Topological dynamics

Conjugacy and semiconjugacy
Examples

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conjugacy

Definition (conjugacy)

- $f : X \to X$

conjugacy

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- $f : X \to X$ and
- $g : Y \to Y$

conjugacy

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- $f : X \to X$ and
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- $g : Y \to Y$ are conjugated
- if $\exists h : X \to Y$ homeomorphism such that

$$\begin{array}{ccccc} & Y & \xrightarrow{g} & Y & \\ h & \downarrow & & \downarrow & h \\ & X & \underset{f}{\to} & X & \end{array}$$

- $f \circ h = h \circ g$

remark

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If $f : X \to X$ and $g : Y \to Y$ are conjugated

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If $f : X \to X$ and $g : Y \to Y$ are conjugated
then they have the same dynamic behavior

proposition 1

proposition 1

- let $f \circ h = h \circ g$ (f and g conjugated)

proposition 1

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- let $f \circ h = h \circ g$ (f and g conjugated)
- p is a periodic point for g

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- let $f \circ h = h \circ g$ (f and g conjugated)
- p is a periodic point for g
- $\Leftrightarrow h(p)$ is a periodic point for f

proposition 2

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- let $f \circ h = h \circ g$ (f and g conjugated)
- x is a dense orbit for g

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- let $f \circ h = h \circ g$ (f and g conjugated)
- x is a dense orbit for g
- $\Leftrightarrow h(x)$ is a dense orbit for f

conjugacy

Definition (conjugacy)

- $g : Y \to Y$

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- $g : Y \to Y$ and
- $f : X \to X$

conjugacy

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- $g : Y \rightarrow Y$ and
- $f : X \rightarrow X$ are semi-conjugated

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h semiconjugacy

conjugacy

Definition (conjugacy)

- $g : Y \to Y$ and
- $f : X \to X$ are semi-conjugated
- if $\exists h : X \to Y$ continuous and surjective such that

$$\begin{array}{ccccc} & Y & \xrightarrow{g} & Y & \\ h & \downarrow & & \downarrow & h \\ & X & \xrightarrow[f]{} & X & \end{array} \qquad h \text{ semiconjugacy}$$

- $f \circ h = h \circ g$

remark

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If there is a semiconjugacy $f \circ h = h \circ g$

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If there is a semiconjugacy $f \circ h = h \circ g$
then the dynamics of g contains the dynamics of f

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- let $f \circ h = h \circ g$ (h semiconjugacy)

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- let $f \circ h = h \circ g$ (h semiconjugacy)
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- let $f \circ h = h \circ g$ (h semiconjugacy)
- p is a periodic point for g
- $\Rightarrow h(p)$ is a periodic point for f

proposition 2

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- let $f \circ h = h \circ g$ (h semiconjugacy)
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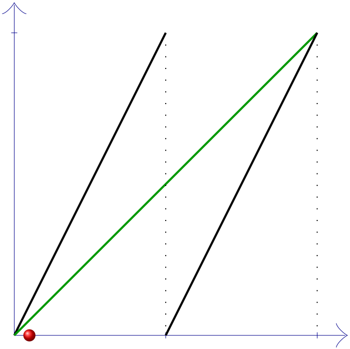
- let $f \circ h = h \circ g$ (h semiconjugacy)
- x is a dense orbit for g
- $\Rightarrow h(x)$ is a dense orbit for f

an example

Consider the map $f : [0,1] \to [0,1]$ such that $f(x) = 2x \mod 1$



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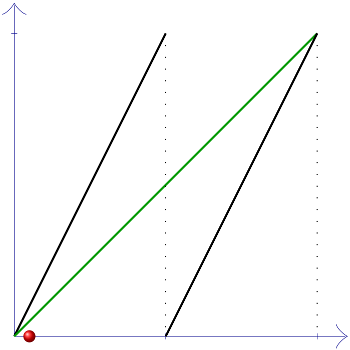

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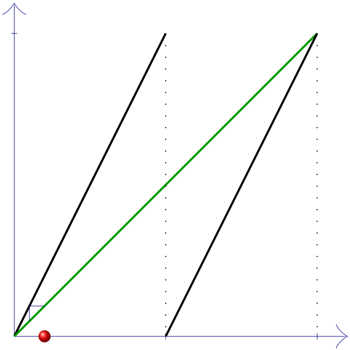

