Biological neuronal networks as deterministic dynamical systems.

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ABSTRACT

The network of $n \geq 2$ synaptically connected neurons can be modeled as a deterministic system, whose actual state is modified by the probabilistic incidence of external excitations. It can be mathematically studied with the theoretical tools of the Dynamical Systems Theory in a qualitative description, rather than using a quantitative method. Also Ergodic Theory known results are applicable.

The abstract mathematical tools provide rigourously proved properties of some n-neurons system models (also for large values of n) and the qualitative tasks of its spike trains. For instance, some systems are mathematically proved to exhibit several characteristic structurally stable limit cycles in the evolution of its internal spikes.
Those limit cycles are not modified by the external small random perturbations, but the system can jump from one cycle to other when an external excitation spikes some of the sensorial neurons of the system. The system has a response capable of processing a large amount of information from the environment, as a probabilistically generated external excitation is added to the system.

It is a known mathematical theorem of the Ergodic Theory that each deterministic (not random) system does define a self characteristic probabilistic distribution know as the measure of maximal entropy or of topological entropy of the system. This probabilistic measure is such that, if the external excitation is added to the system with such a distribution, the system should optimize the information that can internally process.

BIOLOGICAL NEURONAL NETWORKS AS DETERMINISTIC DYNAMICAL SYSTEMS

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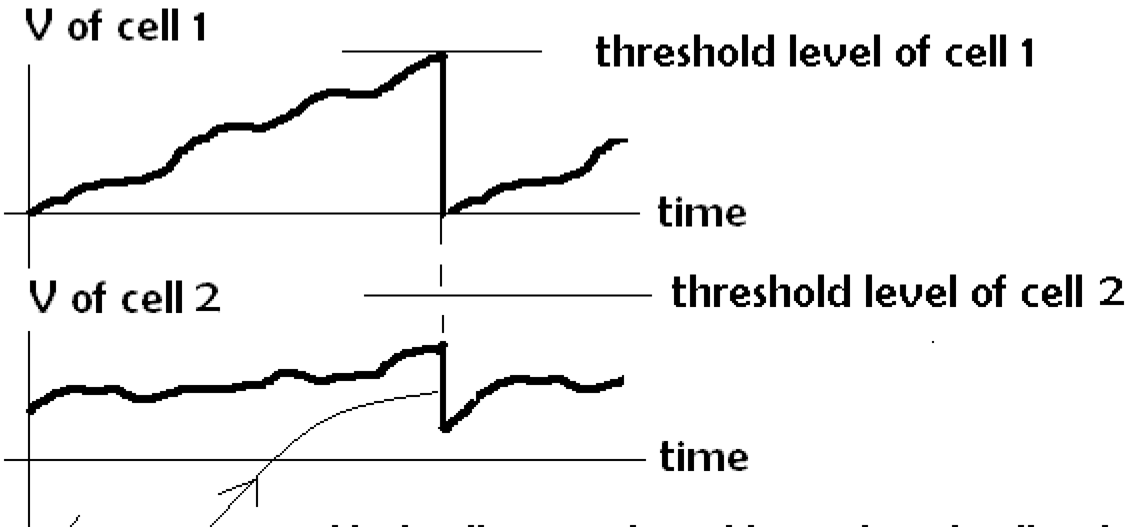
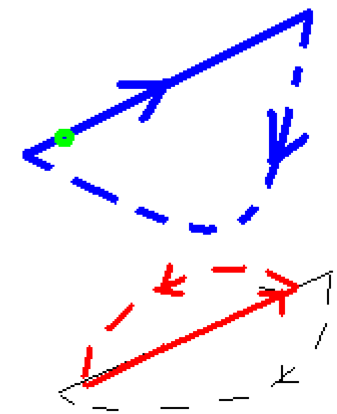
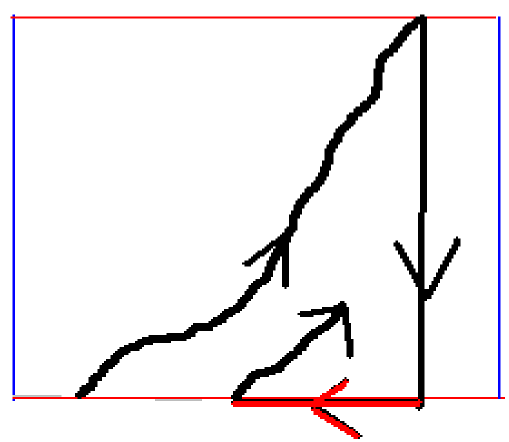
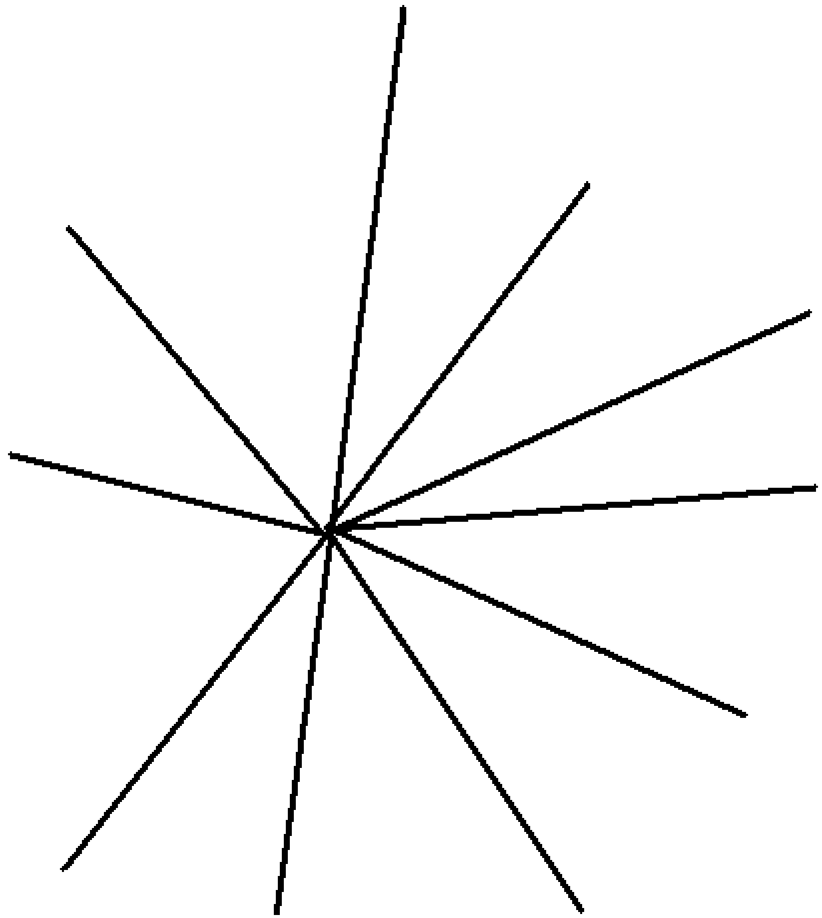




