Synchronized dynamics of on-off oscillators with instantaneous coupling interactions.

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Abstract

We study a dynamical system with discontinuities, obtained as the evolution on time, in a product Riemannian manifold of large dimension, of any finite number $n \ge 2$ of coupled oscillators of the type "on-off". We assume that the interactions (couplings) among each individual dynamical system are instantaneous, and classify them in excitatory, inhibitory or null. We analyze the sync modes, provided that the graph of excitatory interactions is large enough and satisfies some particular topological conditions. The main novelties with respect to previous results, in which we were inspired, are that we do not impose any particular formula for the vector fields governing the dynamics of each individual oscillator during their "off" phases, the number of oscillators in the system may be any $n \ge 2$, and the individual dynamics while are not coupled, do not need to be identical.

1 Statement of the subject of research

We consider a dynamical system $\Phi(\mathbf{x}, t)$ with non isolated discontinuities, evolving with continuous positive time t, from each initial state \mathbf{x} in a product Riemannian manifold M.

$$M = \prod_{i=1}^{n} M_{i}, \quad \phi : M \times \mathbb{R}^{+} \mapsto M$$
$$\Phi(\mathbf{x}, 0) = \mathbf{x}, \ \Phi(\mathbf{x}, t_{1} + t_{2}) = \Phi(\Phi(\mathbf{x}, t_{1}), t_{2}) \ \forall \ t_{1}, t_{2} \in \mathbb{R}^{+}, \ \forall \ \mathbf{x} \in M.$$

The system is obtained from considering $n \geq 2$ coupled oscillators of the "on-off type", that are coupled by instantaneous interactions at the "on" instants. Each oscillator $i \in \{1, 2, ..., n\}$, when uncoupled with the others, and during its "off" phase, is described by the state $x_i \in$ M_i which evolves along the integral curve of a vector field $F_i \in \mathcal{X}^1(M_i)$. The signed and bidirectional interactions among the oscillators introduce discontinuities in Φ , at those points $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in M$ that drop on a set $\Delta \subset M$ which is the union of n codimension-one submanifolds topologically embedded in M, and transversal to the orbits by Φ .

This abstract dynamical system Φ , evolving on an arbitrarily large-dimensional manifold M, and exhibiting non isolated discontinuities along $\Delta \subset M$, comes from other sciences. In fact, they model, among other examples, networks of some types of artificial or biological neurons, and of light coupled electronic oscillators (LCO).

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Each oscillator $i \in \{1, \ldots, n\}$, when uncoupled from the others, is assumed to evolve accordingly to a dynamics defined by a flow without singularities on an open Riemannian manifold M_i , embedded in a compact manifold of larger or equal dimension. It is called an oscillator because it exhibits at least one periodic attractor A_i of period $T_i > 0$. The initial condition, and all the future orbits of each oscillator *i* (even after they are perturbed by the coupling with the other oscillators), are assumed to lay, always, in the union of the open basins of attraction of its periodic attractors. So, each Riemannian manifold M_i is restricted to that open set.

The "on" and the "off" states of each oscillator i are defined, in our abstract context, just conventionally, as an agreement of notation to the purpose of defining later, the instantaneous action from the oscillator i, to any other oscillator j. This action from i to j is produced only at the times when i "turns on". But, in our general context in this work, we are not assuming any particular change in the dynamics of the oscillator i, when it is "turned on", in relation to when it is "turned off". In other words, any additional conditions in the evolution of the oscillator i during its "on" phases, can be added freely. For instance, to apply the results to biological or artificial neurons, it can be assumed that the oscillator "spikes" when it turns on. Other characteristic dynamical behavior can be imposed during when the oscillator turns on, to model biophysical, ecological, mechanical or electronic oscillators, or controlled clock systems applied in communications. The large restriction is the results of this paper resides only in the instantaneous character of the couplings among the oscillators, namely that the interactions between the different dynamical units are assumed to be impulsive.

