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PROPOSITION .2 *Anosov tori in M^3 are incompressible*

PROPOSITION .3 *The only nilmanifold M^3 containing an Anosov torus is $\mathbb{T}^3$*

Case $\Gamma = M$

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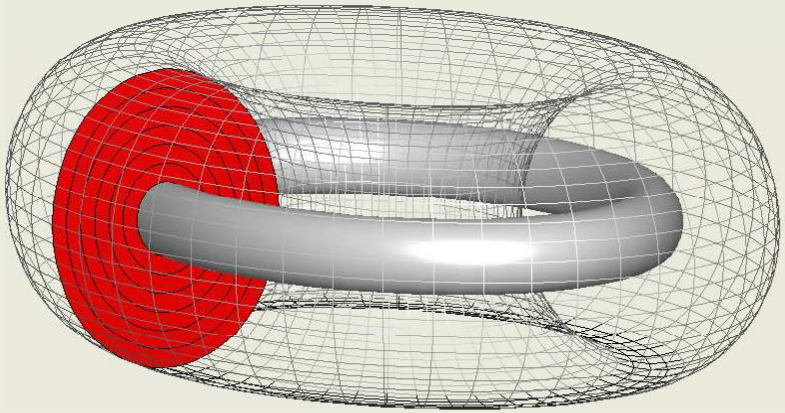

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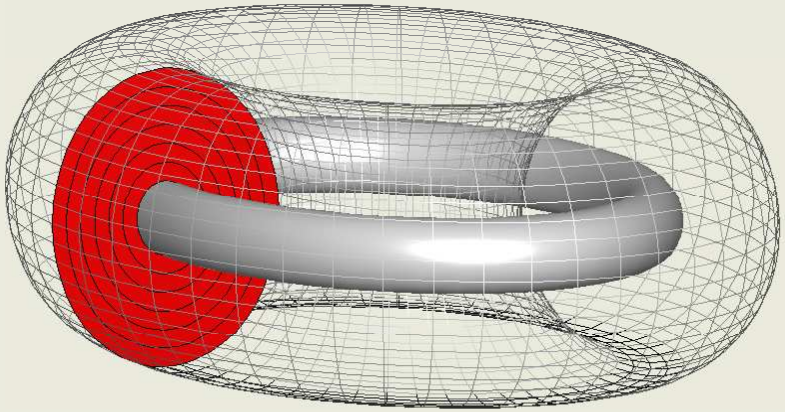

PROPOSITION .6 *$f_* = A$ is hyperbolic on $H_1(N, \mathbb{Z})$ and $h \circ f = A \circ h$ for some $h : N \to \mathbb{T}^2$*

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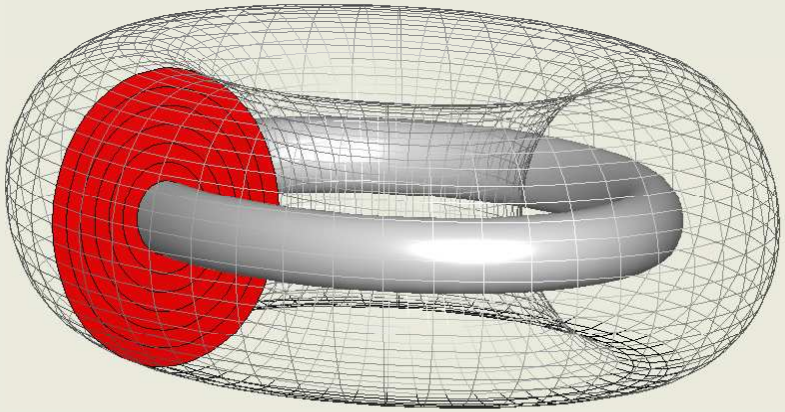


PROPOSITION .6 $f_* = A$ *is hyperbolic on* $H_1(N, \mathbb{Z})$
and $h \circ f = A \circ h$ *for some* $h : N \to \mathbb{T}^2$
+ extra work $\to$ CONTRADICTION

