Subcircuitos predominantemente excitatorios: condiciones matemáticas suficientes para la sincronización de eventos

Eleonora Catsigeras

IMERL - Fac. Ingeniería
Universidad de la República

eleonora@fing.edu.uy

Seminario de Física
Instituto de Física de la Facultad de Ciencias
Universidad de la República
Montevideo, 19 de noviembre de 2015.

Seminario de Física
Facultad de Ciencias
Universidad de la República

Presentación oral
jueves 19 de noviembre de 2015

Eleonora Catsigeras

Título: Sincronización de eventos en subcircuitos neuronales predominantemente excitatorios

Resumen:

Se consideran modelos matemáticos simplificados de subcircuitos neuronales con conexiones sinápticas predominantemente excitatorias.

Se demostrará ciertas desigualdades matemáticas que vinculan algunos de los parámetros del subcircuito como condiciones
suficientes para la presencia recurrente de sincronización de los disparos de todas las neuronas del subcircuito.

Las desigualdades matemáticas encontradas involucran los siguientes parámetros del subcircuito:

cantidad de neuronas excitatorias e inhibitorias en el subcircuito,
configuración topológica del grafo de conexiones sinápticas,
mínimo peso positivo de las excitaciones sinápticas,
y máximo peso negativo de las inhibiciones sinápticas en el subcircuito.

En sentido contrario, mostraremos que aunque las excitaciones sinápticas excitatorias sean predominantes, la sincronización de disparos en forma recurrente no se produce
si los circuitos considerados involucran relativamente pocas neuronas, en relación con la mínima interacción positiva.

ORGANIZATION OF THIS TALK

• Object of study: A mathematical system modeling a circuit of "neurons".
(Abstract – General – Simplified)

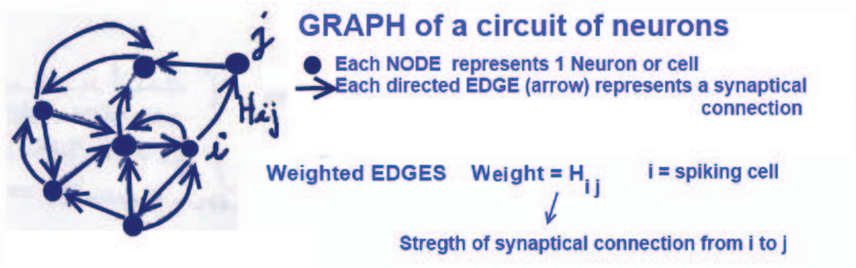
• Questions to research: Quantitative and qualitative dynamics of the mathematical system.

• Methodology of research: Logical deductive proofs (Rigorous proofs).

• Obtained results: Theorems 1 and 2 (their statements and their proofs).

• Mathematical proofs: Unfortunately not included in this talk, but they are the most enjoyable parts of the work.

• Conclusions: How to interpret the statements of Theorems 1 and 2 and their corollaries. They are necessarily true in the (simplified - general - abstract) mathematical model. But ¿do they necessarily hold in the real physical world?



Definition

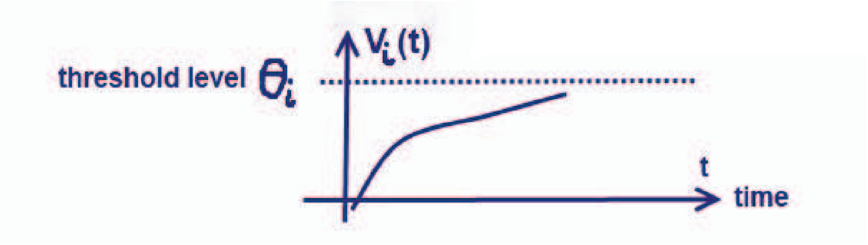
The cell i (node i) is EXCITATORY if $H_{ij} \geq 0 \quad \forall\ j \neq i$.
The cell i (node i) is INHIBITORY if $H_{ij} \leq 0 \quad \forall\ j \neq i$.

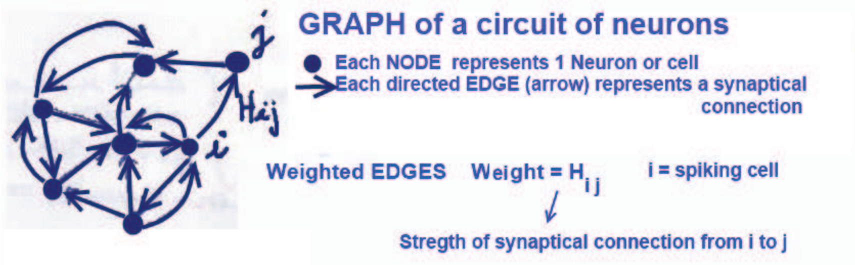
Interspike Regime

(One-dimensional neuron)
Governed by a 1-dimensional differential equation:

$$\frac{dV_i}{dt} = F_i(V_i), \text{ where } F_i > 0, \text{ while } V_i(t) \leq \theta_i$$

The value θ_i is the *THRESHOLD LEVEL*.