In our general context most states $x_i \in M_i$ are, by definition, called "off states" of the oscillator i. On the other hand, the "on" states are defined as follows: there exists a codimensionone embedded submanifold $\Delta_i \subset M_i$, such that $\#(A_i \cap \Delta_i) = 1$, which is called the "threshold level" of the oscillator i, and such that if $x_i(t) \in \Delta_i$, then the oscillator i instantaneously "turns on" at time t. If an oscillator i is coupled to an oscillator $i \neq i$, and at the times t such that $x_i(t) \in \Delta_i$ (namely when i "turns on"), it instantaneously injects a discontinuity jump $\epsilon_{i,j} \in T_{x_j(t)}M_j$ in the state $x_j(t) \in M_j$ of the oscillator j. This instantaneous interaction is void if the respective discontinuity jump $\epsilon_{i,j}: M_j \mapsto T_{x_j(t)}M_j$, which depends on $x_j(t)$, is the identically null map. If an interaction is not void, it is called "excitatory" if the discontinuity jump $\epsilon_{i,j}$ approaches the state x_j of the oscillator j to its "on" state. Namely, the instantaneous injection of the discontinuity $\epsilon_{i,j}$ approaches x_j to the codimension one manifold $\Delta_j \subset M_j$. In other words, if the interaction from i to j is excitatory, then, the on-state of the oscillator iproduces that j reaches sooner its own threshold level, and so j will also "turn on" sooner. Analogously, it is defined the "inhibitory" interaction, when the discontinuity jump $\epsilon_{i,i}$ makes the state x_j to become suddenly farer from its threshold level, and thus the oscillator j will be delayed to turn on.

2 The synchronization theorem.

We analyze the dynamics of the composed dynamical system on M described above, discussing according to the bi-directed and signed graph of interactions among the oscillators. Under certain configurations of the subgraph of excitatory interactions, and under the assumptions that the number n of oscillators which are mutually coupled in that subgraph, is large enough, and that the minimum excitatory interaction is strong enough, we prove the following result:

The composed system synchronizes the on phases, even if the oscillators are mutually very different, and ϵ -quasi synchronizes all the phases, if the periods of all the oscillators are similar.

The " ϵ -quasi synchronization of all the phases" is defined as follows: For each initial condition there exists a transitory time $t_0 \ge 0$, such that for all pair of oscillators i, j, after an homeo-

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morphic rescaling of times $s_{i,j}$ which is ϵC^0 -near the identity, the equality $x_i(t) = x_j(s_{i,j}(t))$ holds for all $t \ge t_0$. We also prove, even if the oscillators are very different, that the composed system acquires its own periodic or quasi periodic behavior, with its own period or quasi-period T. This period, in the case of synchronization of the on states, is usually different and not a multiple of the periods T_i of the individual oscillators, even if, in some particular cases T may be near the period of some "dominant" oscillator of the network.

The route of our proof follows the ideas introduced in [7].

3 Conclusions

The main novelty of our results, in relation to the previous ones [6], [9], [7], [11], in which we were inspired, is that the individual oscillators are not assumed to be identical, and principally that they may evolve in very general manifolds M_i of any finite dimensions d_i , and almost no condition is imposed to the individual dynamics of each of them. In [2], [4], [8], [13], [14], are studied systems composed by two oscillators instantaneously coupled, while in [1], [3], [5], [12], are analyzed the attractors, and in particular the synchronization modes, of large networks of heterogeneous oscillators under some particular hypothesis.

In the present work, each oscillator (if were isolated from the others) is just assumed to behave dynamically as the integral flow of any C^1 vector field $F_i \in \mathcal{X}^1(M_i)$, just provided to exhibit a periodic attractor, with an open local basin, and such that the orbits in this basin are transversal to the codimension one manifold $\Delta_i \subset M_i$ defined above. The second largest difference with some of those previous works, is that in the present communication, we do not assume any particular formulation of the family $\{F_i\}_{1 \leq i \leq n}$ of vector fields, also nor any particular formulation nor regularity, of the interaction maps $\epsilon_{i,j}$. On the contrary, in most of the previous works, the interaction maps are usually assumed to be constants, and also highly regular or tight numerical conditions are imposed to the set of vector fields F_i .

As in [15], we also consider the possible existence of different sync modes with different basins of attractions. In other words, the composed system of the coupled oscillators may exhibit many different periodic sync states, and different periods, depending on the initial condition $\mathbf{x} \in \prod_{i=1}^{n} M_{i}$.

The main restriction in our hypothesis is that the couplings are assumed to have zero delay. It is open the problem to extend the results in the case in which the couplings have a positive delay time, as defined for instance in [10]. It remains open the problem of extending our abstract results, if possible, for networks of coupled on-off oscillators that interact not instantaneously and with some positive delays.

Some of the results so obtained can be also extended if the attractors of the different dynamical units are not periodic. For instance, some of the tools in the proofs of our synchronization result, may be extended to study the synchronization or quasi synchronization, of many dynamical subsystems that exhibit identical or maybe similar non periodic attractors, in particular chaotic attractors, provided that the individual units are coupled by instantaneous interactions.

