PROB 2 : $\frac{1}{\lambda} = \frac{R_\infty Z^2}{1+\frac{m}{M}} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

Átomo de Bohr : $E = -\frac{V(r)}{2} = +\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r}$ $\quad E = T+V(r)$ / $T = +\frac{V(r)}{2}$

$T = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{2r} = \frac{mv^2}{2} \longrightarrow v^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{mr} = \frac{n^2\hbar^2}{m^2r^2} \longrightarrow r = \frac{n^2\hbar^2}{(1/4\pi\varepsilon_0)Ze^2 m}$

$L = mvr = n\hbar \longrightarrow v = \frac{n\hbar}{mr}$ $\quad E = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{(Ze)^2 m}{2n^2\hbar^2}$

$\Delta E = \left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{m}{\hbar^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = h\nu = h\frac{c}{\lambda} \longrightarrow \frac{1}{\lambda} = Z^2\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{m\,2\pi}{\hbar^3 c}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$R_\infty = \left(e^2/4\pi\varepsilon_0\right)^2 \frac{m\,2\pi}{\hbar^3 c}$

b) $\frac{\lambda_D}{\lambda_H} = \frac{(1+m/M_D)}{(1+m/M_H)} \cong (1+m/M_D)(1-m/M_H) = \left[1 + m^2/M_D M_H + m\,(1/M_D - 1/M_H)\right]$

$\frac{\lambda_D}{\lambda_H} \cong 1 + m\,(1/M_D - 1/M_H) \longrightarrow m/M_D = \left(\frac{\lambda_D}{\lambda_H} - 1\right) + \frac{m}{M_H} = (3677)^{-1} \longrightarrow M_D \cong 2M_H$

PROB 4 : $\Psi(x,t) = \varphi(x)\,\tau(t) \longrightarrow -\frac{\hbar^2}{2m}\tau(t)\frac{d^2\varphi}{dx^2} + V(x)\varphi(x)\tau(t) = \frac{\hbar}{i}\varphi(x)\frac{d\tau}{dt}$

$-\frac{\hbar^2}{2m}\frac{1}{\varphi(x)}\frac{d^2\varphi}{dx^2} + V(x) = \frac{\hbar}{i}\frac{1}{\tau(t)}\frac{d\tau}{dt} = E$

(1) $\frac{\hbar}{i}\frac{d\tau}{dt} = \frac{i}{\hbar}E\,\tau(t) \rightarrow \tau(t) \propto e^{\pm iE/\hbar t} = e^{i\omega t}$

(2) $-\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2} + V(x)\varphi(x) = E\varphi(x)$

Región (I) $\varphi_I(x<0) = A e^{ikx} + B e^{-ikx}$ / $k = \sqrt{\frac{2mE}{\hbar^2}}$ $\quad k^2+\gamma^2 = \frac{2mV_0}{\hbar^2}$

Región (II) $\varphi_{II}(x>0) = C e^{-\gamma x}$ / $\gamma = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$

$\varphi_I(x=0) = \varphi_{II}(x=0) \longrightarrow A+B = C \longrightarrow B = C-A$

$\frac{d\varphi_I}{dx}(x=0) = \frac{d\varphi_{II}}{dx}(x=0) \longrightarrow ik(A-B) = -\gamma C \longrightarrow ik(2A-C) = \gamma C$

$C = \frac{2Aik}{\gamma+ik} = \frac{2Aik(\gamma-ik)}{\gamma^2+k^2} = \frac{2A(k^2-ik\gamma)}{\gamma^2+k^2} = \frac{2A}{\sqrt{\gamma^2+k^2}}e^{-i\varphi}$ / $\varphi = \text{Arctg}\left(\frac{\gamma}{k}\right)$

$C = \frac{A\hbar}{mV_0}e^{i\varphi}$ / $\varphi = \text{Arctg}\left(\frac{V_0-E}{E}\right)^{1/2}$

$\Psi(x>0,t) = \frac{A\hbar}{mV_0}e^{-\gamma x}e^{-i(\omega t+\varphi)}$

ΓΚΟΒ 3

a) $\lambda = \frac{h}{p} =$

$E^2 = p^2c^2 + m_0^2c^4$ $\quad K = eV = E - m_0c^2 \rightarrow E = eV + m_0c^2$

$p^2 = \frac{E^2}{c^2} - m_0^2c^2 = \frac{e^2V^2 + m_0^2c^4 - 2m_0c^2eV}{c^2} - m_0^2c^2 = \frac{e^2V^2}{c^2} + \cancel{m_0^2c^2} - 2m_0eV - \cancel{m_0^2c^2}$

$\lambda = \frac{h}{(2m_0eV)^{1/2}\left(1 + \frac{eV}{2m_0c^2}\right)^{1/2}} = \frac{h}{\left(2m_0eV + \frac{\cancel{2m_0}e^2V^2}{\cancel{2m_0}c^2}\right)^{1/2}} = \frac{h}{\left(2m_0eV + \frac{e^2V^2}{c^2}\right)^{1/2}}$ ό.έ.δ.

b) $K = eV = 2MeV$

$\Delta x \Delta p \sim h \rightarrow \Delta p = \Delta\left(2m_0eV + \frac{e^2V^2}{c^2}\right)^{1/2} = \frac{\Delta eV\left(\cancel{2}m_0 + \cancel{2}eV/c^2\right)}{\cancel{2}\left(2m_0eV + \frac{e^2V^2}{c^2}\right)^{1/2}}$

$\Delta p = \frac{\Delta eV\,(m_0 + eV/c^2)}{eV^{1/2}\left(2m_0 + \frac{eV}{c^2}\right)^{1/2}} = \frac{\Delta eV\,(m_0c^2 + eV)}{c\,eV^{1/2}\,(2m_0c^2 + eV)^{1/2}} = \frac{\Delta eV\,(2{,}5\cancel{M})}{\sqrt{2}\,(1+2)^{1/2}\cancel{M}\,c}$

$m_0c^2 = 0{,}5\,MeV$

$\Delta p = 1{,}316\,\frac{\Delta eV}{c}$

$\Delta x \Delta p \sim h \rightarrow \Delta x\, 1{,}316\,\frac{\Delta eV}{c} \sim h \rightarrow \Delta eV \sim \frac{hc}{1{,}3\times10^{-5}} \approx \frac{6{,}6\times10^{-34}\times 3\times10^{8}}{1{,}3\times10^{-5}}$

$\Delta eV \sim 1{,}5\times10^{-20}\ J = 0{,}1\,eV$ $\quad \Delta V = 0{,}1\,V$