Some advances on partially hyperbolic and non-uniformly hyperbolic dynamics

Jana Rodriguez Hertz

Universidad de la República
Uruguay

Dijon 2010

setting

- M compact Riemannian manifold

setting

- M compact Riemannian manifold
- $f : M \to M$ partially hyperbolic diffeomorphism

entropy maximizing measures

Definition

μ is an entropy maximizing measure if

entropy maximizing measures

Definition

μ is an entropy maximizing measure if

- $h_{top}(f) = h_\mu(f)$

entropy maximizing measures

Definition

μ is an entropy maximizing measure if

- $h_{top}(f) = h_{\mu}(f)$
- μ is ergodic

problem

problem

problem

problem

- existence

problem

problem

- existence
- uniqueness/finiteness

problem

problem

- existence
- uniqueness/finiteness

of e.m.m.

advances

Theorem (RHRHTU(2009):∃& uniqueness/finiteness)

If $f \in \mathcal{P}H(M^3)$ is such that

advances

Theorem (RHRHTU(2009):∃& uniqueness/finiteness)

If $f \in \mathcal{P}H(M^3)$ is such that

- *f is accessible*

advances

Theorem (RHRHTU(2009):∃& uniqueness/finiteness)

If $f \in \mathcal{P}H(M^3)$ is such that

- *f is accessible*
- *f is dynamically coherent*

advances

Theorem (RHRHTU(2009):∃& uniqueness/finiteness)

If $f \in \mathcal{P}H(M^3)$ is such that

- *f is accessible*
- *f is dynamically coherent*
- *f is smooth*

advances

Theorem (RHRHTU(2009):∃& uniqueness/finiteness)

If $f \in \mathcal{P}H(M^3)$ is such that

- *f is accessible*
- *f is dynamically coherent*
- *f is smooth*
- *center leaves are circles*

advances

Theorem (RHRHTU(2009): $\exists$& uniqueness/finiteness)

If $f \in \mathcal{P}H(M^3)$ is such that

- *f is accessible*
- *f is dynamically coherent*
- *f is smooth*
- *center leaves are circles*

then, either:

- $\lambda_c(\mu) = 0$ *for some* μ

advances

Theorem (RHRHTU(2009):∃& uniqueness/finiteness)

If $f \in \mathcal{P}H(M^3)$ is such that

- *f is accessible*
- *f is dynamically coherent*
- *f is smooth*
- *center leaves are circles*

then, either:

- $\lambda_c(\mu) = 0$ *for some* μ
- $\#\{e.m.m.\} = 1$

advances

Theorem (RHRHTU(2009):∃& uniqueness/finiteness)

If $f \in \mathcal{P}H(M^3)$ is such that

- *f is accessible*
- *f is dynamically coherent*
- *f is smooth*
- *center leaves are circles*

then, either:

- $\lambda_c(\mu) = 0$ *for some* μ
- $\#\{e.m.m.\} = 1$
- $\sim$ *isometric extension of Bernoulli*

advances

Theorem (RHRHTU(2009):∃& uniqueness/finiteness)

If $f \in \mathcal{P}H(M^3)$ is such that

- *f is accessible*
- *f is dynamically coherent*
- *f is smooth*
- *center leaves are circles*

then, either:

- $\lambda_c(\mu) = 0$ *for some* μ
- $\#\{e.m.m.\} = 1$
- $\sim$ *isometric extension of Bernoulli*

or:

- $\lambda_c(\mu) > 0$ *for some* μ

advances

Theorem (RHRHTU(2009):∃& uniqueness/finiteness)

If $f \in \mathcal{P}H(M^3)$ is such that

- *f is accessible*
- *f is dynamically coherent*
- *f is smooth*
- *center leaves are circles*

then, either:

- $\lambda_c(\mu) = 0$ *for some* μ
- $\#\{e.m.m.\} = 1$
- $\sim$ *isometric extension of Bernoulli*

or:

- $\lambda_c(\mu) > 0$ *for some* μ
- $\#\{e.m.m\} = 2k$

advances

Theorem (RHRHTU(2009):∃& uniqueness/finiteness)

If $f \in \mathcal{P}H(M^3)$ is such that

- *f is accessible*
- *f is dynamically coherent*
- *f is smooth*
- *center leaves are circles*

then, either:

