben (BioMathematics) Chairman
Caputi, Angel (NeuroPhysiology) Chairman
Gómez, Leonel (NeuroPhysiology) Chairman
Aguilera, Pedro (NeuroPhysiology)
Curti, Sebastian (NeuroPhysiology)
Macadar, Omar (NeuroPhysiology)
Migliaro, Adriana (NeuroPhysiology)
Mizraji, Eduardo (BioPhysics)
Lewowicz, Jorge (Mathematics)
Morales, Francisco (NeuroPhysiology)

REGIONAL COMMITTEE

Tamarit, Francisco (Córdoba, AR)
Samengo, Inés (Bariloche, AR)
Balenzuela, Pablo (Buenos Aires, AR).
Kohn, Andre (Sao Paulo, BR).
Robles, Luis (Santiago, CH).
Quillfeldt, Jorge (Rio Grande do Sul, BR)
Roque, Antonio (Universidad de Sao Paulo, Brasil)

INTERNATIONAL COMMITTEE:

Braun, Hans (Marburg, DE)
Borisyuk, Roman (Plymouth, UK)
Bugmann, Guido (Plymouth, UK)
Chialvo, Dante (Chicago, US)
Eckhorn, Reinhard (Marburg, DE)
Kohn, Andre (Sao Pablo, BR)
Lansky, Petr (Prague, CZ)
Longtin, André (Ottawa, CA)
Moss, Frank (St. Louis, US)
Poon, Paul (Tainan,Taiwan R.O.C.)
Richmond, Barry (Washington, US)
Rinzel, John (New York, US)
Rospars, Jean-Pierre (Versailles, FR)
Segundo, José (Montevideo, UR)
Villa, Alessandro (Grenoble, FR)
Moore, George P. (Los Angeles, US)
Rosenberg, Jay (Glasgow, UK)
Stiber,Michael (Bothell, US)
Sato, Shunsuke. (Osaka, JP)

Contact: mariana@neuralcoding2007.edu.uy

Sponsors

- Instituto de Investigaciones Biológicas Clemente Estable (IIBCE)
- Intendencia Municipal de Montevideo (IMM)
- Internacional Brain Research Organization (IBRO)
- Latin American Brain Research Organization (LABRO)
- National Science Fundation (NSF)
- CSIC, Universidad de la República Oriental del Uruguay
- Programa de Desarrollo de Ciencias Básicas (PEDECIBA)

Foreword

NEURAL CODING symposia series are multidisciplinary workshops of scientists specialized in neural coding. The 2007 edition will be held for the first time in Latin America. This workshop will also be celebrated in honour of Prof. José Pedro Segundo, founder and leading scientist in the field, and active participant since their inception.

Our aim is to held the VII Neural Coding Workshop in the same spirit that made the previous meetings, that traditionally are relatively small and highly interdisciplinary. Their major emphasis is the search for common principles in neural coding, though without neglecting functionally relevant differences between systems. Previuos workshops were held in Prague (CZ, 1995), Versailles (F, 1997), Osaka (JP, 1999), Plymouth (UK, 2001), Aulla (Italy, 2003) and Marburg (Germany, 2005). In this last meeting in Margurg and considering the growing importance of the field in Latin America's south cone, Montevideo, Uruguay was elected as the next location.

Neural Coding symposia bring together neuroscientists from different fields with the conviction that multidisciplinary approaches are essential for better understanding neural coding mechanisms as well as their disturbances in clinical cases. Hence, the attendees of the Neural Coding Workshop should be prepared to cross the borders of their own disciplines. A particular aim of the workshop is to compare and integrate results from different functional levels, including subcellular, single-neuron, neuronal network and systemic levels. It is also the intention to assert between-level interdependencies and their implications for sensation, cognition, autonomous control and action.