The End

definitions

Anosov torus

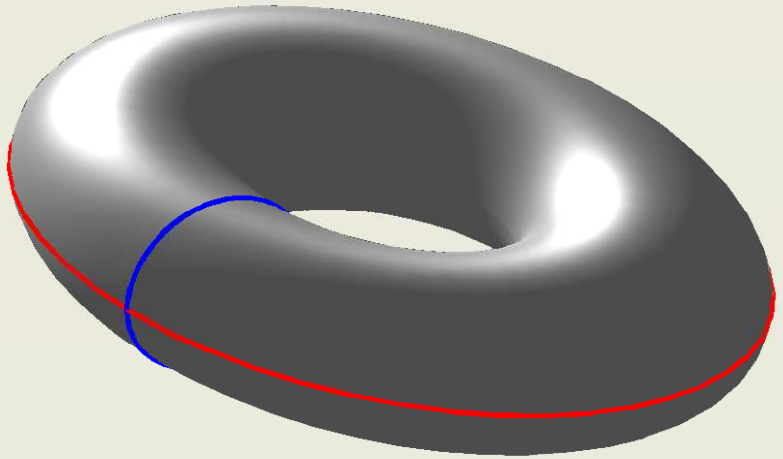
T is an Anosov torus if there is

$$g : M \to M$$

such that $g|T$ is Anosov.

incompressible

A torus $T \in M$ is incompressible if $i_{\#} : \pi_1(T) \to \pi_1(M)$ is injective



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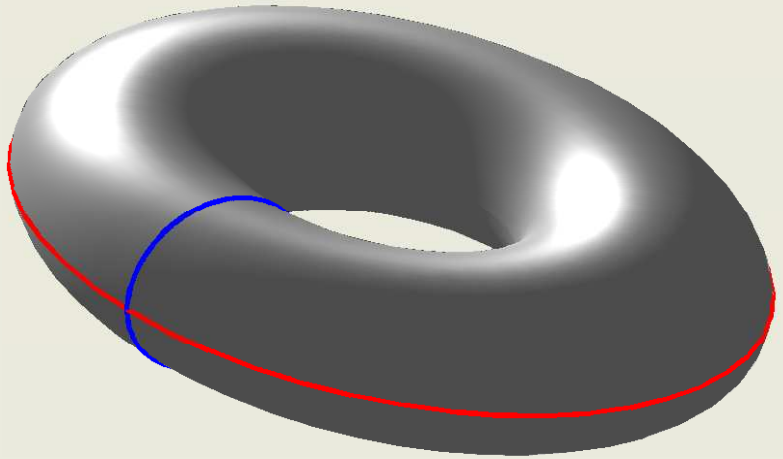

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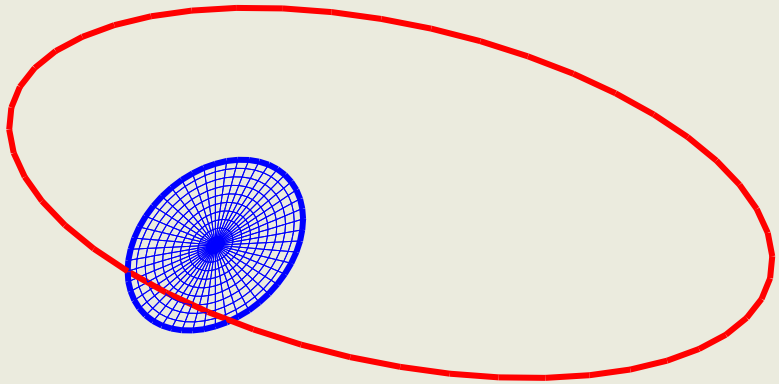

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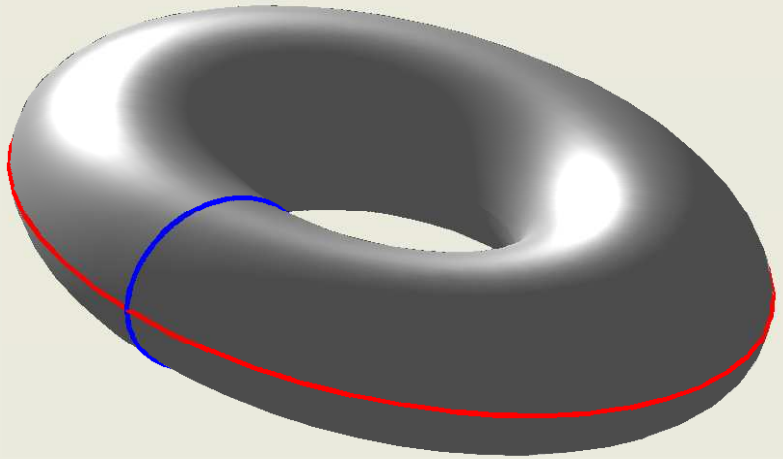