- $\lambda_c(\mu) = 0$ *for some* μ
- $\#\{e.m.m.\} = 1$
- $\sim$ *isometric extension of Bernoulli*

or:

- $\lambda_c(\mu) > 0$ *for some* μ
- $\#\{e.m.m\} = 2k$
- $\sim$ *finite extension of Bernoulli*

remark

remark

In particular we get $f \in \mathrm{Diff}^{\infty}(M)$ such that

remark

remark

In particular we get $f \in \mathrm{Diff}^{\infty}(M)$ such that

- $h_{top}(f) > 0$

remark

remark

In particular we get $f \in \mathrm{Diff}^{\infty}(M)$ such that

- $h_{top}(f) > 0$
- f robustly transitive

remark

remark

In particular we get $f \in \mathrm{Diff}^{\infty}(M)$ such that

- $h_{top}(f) > 0$
- f robustly transitive
- $\#\{\text{e.m.m.}\} > 1$

remark

remark

In particular we get $f \in \mathrm{Diff}^{\infty}(M)$ such that

- $h_{top}(f) > 0$
- f robustly transitive
- $\#\{\text{e.m.m.}\} > 1$

This answers question by Buzzi-Fisher (2009)

advances

Theorem (Ures (2010))

If $f \in \mathcal{P}H(\mathbb{T}^n)$ is such that

advances

Theorem (Ures (2010))

If $f \in \mathcal{P}H(\mathbb{T}^n)$ is such that

- *f absolutely partially hyperbolic*

advances

Theorem (Ures (2010))

If $f \in \mathcal{P}H(\mathbb{T}^n)$ is such that

- *f absolutely partially hyperbolic*
- $c = 1$

advances

Theorem (Ures (2010))

If $f \in \mathcal{P}H(\mathbb{T}^n)$ is such that

- *f absolutely partially hyperbolic*
- *$c = 1$*
- *f homotopic to an Anosov diffeomorphism*

advances

Theorem (Ures (2010))

If $f \in \mathcal{P}H(\mathbb{T}^n)$ is such that

- *f absolutely partially hyperbolic*
- *$c = 1$*
- *f homotopic to an Anosov diffeomorphism*
- *$\mathcal{F}^s$, $\mathcal{F}^u$ quasi-isometric*

advances

Theorem (Ures (2010))

If $f \in \mathcal{P}H(\mathbb{T}^n)$ is such that

- *f absolutely partially hyperbolic*
- *$c = 1$*
- *f homotopic to an Anosov diffeomorphism*
- *$\mathcal{F}^s, \mathcal{F}^u$ quasi-isometric*

then

$$\#\{e.m.m.\} = 1$$

advances

Corollary

If $f \in \mathcal{P}H(\mathbb{T}^3)$ *is*

advances

Corollary

If $f \in \mathcal{P}H(\mathbb{T}^3)$ *is*

- *absolutely partially hyperbolic*

advances

Corollary

If $f \in \mathcal{P}H(\mathbb{T}^3)$ *is*

- *absolutely partially hyperbolic*
- *homotopic to an Anosov diffeomorphism*

advances

Corollary

If $f \in \mathcal{P}H(\mathbb{T}^3)$ *is*

- *absolutely partially hyperbolic*
- *homotopic to an Anosov diffeomorphism*

then

$$\#\{e.m.m.\} = 1$$

example

Example (BFSV(2009))

$\exists \mathcal{U} \subset \mathcal{P}H(\mathbb{T}^n)$ open with $c = 2$ such that

example

Example (BFSV(2009))

$\exists \mathcal{U} \subset \mathcal{P}H(\mathbb{T}^n)$ open with $c = 2$ such that

$$\#\{\text{e.m.m.}\} = 1$$

advances

Theorem (Díaz-Fisher (2009))

If $f \in \mathcal{P}H(M)$ is such that

advances

Theorem (Díaz-Fisher (2009))

If $f \in \mathcal{P}H(M)$ is such that

- $c = 1$

advances

Theorem (Díaz-Fisher (2009))

If $f \in \mathcal{P}H(M)$ is such that

- $c = 1$

then

$$\#\{e.m.m.\} \geq 1$$

advances

Announcement (DFPV)