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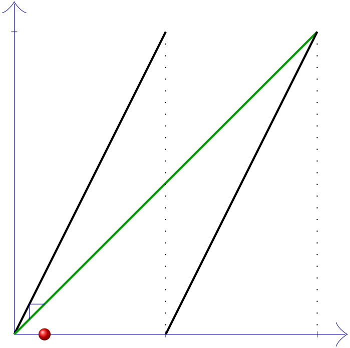

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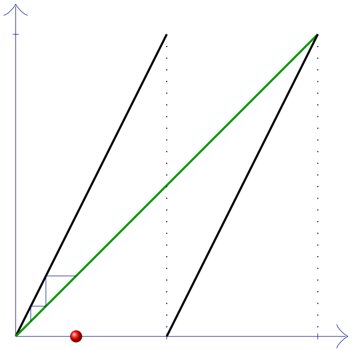

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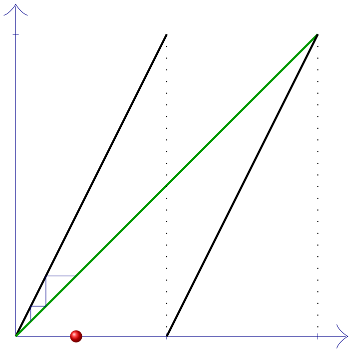

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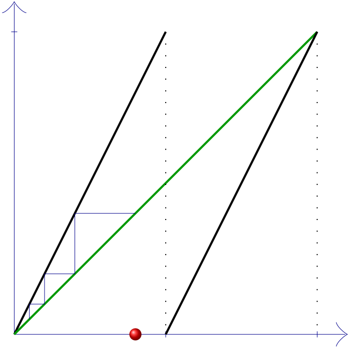

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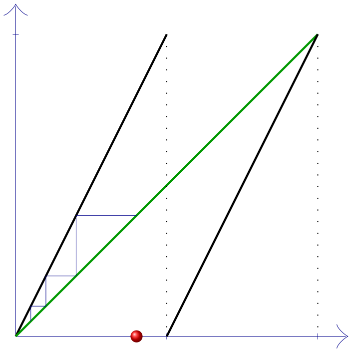

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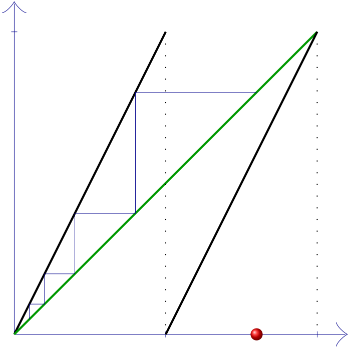

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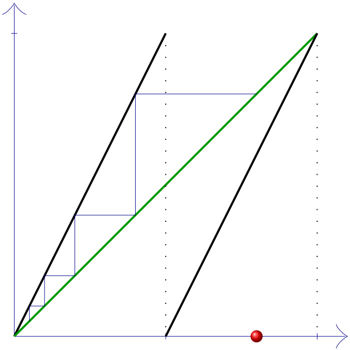

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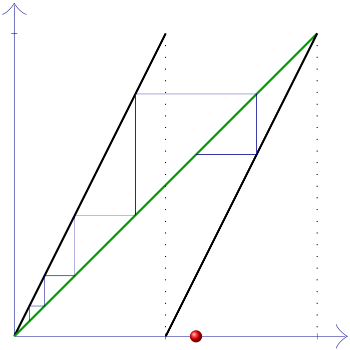

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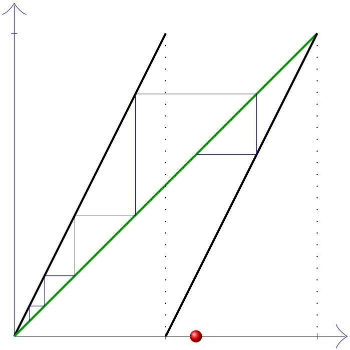

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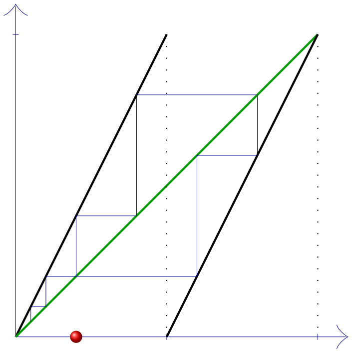