Expected and encouraged will be the discussion (intense and, of course controversial) about the experimental, modelling and analytical approaches and also about their promises (in solo or combined) for a better understanding of neural coding mechanisms. The possible role of these techniques for unraveling the pathophysiology and for designing treatments of neurological and related diseases should not be neglected. Major emphasis shall be laid on biologically inspired formal and computer-implemented models which could elucidate the functionally relevant dynamics of the neural coding mechanisms involved.

We accepted 71 abstracts, from which 40 were selected to be presented orally. Two special sessions were programmed: 1) Session "Donald Perkel" on Spike Train Analysis, and 2) Session in honor of Jose P. Segundo, with the scientific presentations organized by Dante Chialvo.

We receive 95 inscriptions: 28 from Europe, 15 from Asia, 45 from the South Cone, 16 from the rest of America and 1 from Africa; 36 of them are graduate students.

Ruben Budelli   Angel Caputi   Leonel Gómez

Spatio temporal patterns

Thursday 8th

Montevideo City hal

04:00 pm A spatiotemporal coding in the hippocampal CA3-CA1 networks
Minoru Tsukada, Yasuhiro Fukushima, Hiroshi Kojima, Ichiro Tsuda, Yutaka Yamaguti, Shigeru Kurod
p.43

04:30 pm Spatiotemporal Response of Neurons to Time-Dependent Spectral Features
Pablo Balenzuela, Jordi Garcia-Ojalvo.
p.46

05:00 pm Conditions for divergency in the connectivity patterns of synfire chains emerging from random graphs with embedded spike timing dependent plasticity
Tatyana S. Turova, Javier Iglesias, Alessandro E.P. Villa
p.48

05:30 pm Effect of gap junction coupling on the dynamic range of a model of the rod photoreceptor layer
Rodrigo Publio, Rodrigo Oliveira, Antônio C. Roque
p.50

06:00 pm Successive filtering of the spike train with deterministic temporal structure by spiking neurons
Yoshiyuki Asai, Takashi Yokoi, Alessandro E.P. Villa
p.53

Oscillatory Processes

Friday 9th

Casapueblo, Punta del Est

10:00 pm Coupling nonlinear oscillators for fun and profit
Adi Bulsara
p.59

10:30 pm Identification of oscillators that generate Mauthner cell inhibitory synaptic noise
Fabio Marti, Henri Korn, Philippe Faure
p.60

11:00 pm Interdependencies of neural impulse pattern and synchronization: implications for physiological functions and disease
Hans A. Braun, Horst Schneider, Svetlana Postnova
p.62

Session Donald Perkel

Saturday 10th

Montevideo City hall

09:00 am Temporal encoding of electrosensory information: decoding and ambiguities
Curtis Bell

09:30 am Notes on neural coding. Spike trains and point processes
José Pedro Segundo
p.67

10:00 am GABAergic drive of thalamic neurons in a songbird circuit essential for vocal learning
David Perkel
p.68

11:00 am Mathematical models of spikes codes as deterministic self-adapting dynamical systems
Eleonora Catsigeras
p.69

11:30 am Modelling the response of cat V1 neurons to drifting gratings
Guido Bugmann
p.70

Information

Saturday 10th

Montevideo City hall

12:00 pm Examining the joint neural code of latency and firing rate by Bayesian Binning
Dominik Endres, Johannes Schindelin, Peter Földiák and Mike Oram
p.73

12:30 pm Upper and lower bounds to the estimation of Shannon information from spike train data
Marcelo A. Montemurro, Riccardo Senatore, Stefano Panzeri
p.75

Motor Control

Saturday 10th

Montevideo City hall

02:30 pm Controlling precise movement with stochastic signals
Jianfeng Feng
p.79

03:00 pm Coding of Movement in Parkinson's Disease
George P. Moore
p.81

Sensory Coding

Saturday 10th

Montevideo City hall

04:00 pm Contrast, stimulus selectivity and neuronal responses
Mike W. Oram
p.84

04:30 pm Can models of human sound localization be any useful to owls?
José Luis Peña
p.88