If $f \in \mathcal{P}H(M)$ is such that

advances

Announcement (DFPV)

If $f \in \mathcal{P}H(M)$ is such that

$$E^c = E^1 \oplus E^2 \oplus \ldots \oplus E^n$$

advances

Announcement (DFPV)

If $f \in \mathcal{P}H(M)$ is such that

$$\begin{array}{ccccccccc} E^c = & E^1 & \oplus & E^2 & \oplus & \dots & \oplus & E^n \\ & \uparrow & & \uparrow & & \dots & & \uparrow \\ & \text{1-dim} & & \text{1-dim} & & \dots & & \text{1-dim} \end{array}$$

advances

Announcement (DFPV)

If $f \in \mathcal{P}H(M)$ is such that

$$\begin{array}{ccccccccc} E^c = & E^1 & \oplus & E^2 & \oplus & \dots & \oplus & E^n \\ & \uparrow & & \uparrow & & \dots & & \uparrow \\ & \text{1-dim} & & \text{1-dim} & & \dots & & \text{1-dim} \end{array}$$

then

$$\#\{\text{e.m.m.}\} \geq 1$$

advances

Example (Buzzi-JRH)

There exists $f \in \mathcal{P}H(M)$ such that

advances

Example (Buzzi-JRH)

There exists $f \in \mathcal{P}H(M)$ such that

$$\#\{\text{e.m.m.}\} = 0$$

remark

remark (Newhouse, Yomdim)

If $f \in C^\infty(M)$, then

remark

remark (Newhouse, Yomdim)

If $f \in C^\infty(M)$, then

$$\#\{\text{e.m.m.}\} \geq 1$$

question

question

If $f \in \mathcal{P}H(M)$ is such that

question

question

If $f \in \mathcal{P}H(M)$ is such that

- $c = 1$

question

question

If $f \in \mathcal{P}H(M)$ is such that

- $c = 1$
- f is accessible

question

question

If $f \in \mathcal{P}H(M)$ is such that

- $c = 1$
- f is accessible

Is

$$\#\{\text{e.m.m.}\} < \infty?$$

question

question

If $f \in \mathcal{P}H(M)$ is such that

- $c = 1$
- f is accessible

Is

$$\#\{\text{e.m.m.}\} < \infty?$$

question

any extra hypothesis?

question

question

Let $f \in \mathcal{P}H(M)$ such that

question

question

Let $f \in \mathcal{P}H(M)$ such that

- $c \geq 2$

question

question

Let $f \in \mathcal{P}H(M)$ such that

- $c \geq 2$
- f is accessible

question

question

Let $f \in \mathcal{P}H(M)$ such that

- $c \geq 2$
- f is accessible
- extra hypothesis?

question

question

Let $f \in \mathcal{P}H(M)$ such that

- $c \geq 2$
- f is accessible
- extra hypothesis?

Is

$$\#\{\text{e.m.m.}\} < \infty?$$

setting

- $f : M \to M$

setting

- $f : M \to M$
- $f \in \mathrm{Diff}^2(M)$

setting

- $f : M \to M$
- $f \in \mathrm{Diff}^2(M)$
- f conservative

problem

problem

Identify

problem

problem

Identify

- hyperbolic

problem

problem

Identify

- hyperbolic
- ergodic

problem

problem

Identify

- hyperbolic
- ergodic

components of f

Phc

Theorem (RHRHTU (2009))

$\forall\Lambda$ *hyperbolic ergodic component of* $f \in \mathrm{Diff}^2_m(M)$

Phc

Theorem (RHRHTU (2009))

$\forall\Lambda$ hyperbolic ergodic component of $f \in \mathrm{Diff}^2_m(M)$

- *$\exists p \in \mathrm{Per}(f)$ hyperbolic such that*

Phc

Theorem (RHRHTU (2009))

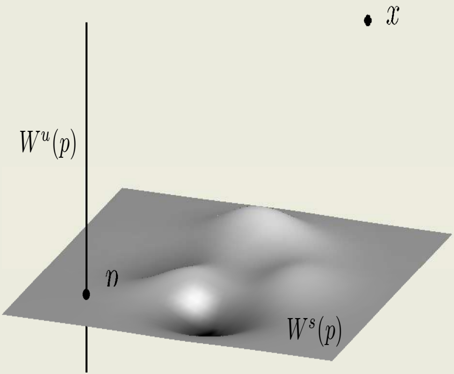
$\forall \Lambda$ *hyperbolic ergodic component of* $f \in \mathrm{Diff}^2_m(M)$

