ATRACTORES ESTADÍSTICOS Y MEDIDAS PSEUDO-FÍSICAS

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Resumen

Revisaremos la definición de atractor estadístico de Ilyashenko agregando la condición de minimalidad α-observable. Demostraremos la existencia de estos atractores, y su caracterización como el mínimo soporte compacto de todas las medidas pseudo-físicas de las órbitas en su cuenca de atracción estadística.

Finalmente, probaremos que, dado $\alpha > 0$ fijo, la variedad queda Lebesgue-c.t.p. partida en una cantidad finita de cuencas de atracción estadística de atractores de Ilyashenko α-observable minimales.

1. Introducción

La presentación sigue las diapositivas adjuntas. Su contenido corresponde a los resultados publicados en el artículo [6].

Más abajo incluimos las referencias bibliográficas citadas en dicho artículo.

Referencias

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$f : M \mapsto M$ Borel measurable map; M compact finite-dim. Riemannian manifold; m normalized Lebesgue measure $m(M) = 1$.

Definition

Ilyashenko's or Statistical Attractor is a compact set $\emptyset \neq K \subset M$ such that:

a) $m(B(K)) > 0$,

where the basin $B(K)$ of statistical attraction of K is

$$B(K) := \Big\{ x \in M : \lim_{n \to +\infty} \frac{1}{n} \#\{0 \leq j \leq n-1 : \ \mathrm{dist}(f^j(x), K) < \epsilon\} = $$

$$= 1 \quad \forall \, \epsilon > 0 \Big\}.$$

b) K is minimal with respect to its basin, i.e.:

$$\emptyset \neq K' \text{ compact } \subset K, \quad B(K') = B(K) \ m-\text{a.e.} \qquad \Rightarrow \qquad K' = K$$

COMMENTS: Global $m(B(K)) = 1$, existence; non global Karabacak-Ashwin; statistical attraction, physical measures, Milnor's attractors; examples Hu-Young, Quas, Eight, Bowen; non necessary f-invariance.

$0 < \alpha \leq 1; \quad \emptyset \neq K \subset M$ compact.

Definition

α-observability and α-obs. minimality
K is α-observable if $m(B(K)) \geq \alpha$.
K is minimally α-obs. if it is α-obs. and any proper compact subset of K is not α-obs.

Proposition

K is a Ilyashenko's statistical attractor if and only if it is a compact set minimally α-obs. for **some** *value of $\alpha > 0$.*

REMARKS Previously fixed $\alpha \in (0, 1]$: Ilyashenko's statistical attractor is not nec. α-observable, and if yes, then it is not nec. minimally α-obs. (Examples)
Maybe $\nexists$ minimal Stat. Attr. among all (Example)

Theorem

(Existence) *For any* $\alpha \in (0,1]$ *there exists an* α*-obs. minimal statistical attractor. If* $\alpha = 1$*, then it is unique.*

PROOF (on the board)

Recall Definition of SRB-like (or pseudo-physical) probability measure, and the theorem of their necessary existence (for any Borel measurable map $f : M \mapsto M$).

Theorem

(SRB-like characterization) *Any statistical attractor* K *is the minimal compact support of all the SRB-like probability measures of the map* $f|_{B(K)}$.
Conversely, if B *is a* m*-positive* f*-invariant set and if* K *is the minimal compact support of all the SRB-like measures of* $f|_{B(K)}$ *then* K *is a statistical attractor.*

PROOF (on the board)

Finite Lebesgue-decomposition of M : $\{B_1, B_2, \ldots, B_k\}$ such that $m\big(M \setminus (\cup B_i)\big) = 0$ and $m(B_i \cap B_j) = 0$ for all $i \neq j$.

Theorem

Decomposition of the space *For any Borel measurable map $f : M \mapsto M$ and for any $\alpha \in (0, 1]$ there exists a* **finite** *Lebesgue-decomposition of M into the basins $B(K_i)$ of α_i-obs. minimal statistical attractors, with $\alpha_i = \alpha$ for all i except at most one.*

PROOF (on the board)