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XII Latin American Workshop on Nonlinear Phenomena (LAWNP-2011)

October 10 to October 14, 2011, San Luis Potosi, Mexico

BOOK OF ABSTRACTS

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The XII Latin American Workshop on Nonlinear Phenomena (LAWNP2011), will be held in San Luis Potosi, Mexico, from October 10 to October 14, 2011.

This international conference will be the twelfth in a series of Latin American Workshops that have taken place every two years to discuss different aspects of nonlinear phenomena. The previous meetings took place in Buzios, Brazil in 2009, Arica, Chile in 2007, Bariloche, Argentina in 2005, Salvador, Brazil in 2003, Cocoyoc, Mexico in 2001, Cordoba, Argentina in 1999, Canela, Brazil in 1997, Bariloche, Argentina in 1995, Mar del Plata, Argentina in 1993, Santiago, Chile in 1990, and Mar del Plata, Argentina in 1988.

The workshop will center on nonlinear dynamics, in particular spatially-extended dynamics, and out-of-equillibrium phenomena. A partial lists of subjects follows: space-time chaos, synchronization, pattern formation, coherent structures, morphogenesis and developmental biology, far from equilibrium phase transitions, granular materials, inelastic gases, coarsening, aging, nanomachines, reaction kinetics, instabilities and bifurcations, nonlinear fluid dynamics, dynamics of complex systems, and dynamics on complex networks.

The Local Organizing Committee acknowledges the help and support of the members of the International Advisory Committee and of all the participants. The support of the following is gratefully acknowledged:

- Universidad Autónoma de San Luis Potosí, San Luis Potosí, Mexico
- Centro de Ciencias de la Complejidad through the Thematic Network on Complexity, Science and Society (C3)
- Centro Latino-Americano de Física (CLAF)
- Sociedad Mexicana de Física (SMF)
- Proyecto Universitario de Fenómenos no Lineales y Mecánica (FENOMEC), Universidad Nacional Autónoma de México, Mexico
- Consejo Nacional de Ciencia y Tecnología (CONACyT)
- Centro dde Investigación en Energía, Universidad Nacional Autónoma de México, Mexico

Invited Talks

Afraimovich, Valentin

Universidad Autónoma de San Luis Potosí, México. Sequential behavior in dynamical networks

Complex networks such as the neuronal ones composed of neurons coupled by chemical synapses are known to exhibit a large variety of activity forms. Recent neurophysiological experiments have shown that neuronal processes are often accompanied by short transitive activity of individual elements or small groups of elements. Such a behavior is called the sequential dynamics. In the framework of dynamical systems theory this behavior is related to the existence of a collection of metastable invariant sets joint by heteroclinic trajectories in the phase space. The sequential dynamics can be treated as a process of successive switching among these sets. Such a treatment allows one to explain the temporal order in which elements become activated and to single out the parameters of the system responsible for its prediction (see cited articles and the references therein). In the talk it is supposed to tell about both the situation where metastable sets are just equilibrium points or limit cycles and where they are more complex sets. References:

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Albano, Ezequiel[1,2] and Baglieto, Gabriel[1]

[1] Instituto de Física de Líquidos y Sistemas Biológicos. CCT La Plata. CONICET, UNLP, La Plata, Argentina. [2] Departamento de Física, F. de Ciencias Exactas, UNLP, La Plata, Argentina. **On the collective behavior of self-driven individuals**

The study of flocking behavior has attracted interdisciplinary interest due not only to their fascinating characteristics and their ubiquity in all scales, but also for their complex nature. Modeling of swarming and flocking contributes to the understanding of natural phenomena and becomes relevant for many practical and technological applications, e.g. collective robotic motion, design and control of artificial micro swimmers, etc. [1-7]. Within this broad context, the Vicsek Model (VM) [2], which considers individuals that try to adopt the direction of movement of their interacting neighbors, under the influence of some noise, e.g. due to the environment, has gained large popularity becoming an archetypical model for the study of the onset of order upon the interactive displacement of self-driven individuals. The simple rules of the VM guarantee the observation of a rather complex and interesting critical behavior: an ordered phase of collective motion is found for low enough levels of noise, while a disordered phase is observed at high noise. However, the nature of the phase transition between those phases could be of first-or secondorder, depending on the type of considered noise [4,5,7]. The aim of this paper is to investigate the structural characteristics of the networks formed among the self-driven individuals during the farm-from equilibrium stationary states of the VM. We expect that the proposed study will shed some light on some poorly understood characteristics of the VM, such as the origin of