- $\exists p \in \mathrm{Per}(f)$ *hyperbolic such that*

$$\Lambda = \mathrm{Phc}(p)$$

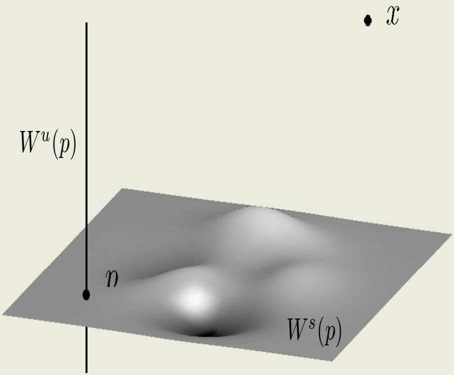
Phc

where, given $p \in \mathrm{Per}(f)$ hyperbolic



Phc

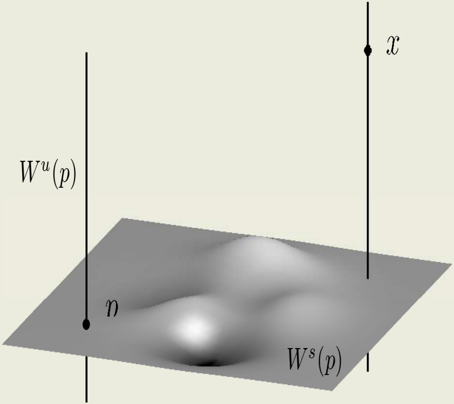
where, given $p \in \mathrm{Per}(f)$ hyperbolic