The cell i (node i) is excitatory if $H_{ij} \geq 0 \quad \forall\, j \neq i$.
The cell i (node i) is inhibitory if $H_{ij} \leq 0 \quad \forall\, j \neq i$.

Synaptical Rule

When neuron i spikes (at instant t_0), the potential V_j of neuron $j \neq i$ suffers and "instantaneous" jump:

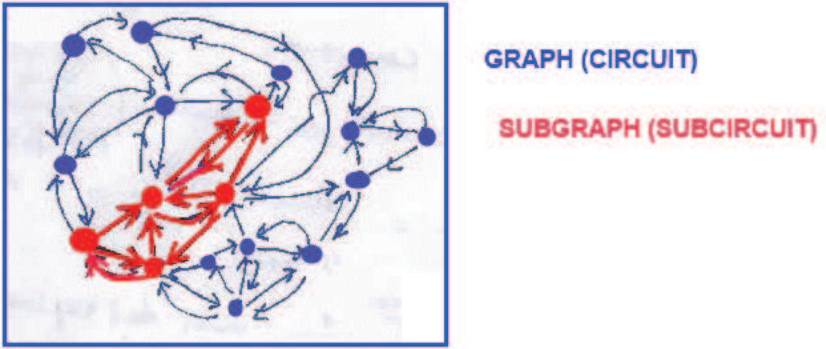
$$V_j(t_0) = V_j(t_0^-) + H_{i,j} \text{ if this number is } < \theta_j, \text{ or}$$

$$V_j(t_0) = 0 \text{ otherwise, and if so, also } j \text{ spikes at instant } t_0.$$

This math. model of the synaptical actions is SIMPLIFIED so:

- Instantaneous jump in the postsynaptical potential V_j.
- The REFRACTORY PHENOMENON holds for the spiking cells.

Graph of a circuit containing the graph of a subcircuit (in red) which is called a SUBGRAPH



Parameters' Space

Which are the "parameters" of the mathematical system modeling the circuit of neurons? Their values of some of them are NOT numbers but FUNCTIONS or other non numerical MATHEMATICAL STRUCTURES.

For the relaxation and spiking regime of the neurons:

$$(m, F_1, F_2, F_3, \ldots, F_m, \theta_1, \theta_2, \theta_3, \ldots, \theta_m), \text{ where}$$

- m is the number of neurons in the circuit
- F_i is the (vectorial) function at the second member of the system of differential equations $d\overline{x}_i/dt = F_i(\overline{x}_i)$ governing the relaxation regime of the neuron i.
- θ_i is a real number: the threshold level of neuron i.

For the synaptical connections:

$$(G, \ H_{1,2}, H_{1,3}, \ldots, H_{m-1,m}), \text{ where}$$

- G is a Graph Structure: the graph of the circuit, with m nodes $i \in \{1, 2, \ldots, m\}$ and directed and weighted edges $(i, j) : \ i \neq j$
- The weights $H_{i,j}$ of the edges of the graph G: they are (positive or negative or zero) real numbers.

Questions of Research: Dynamics of the Network (circuit or graph)

Which are the qualitative and quantitative mathematical properties that one may obtain, BY LOGICAL DEDUCTION, from the general mathematical model defined above?

EXAMPLES:

- Sequence of spiking instants
- ISI (interspike intervals) of a cell or of a subcircuit.
- Attractors and their basins - periodic orbits - limit cycles
- Synchronization of the spikes of several cells (periodic or non periodic synchronization)
- Waiting times until synchronization
- Recurrence

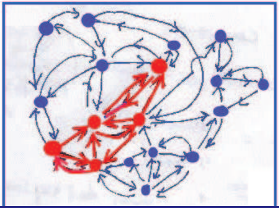
Definition

Recurrent synchronization of spikes in a subcircuit S
if there exists instants $t_1, t_2, \ldots, t_n, t_{n+1}, \ldots, \ldots$
such that at instant t_n all the cells of the subcircuit S spike simultaneously.

In Game Theory the phenomenon of synchronization is called "Grand Coalition".

Remark: Between the simultaneous synchronizations at instants t_n and t_{n+1}, some neurons of the subcircuit may spike.

Sufficient mathematical conditions for recurrent synchronization of the spikes in a subcircuit.



GRAPH (CIRCUIT)

SUBGRAPH (SUBCIRCUIT)

Theorem 1

If

- *S is complete and excitatory: $H_{i,j} > 0 \ \ \forall i \neq j$ in S,*
- *at least one cell in S is pacemaker,*
- *the number m of cells in S is large enough in relation to the mininum excitatory weight:*

$$\sqrt{m} \ \geq \ \frac{\max_{j \in S} \theta_j}{\min_{i \neq j \text{ in } S} H_{i,j}},$$

then

all the cells of the subcircuit S recurrently synchronize spikes while S does not receive inhibitions from the cells outside S.