negative jump on V of cell 2, produced by spike of cell 1, thorugh inhibitory synapsis



Dynamical system of 2 cells: flux in a 2-dimensional TORUS: Discontinuity in the V level of cell 2 (red) when cell 1's spike is produced.



n neurons:

n independent axis = n dimensions.

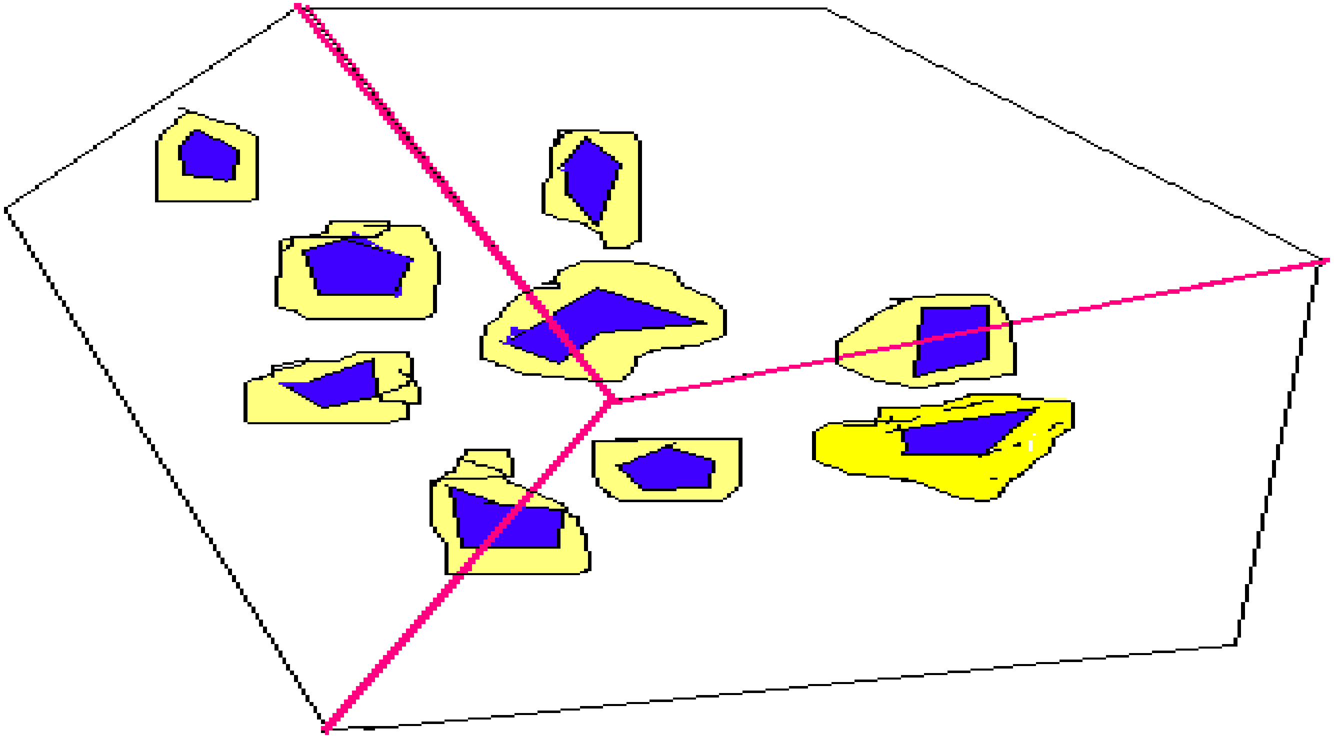
flux in a n-dimensional TORUS
(Budelli 1998)