$x \in \mathrm{Phc}(p)$ if

Phc

where, given $p \in \mathrm{Per}(f)$ hyperbolic

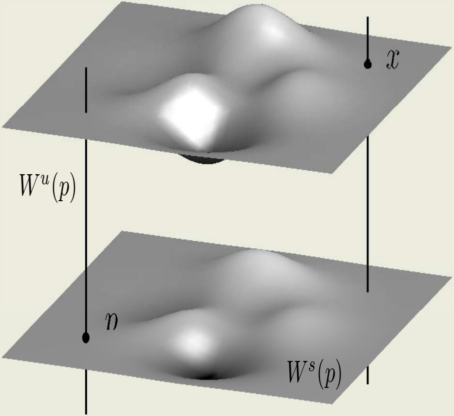


$x \in \mathrm{Phc}(p)$ if

$$W^u(x) \pitchfork o(W^s(p)) \neq \emptyset$$

Phc

where, given $p \in \mathrm{Per}(f)$ hyperbolic



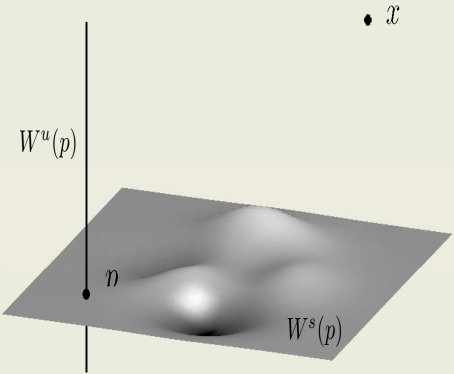
$x \in \mathrm{Phc}(p)$ if

$$W^u(x) \pitchfork o(W^s(p)) \neq \emptyset$$

$$W^s(x) \pitchfork o(W^u(p)) \neq \emptyset$$

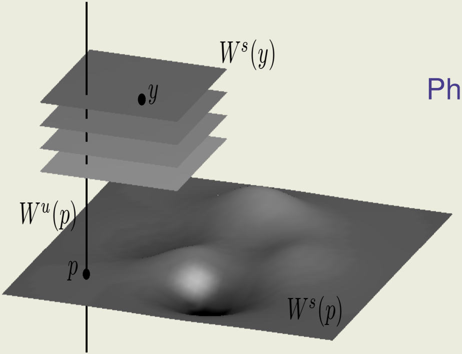
stable and unstable Phc

$$\mathrm{Phc}(p) = \mathrm{Phc}^s(p) \cap \mathrm{Phc}^u(p)$$



stable and unstable Phc

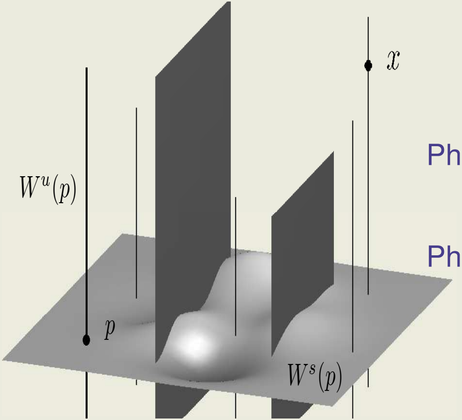
$$\mathrm{Phc}(p) = \mathrm{Phc}^s(p) \cap \mathrm{Phc}^u(p) \quad \text{where}$$



$$\mathrm{Phc}^s(p) = \{x : W^s(x) \pitchfork o(W^u(p)) \neq \emptyset\}$$

stable and unstable Phc

$$\text{Phc}(p) = \text{Phc}^s(p) \cap \text{Phc}^u(p) \quad \text{where}$$



$$\text{Phc}^s(p) = \{x : W^s(x) \pitchfork o(W^u(p)) \neq \emptyset\}$$

$$\text{Phc}^u(p) = \{x : W^u(x) \pitchfork o(W^s(p)) \neq \emptyset\}$$

advances

Theorem (RHRHTU(2009))

If $f \in \mathrm{Diff}^2(M)$, $p \in \mathrm{Per}(f)$ *hyperbolic, and*

advances

Theorem (RHRHTU(2009))

If $f \in \mathrm{Diff}^2(M)$, $p \in \mathrm{Per}(f)$ *hyperbolic, and*

- $m(\mathrm{Phc}^s(p)) > 0$

advances

Theorem (RHRHTU(2009))

If $f \in \mathrm{Diff}^2(M)$*,* $p \in \mathrm{Per}(f)$ *hyperbolic, and*

- $m(\mathrm{Phc}^s(p)) > 0$
- $m(\mathrm{Phc}^u(p)) > 0$

advances

Theorem (RHRHTU(2009))

If $f \in \mathrm{Diff}^2(M)$, $p \in \mathrm{Per}(f)$ *hyperbolic, and*

- $m(\mathrm{Phc}^s(p)) > 0$
- $m(\mathrm{Phc}^u(p)) > 0$

then

advances

Theorem (RHRHTU(2009))

If $f \in \mathrm{Diff}^2(M)$, $p \in \mathrm{Per}(f)$ *hyperbolic, and*

- $m(\mathrm{Phc}^s(p)) > 0$
- $m(\mathrm{Phc}^u(p)) > 0$

then

- $\mathrm{Phc}(p) = \mathrm{Phc}^s(p) = \mathrm{Phc}^u(p)$

advances

Theorem (RHRHTU(2009))

If $f \in \mathrm{Diff}^2(M)$, $p \in \mathrm{Per}(f)$ *hyperbolic, and*

- $m(\mathrm{Phc}^s(p)) > 0$
- $m(\mathrm{Phc}^u(p)) > 0$

then

- $\mathrm{Phc}(p) = \mathrm{Phc}^s(p) = \mathrm{Phc}^u(p)$
- $\mathrm{Phc}(p)$ *hyperbolic ergodic component of* m

applications

applications

applications

applications

- stable ergodicity in $\mathcal{P}H(M)$ context ($c = 2$) - RHRHTU

applications

applications

- stable ergodicity in $\mathcal{P}H(M)$ context ($c = 2$) - RHRHTU
- ergodicity in Nuh context - Avila-Bochi

applications

applications

- stable ergodicity in $\mathcal{P}H(M)$ context ($c = 2$) - RHRHTU
- ergodicity in Nuh context - Avila-Bochi
- ergodicity with one Lyapunov exponent $\neq 0$ in M^3 - JRH

questions

- Other contexts?

merci

- Merci!