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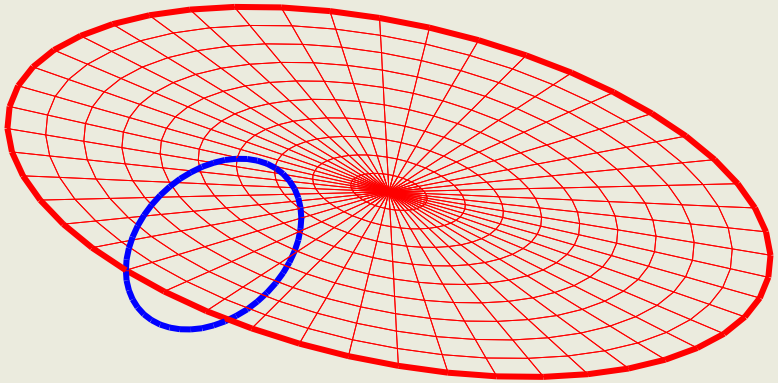

*

minimal foliation

$\mathcal{F}$ is a minimal foliation if every leaf is dense

propositions

proposition 1 - case $\emptyset \subsetneq \Gamma \subsetneq M$

- $\mathrm{Per}(f)$ is dense in $\partial^c\Gamma$ (RH,RH,U '05)

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- If $\pi_1(M)$ is nilpotent, then there is L that is an Anosov torus

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proposition 4 - case $\Gamma = M$

- $\overline{\{f^n(L) : n \in \mathbb{Z}\}} = M$ for all leaves L of $\mathcal{F}^{su}$

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proposition 5 - case $\Gamma = M$

- $\exists \mu$ such that $\mathrm{supp}\mu = \mathrm{M}$ (Plante '75)

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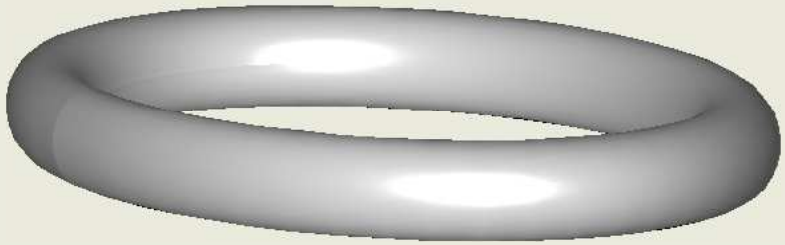
-  T incompressible torus

$\mathbb{T}^2 \times [0,1]$

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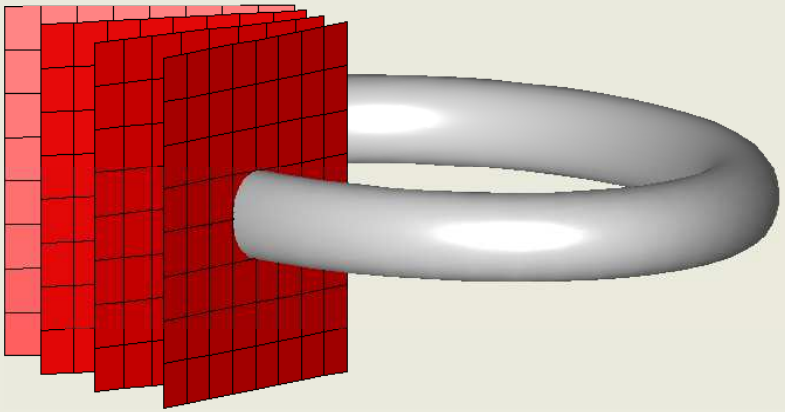
-  Roussarie '74 $\Rightarrow \mathcal{F}^{su} \pitchfork T$

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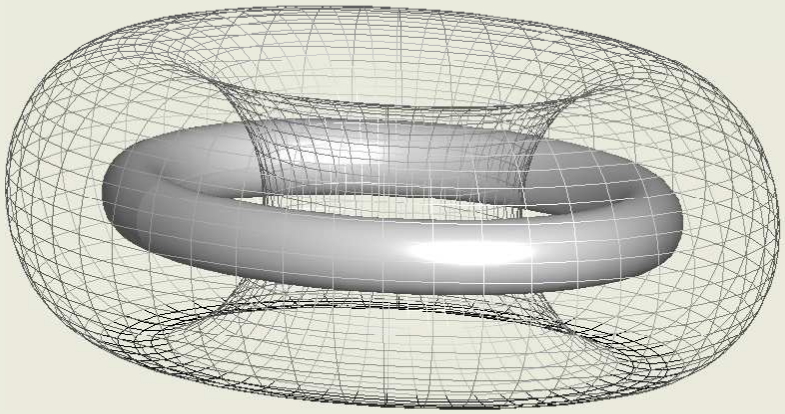
- 