Theorem 1

If

- *S is complete and excitatory: $H_{i,j} > 0 \ \ \forall i \neq j$ in S,*
- *at least one cell in S is pacemaker,*
- *the number m of cells in S is large enough in relation to the mininum excitatory weight:*

$$\sqrt{m} \ \geq \ \frac{\max_{j \in S} \theta_j}{\min_{i \neq j \text{ in } S} H_{i,j}},$$

then
all the cells of the subcircuit S recurrently synchronize spikes while S does not receive inhibitions from the cells outside S.

Remark This theorem holds:

- For any initial state of the cells
- Disregarding which are the functions F_i, and if they are similar or mutually very different, and which are the dimensions of vectorial states of the cells.
- No matter if the cells are mutually very different
- No matter if the interactions are mutually very different
- Disregarding how short or long are the refractory periods (but the refractory phenomenon must exist).

Theorem 1

If

- S *is complete and excitatory:* $H_{i,j} > 0 \;\; \forall i \neq j$ *in* S*,*
- *at least one cell in* S *is pacemaker,*
- *the number* m *of cells in* S *is large enough in relation to the mininum excitatory weight:*

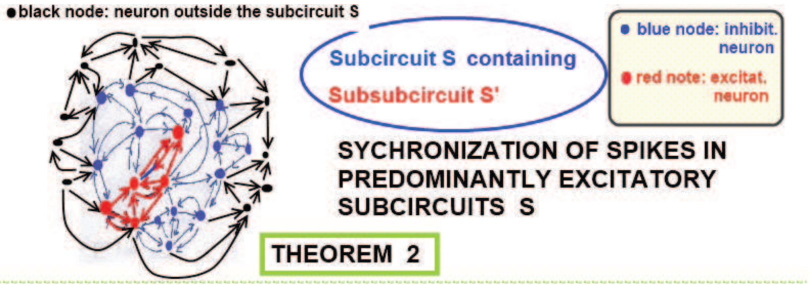
$$\sqrt{m} \geq \frac{\max_{j \in S} \theta_j}{\min_{i \neq j \text{ in } S} H_{i,j}},$$

then

all the cells of the subcircuit S *recurrently synchronize spikes while* S *does not receive inhibitions from the cells outside* S*.*

CHANGING THE CONNECTIONS OF THE SUBCIRCUIT S to be non complete, but still excitatory, provided the number of non null connections is large enough, still produce recurrent synchronization, if certain other mathematical conditions and inequalities hold (work in progress).

Some HISTORY:
1992 Mirollo-Strogatz
1996 Bottani



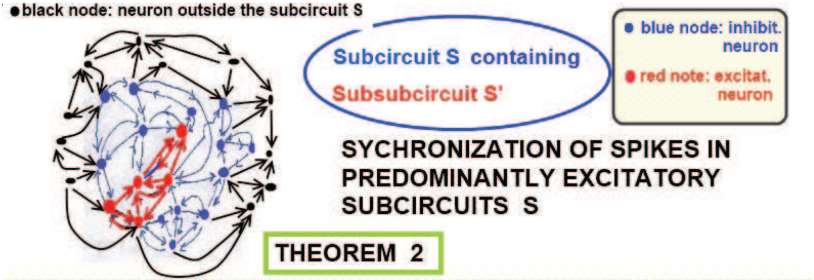
Theorem 2

If the subcircuit S contains a sub-subcircuit S' such that

- S' *is complete and excitatory*
- *all the cells in* $S \setminus S'$ *are inhibitory*
- *at least one neuron of S' is a pacemaker*
- *the number* m' *of excitatory neurons in* S' *satisfies the following inequality:*

$$\sqrt{m'} \geq \frac{\left(\max_{j\in S}\theta_j\right) + \left(\max ISI_{S'}/\min_{j\in S} ISI_j\right)\cdot\left(\min_{j\in S\setminus S', i\in S'}|H_{j,i}|\right)}{\min_{i\neq j, i\in S', j\in S} H_{i,j}},$$

then all the cells of the subcircuit S recurrently synchronize spikes, while they do not receive inhibitions from the cells outside S.



CONCLUSIONS

1. To avoid recurrent synchr. of spikes in the subcircuit S' composed by excitatory cells connect the nodes of S' with edges coming from inhibitory cells, BUT:
2. If the inhibitory cells connected to S' are themselves excited by the cells of S, then
   - they do not avoid the recurrent synchronization of S'.
   - Worst, the inhibitory cells also synchronize spikes with the excitatory cells of S'.
   - So, due to the refractory phenomenon, the inhibitory cells spiking simultaneously with those of S' do not inhibit them.
3. Other cells outside S (in black in the figure), that are inhibited by S but not excited by S', may turn off (do not spike).