

① a) $x < 0$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = +i\hbar \frac{\partial \psi}{\partial t} \rightarrow \psi(x,t) = \psi(x) \tau(t)$$

$$\tau(t) = e^{-iEt/\hbar}$$

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} = -E \rightarrow \psi_1(x) = A e^{ikx} + B e^{-ikx} \quad / \quad k = \frac{\sqrt{2mE}}{\hbar}$$

$x > 0$ $\psi_2(x) = C e^{ikx}$

b) $\psi_1(x=0) = \psi_2(x=0) \rightarrow A+B=C \rightarrow B=C-A$

$$\left. \frac{d\psi_1}{dx} \right|_{x=0^-} = ik(A-B)$$

$$\left. \frac{d\psi_2}{dx} \right|_{x=0^+} = ikC$$

$$ik(C-A+B) = -\frac{2mS}{\hbar^2} C \rightarrow ik(C-A) = -\frac{2mS}{\hbar^2} C \rightarrow C = \frac{ikA}{\frac{mS}{\hbar^2} + ik}$$

Usando $k = \frac{mS}{\hbar^2} \rightarrow C = \frac{ikA(\alpha - ik)}{\alpha^2 + k^2} = \frac{A k (\alpha - i k)}{\alpha^2 + k^2}$

$$T(E) = \frac{C^* C}{A^* A} = \frac{k^2 (\alpha^2 + k^2)}{(\alpha^2 + k^2)^2} = \frac{k^2}{\alpha^2 + k^2} = \frac{1}{\frac{\alpha^2}{k^2} + 1} \quad / \quad \frac{\alpha^2}{k^2} = \frac{m^2 S^2}{\hbar^4} \frac{\hbar^2}{2mE}$$

$$T(E) = \left(1 + \frac{mS^2}{2\hbar^2 E} \right)^{-1}$$

② $\gamma = 1,25$

$\Delta t' = 2h$

a) $\Delta t = \gamma (\Delta t' + \frac{v}{c^2} \Delta x')$; $\Delta x = \gamma (\Delta x' + v \Delta t')$

$$\Delta t_p = \gamma \Delta t' - \frac{\Delta x}{c}$$

$$\Delta t_p = \gamma \Delta t' - \frac{\gamma}{c} (\Delta x' + v \Delta t')$$

$$\Delta t_p = \gamma (1 - v/c) \Delta t' = \frac{(1 - v/c) \Delta t'}{(1 - v/c)^{1/2} (1 + v/c)^{1/2}} = \sqrt{\frac{1 - v/c}{1 + v/c}} \Delta t' = 1 \text{ hora}$$

c) Trayecto recorrido por la nave durante el examen: $\Delta x_A = v \Delta t = \gamma v \Delta t' = 1,5c$

Trayecto recorrido por la nave mientras llega señal a Tierra: $\Delta t_B = \frac{\Delta x_A}{c} = 1,5h$

$$\Delta x_B = v \Delta t_B = 0,9c$$

Trayecto recorrido por la nave mientras corrigen ($\Delta t = 2h$): $\Delta x_C = v \Delta t = 1,2c$

Trayecto recorrido por la nave mientras llegan resultados: $\Delta x_D = v \Delta t_D$

$$\Delta t_D = \Delta x_A + \Delta x_B + \Delta x_C + \Delta x_D \Rightarrow \Delta t_D = 9h \quad \Delta t_T = 12,5h \quad \Delta t_T = 10 \text{ horas}$$