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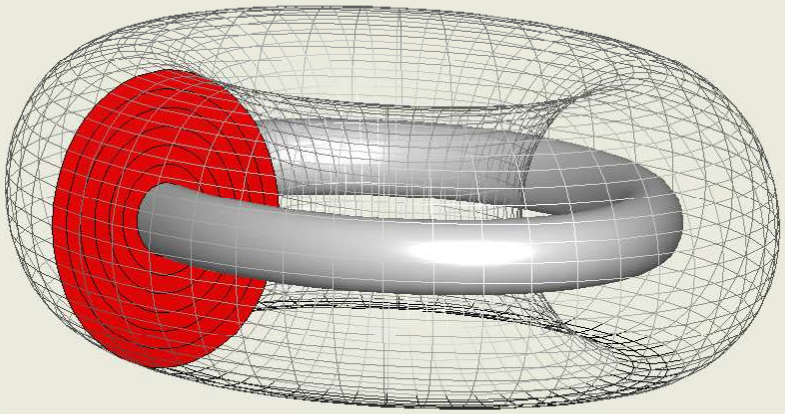
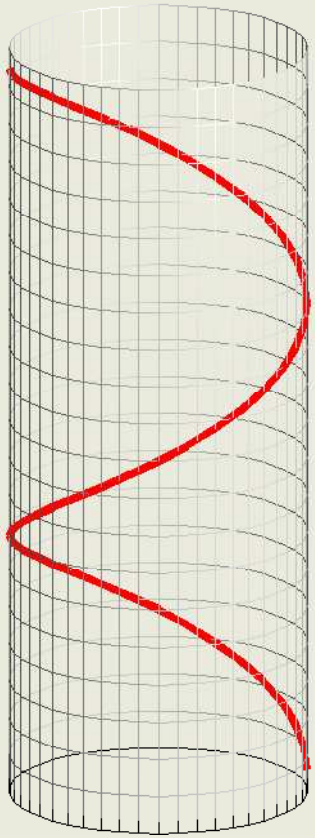
- 

$$N = \mathbb{T}^2 \times [0,1]/(x,0) \sim (Ax,1)$$

where $A = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$

*

extra work



$$h(W^u(x)) = W^u(h(x))$$

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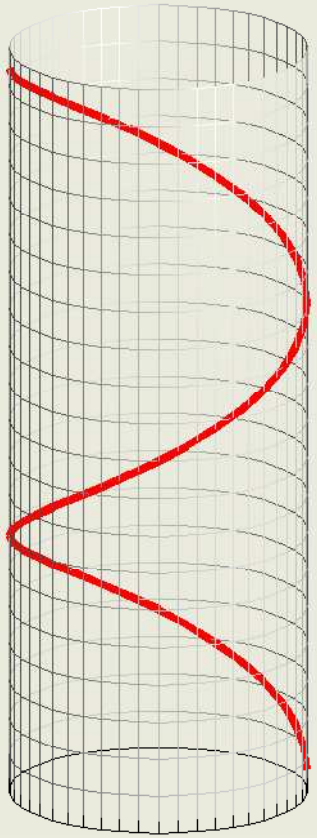