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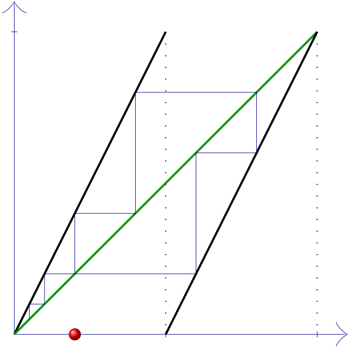

$\underline{x} = 0000110$

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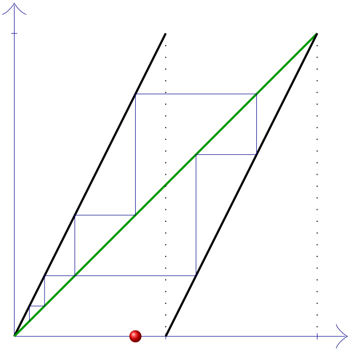

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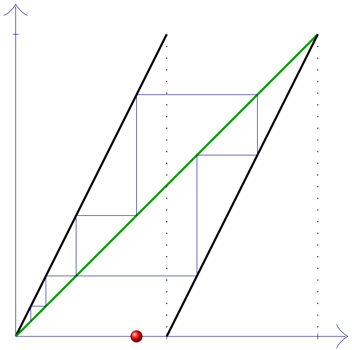

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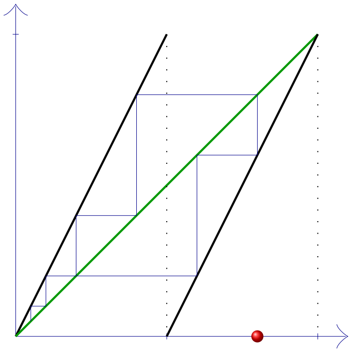

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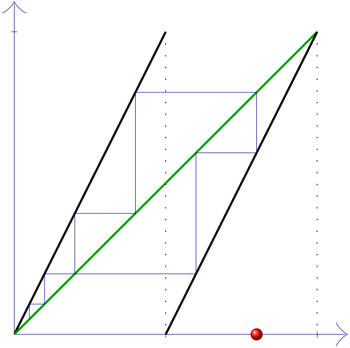

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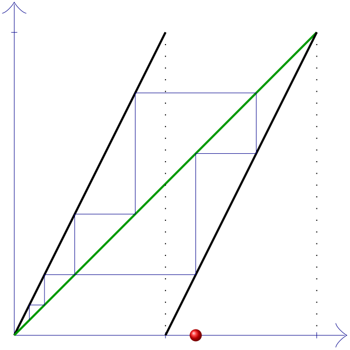

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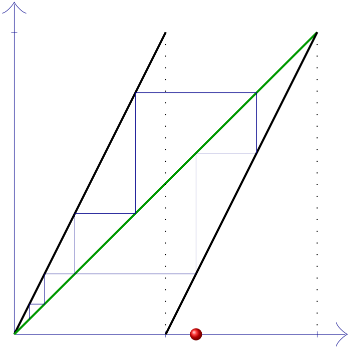

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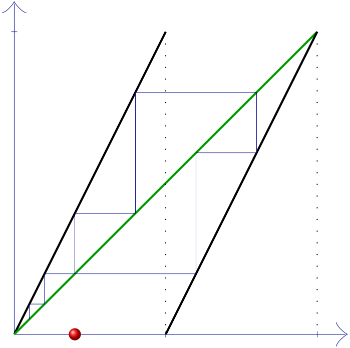

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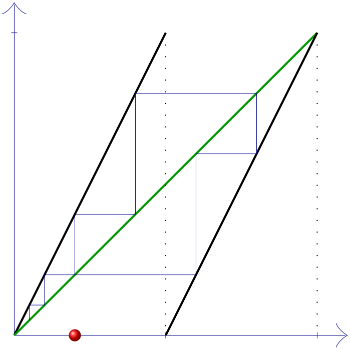

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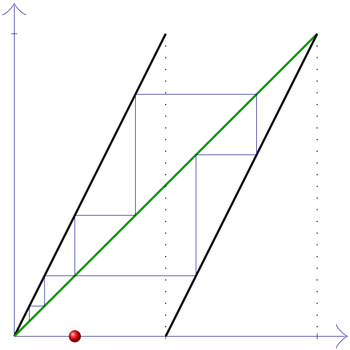

$\underline{x} = 00001100110011\ldots$