05:00 pm Non-auditory responses in the auditory cortex of behaving chinchillas
Luis Robles, Paul H. Delano, Diego Elgueda and Pedro E. Maldonado
p.89

05:30 pm Envelope encoding in neurons of the ventral cochlear nucleus
Manuel C. Eguia, Guadalupe C. Garcia and Sebastian Romano
p.90

06:00 pm Should neuronal spikes be given equal weighting in the generation of spectral temporal receptive field?
Paul W.F. Poon, T.R. Chang, T.W. Chiu, P.C. Chung
p.91

Cognitive Processes

Sunday 11th

Montevideo City hall

09:00 am Learning invariant object representations from the spatial-temporal correlations of dynamic retinal images in a network of spiking neurons
Frank Michler, Reinhard Eckhorn, Thomas Wachtler
p.95

09:30 am Tackling complex problems through simple steps: the roles of the M4 subsystem in the cholinergic memory modulation system, and how this may relate to the nature of the engram
J.A. Quillfeldt, V.B. Lanziotti, F. Diehl, L.O. Alvares, G. Sánchez, L. Fürstenau, T. Mello e Souza, E. Kornisiuk, D. Jerusalinsky
p.96

10:00 am Self-control with spiking neural networks playing games
Chris Christodoulou, Gaye Banfield
p.97

10:30 am Different molecular cascades in different brain sites underlie memory consolidation
Iván Izquierdo, Martín Cammarota, Weber |C. Da Silva, Lia R.M. Bevilaqua, Janine Rossato, Juliana Bonii, and Jorge H. Medina
p.99

Session in Honor of J. P. Segundo

02:00 pm Functionality and the structure of neural codes
Barry J. Richmond
p.103

02:30 pm Timing computations in the auditory brain stem
John Rinzel
p.105

03:00 pm Synaptic "noise" in a model thalamocortical network increases information transfer from sensory input to cortex
Leonel Gomez-Sena and Thierry Bal
p.106

03:30 pm Burst firing is a neural code in an insect auditory system
Inés Samengo, Hugo G. Eyherabide
p.107

04:00 pm Brain dynamics at rest: not an oxymoron
Dante R. Chialvo
p.108

05:30 pm *Dr.Ricardo Ehrlich, Major of Montevideo*

Dr.Rodrigo Arocena Chancelor of the Universidad de la República

José Pedro Segundo

Oral Session Donald Perkel
Spike Train Analysis

Mathematical models of spikes codes as deterministic self-adapting dynamical systems

Eleonora Catsigeras

Instituto de Matemática. Facultad de Ingeniería.
Universidad de la República. Montevideo. URUGUAY.
eleonora@fing.edu.uy
www.fing.edu.uy/eleonora

ABSTRACT

In a scale level of a whole system the network of several interacting neurons can be modeled as a deterministic system, whose actual state is modified by the probabilistic incidence of external excitations. It can be mathematically studied with the theoretical tools of the Dynamical Systems Theory in a qualitative description, rather than using a quantitative method. Also Ergodic Theory known results are applicable.

Those abstract mathematical tools provide rigorously proved properties of some *n*-neurons system models (with *n* as large as wanted) and the qualitative tasks of its spike trains. For instance, some systems are mathematically proved to exhibit several characteristic structural stable limit cycles while its evolution is just remaining from its internal spikes, in the meanwhile it is not modified by the external excitation spikes of the sensorial neurons of the system. But at the same time, the same system can have a response capable of processing a large amount of information while a probabilistically generated external excitation is added to the system.

It is a known mathematical theorem of the Ergodic Theory that each deterministic (not random) system does define a self characteristic probabilistic distribution (known as "measure of maximal entropy or of topological entropy of the system") such that, if external excitation were added to the system according to that theoretical probabilistic distribution, the system should optimize the information that is internally processed.