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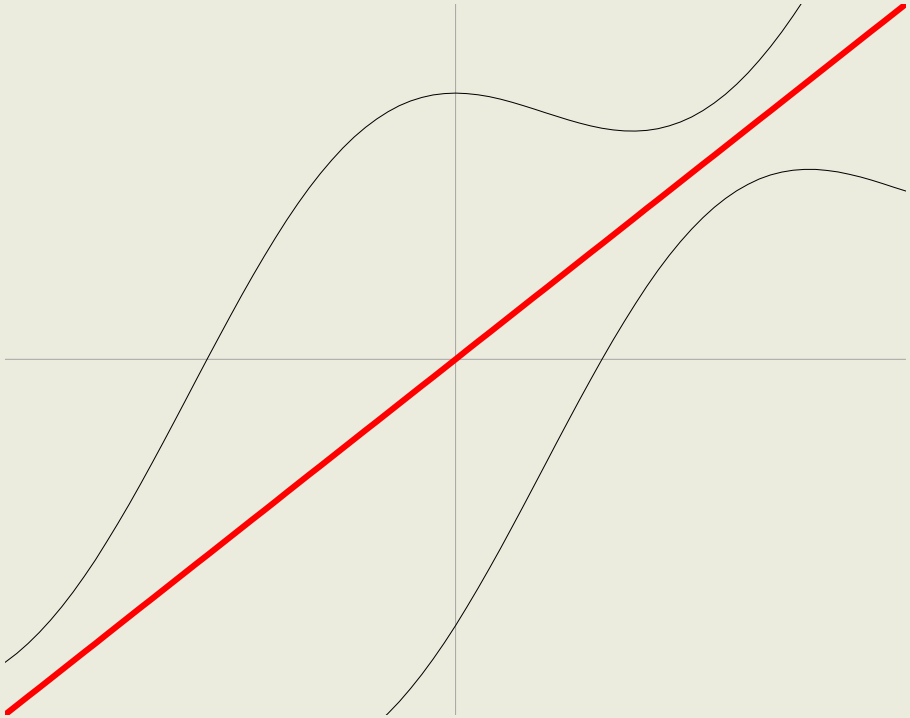

$h : F^{su}(x) \longrightarrow \mathbb{T}^2$

$\tilde{h} : F^{su}(x) \longrightarrow \mathbb{R}^2$

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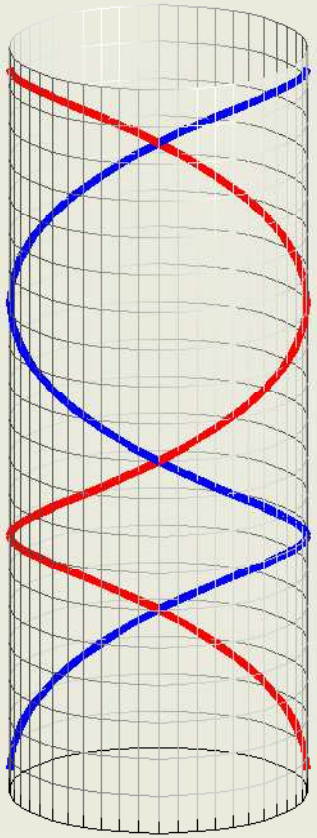

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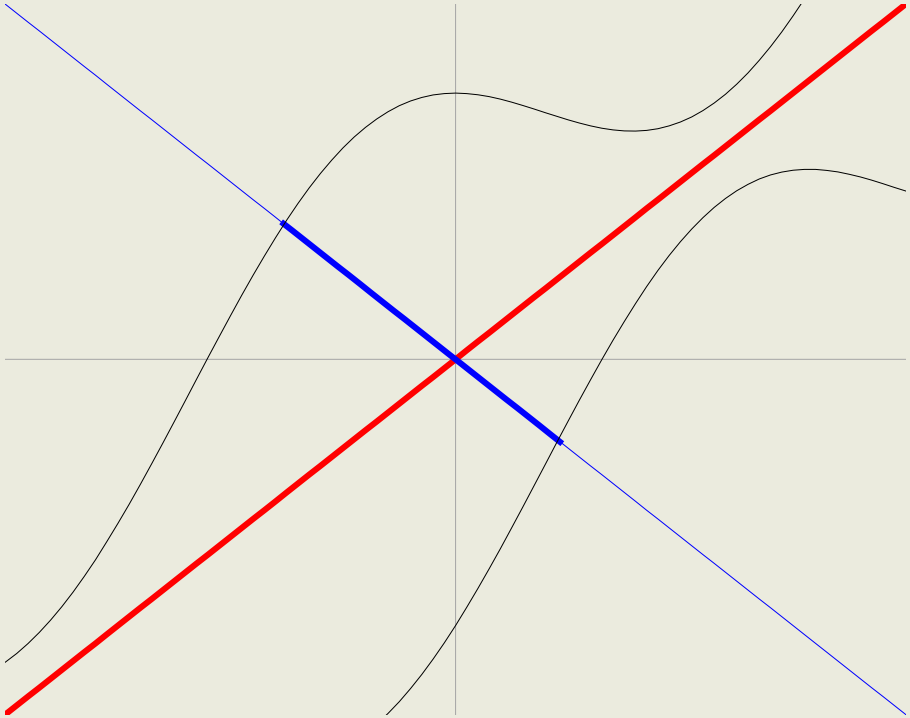

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