Talks and posters

Almendral Sanchez, Juan Antonio

Unveiling Protein Functions through the Dynamics of the Interaction Network

Cutting-edge technologies are adding sequences to the databases faster than the pace at which insights into their function can be gained. As a consequence, the vast majority of known proteins have not been characterized experimentally, and their function is yet unknown. To predict the function of a protein, two main strategies have been followed so far. The first one relies on the analysis of the protein itself, and the second is based on high-throughput techniques. Highthroughput protein-protein interactions detection experiments allow nowadays a representation of the global cell functioning in terms of a network, with nodes representing proteins and edges representing the detected mutual interactions. Notwithstanding the accomplishments of these analyses, it is important to highlight that most high-throughput methods can suffer from high false positive and false negative rates and, therefore, functional assignments that are based on these tools may lead to misclassifications. Several past studies attempted already to determine to what extent the function of a protein depends on the way it is interacting with the others in the protein interaction network (PIN). However, the use of such network representation for prediction requires the determination of the specific scale of the PIN that one has to consider for unveiling the individual protein function. And, in this latter framework, the current state of the art includes two types of approaches. From one side, several direct annotation schemes have been devised, with the common inspiration of analyzing the local scale features of the PIN. From the other side, more recent module assisted techniques have attempted to use the extra knowledge arising from the mesoscale of clustered structures of the PIN, by first identifying dense agglomerates in the network that are loosely connected to other areas of the graph, and then to use this topological information for predictions on the protein specific function. The approach we put forward constitutes a third, novel, strategy. We provide evidence that an alternative source of information is, in fact, the one arising from the analysis of how the modular PIN structure actually organizes the synchronization dynamics of an ensemble of oscillators. In particular, we show how the combination of synchronization features emerging in the PIN structure with a rudimentary classification of proteins based on expert manual assignment, allows, indeed, to gather information on misclassification problems, as well as to offer a more accurate function assignment that is consistent with more recent (and better refined) manual annotation of these protein functions. Not less important is the ability of the approach we introduce to assess the coupling of different functional categories, to determine how closely associated they are, and which proteins participate in both of them.

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dom are coupled through a mean field that evolves self-consistently. Based on the linear stability of period-one and period-two orbits of the coupled maps, we construct coherent states in which the degrees of freedom are synchronized and the mean field stays fixed. Nontwist systems exhibit global bifurcations in phase space known as separatrix reconnection. Here we show that the mean-field coupling leads to dynamic, self-consistent reconnection in which transport across invariant curves can take place in the absence of chaos due to changes in the topology of the separatrices. In the context of self-consistent chaotic transport we study two novel problems: suppression of diffusion and breakup of the shearless curve. For both problems we construct a macroscopic effective diffusion model with time-dependent diffusivity.

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Catsigeras García, Eleonora

Synchronized dynamics of on-off oscillators with instantaneous coupling interactions.

We study a dynamical system with discontinuities, obtained as the evolution on time, in a product Riemannian manifold of large dimension, of any finite number n^2 of coupled oscillators of the type "on-off". We assume that the interactions (couplings) among each individual dynamical system are instantaneous, and classify them in excitatory, inhibitory or null. We analyze the sync modes, provided that the graph of excitatory interactions is large enough and satisfies some particular topological conditions. The main novelties with respect to previous results, in which we were inspired, are that we do not impose any particular formula for the vector fields governing the dynamics of each individual oscillator during their "off" phases, the number of oscillators in the system may be any n^2 , and the individual dynamics while are not coupled, do not need to be identical.

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Cervantes, Fernando

Multifractal characterization of the porous structure of sodium silicate gels

A distinctive feature of silicate gels is the porosity of its structure, in these gels the connectivity, spatial and size distribution of pores depend on the complexity of the conditions during the polymerization process. In general, the porous structure of the gel does not exhibit an obvious order [1]. However, through multifractal analysis, one can determine a number of correlations in it. These correlations generally can be described by scaling relations, i.e., they show a power law dependence. As reported in the literature structures resulting from aggregation processes show a complex multifractal structure in general [2,3]. In this work we characterize the complexity of the porous structure in sodium silicate gels through the mass fractal dimension, multifractal spectrum, lacunarity index, etc. Silicate gels were prepared by neutralizing a solution of sodium metasilicate with another solution of acetic acid, both in concentration 1M. Additionally, ferromagnetic particles were dispersed in these gels and a static magnetic field was applied during the polymerization process. The digital image analysis showed that the porous structure of the samples presents multifractal characteristics.