POINCARÉ SECTION: FIRST
RETURN MAP:

it is a descrete dynamical system
in a n-1 dimensional ball with
discontinuities over a n-2
dimensional surface, of finite jumps.

A PIECEWISE CONTINUOUS DISCRETE (n-1) dimensional DYNAMICAL SYSTEM in a topological ball.

- For n neurons: dynamics in a n-dimensional cube.
- G = First return map to the backward faces of the cube.
- F= G o G = Second return map.
- F^k = 2k-th. return map.



In blue: Atoms of generation k of the return map F

In yellow: Atoms of an extension of F. Any map perturbed from F has its atoms in the blue and yellow regions.

In red: the discontinuity lines of F.

- F is piecewise continuous (discontinuity (n-2)-dimensional surfaces).

- F is contractive in each continuous piece (with a well chosen metric in the (n-1) dimensional dominium of F). This is due to inhibitory synapsis hypothesis.

- F has the separation property:

  Atoms of the same generation are pairwise disjoint, so there is a positive minimum distance between them (if the neurons are not very different).

THEOREM 1 (Budelli-Catsigeras-Rovella 2005):

Contractive piecewise continuous maps

F in n -1 ≥ 2 dimensions

that have the separation property

generically exhibit persistent limits cycles

(i.e. a periodic attractors that persists under small perturbations of the map F).

COROLLARY:

Generic networks of n ≥ 3 inhibitory neurons

have <u>persistent</u> periodic behaviours

(up to many periodic attractors (limit cycles), and of finite but “unbounded!!!” period).

CONCLUSIONS:

1) **EXTERIOR SIGNALS DO CHANGE THE BEHAVIOR** (spike train codes) OF THE SYSTEM of n neurons, and allow:

- **categorize the inputs**
- **make decissions**

between large number of possibilities if the system has n neurons (n ~ 10 exp 12),

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2) **Period of spike train codes are theoretically as large as needed (finite but UNBOUNDED)** so, in practice (not assympthotically) they may be VIRTUALLY chaotic.

Long periods of LIMIT CYCLES OF SPIKE TRAINS allow the system:

- **long term memory functions**
- **categorize many different inputs**
- **have a scenario of many (finitely many but a large number) of decissions**

CONCLUSIONS:

3) INSIGNIFICATIVE PERTURBATIONS OF THE SYSTEM DO NOT CHANGE THE NUMBER NOR THE CHARACTERISTICS OF SPIKE TRAIN PERIODIC LIMIT CYCLES OF THE SYSTEM. **(THE SYSTEM IS STRUCTURABLY STABLE)**

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4) SIGNIFICATIVE CHANGES IN THE PARAMETERS OF THE SYSTEMS for instance: some decreasing in the number of neurons,
or in the number of synapsis, (as those occuring during childhood),
also some changes of the numerical coefficients modeling the chemical reactions in the membranes of the cells, etc

DO CHANGE

the quantity, characteristics, size of its basins, and periods of the limit cycles of spike trains, and thus also CHANGE

the **maximal entropy** capability of the system, and
the INTERNAL PROBABILITY DISTRIBUTION OF MAXIMAL ENTROPY
and the THEORETICAL INPUT SIGNALS that the system can process optimally.
