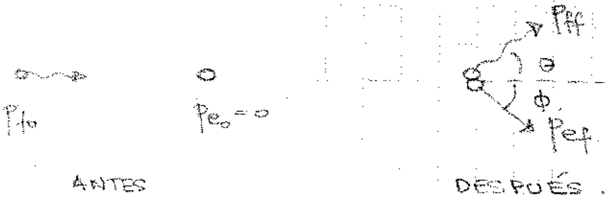
EXAMEN FIS MOD

27/02/09

(1)

a)



* $p_{fo} = p_{ff} \cos\theta + p_{ef} \cos\phi$ $\quad E_{fo} = \frac{hc}{\lambda}$; $E_{ff} = \frac{hc}{\lambda'}$; $E_{ef} = \sqrt{p_{ef}^2 c^2 + m_o^2 c^4}$

* $p_{ff} \operatorname{sen}\theta = p_{ef} \operatorname{sen}\phi$ $\quad p_{fo} = \frac{h}{\lambda}$; $p_{ff} = \frac{h}{\lambda'}$

* $E_{fo} + m_o c^2 = E_{ff} + E_{ef}$

b) $E_{ef} = E_{fo} - E_{ff} + m_o c^2 = \text{máx} \longrightarrow E_{ff} = \text{mín} = \frac{hc}{\lambda'} = \text{mín} \longrightarrow \lambda' = \text{máx}$

$$\lambda' = \lambda + \frac{h}{m_o c}(1 - \cos\theta) \rightarrow \cos\theta = -1 \rightarrow \theta = 180°$$

$$\lambda' = \lambda + 2\frac{h}{m_o c} \rightarrow \lambda' - \lambda = \frac{2h}{m_o c} = 2 \times 0{,}0243 \times 10^{-10} = 4{,}86 \times 10^{-12} \text{ m.}$$

$$E_{fo} - E_{ff} = 45 \text{ keV} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = 1{,}242 \times 10^{-6} \text{ eV m} \left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$h = 4{,}14 \times 10^{-15}$ eV.s.
$c = 3{,}0 \times 10^8$ m/s.

$$\frac{1}{\lambda} - \frac{1}{\lambda'} = 3{,}623 \times 10^{10} \text{ m}^{-1}$$

$$\lambda' - \lambda = 4{,}86 \times 10^{-12} = 3{,}623 \times 10^{10} \lambda\lambda' \rightarrow \lambda\lambda' = 1{,}341 \times 10^{-22} \text{ m}^2$$

$$\lambda' = \frac{1{,}341 \times 10^{-22}}{\lambda} \rightarrow \frac{1{,}341 \times 10^{-22}}{\lambda} - \lambda = 4{,}86 \times 10^{-12}$$

$$\lambda^2 + 4{,}86 \times 10^{-12}\lambda - 1{,}341 \times 10^{-22} = 0 \qquad \lambda = 0{,}094 \text{ Å}$$

(2) a) $\mathcal{E} = -\frac{dV}{dr} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{4/3\,\pi R^3} \times 4/3\,\pi r^3 \frac{1}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{Qr}{R^3}$

$U(r) = (-e)\,V(r) = -\frac{k_0 Q e}{R}\left[\frac{3}{2} - \frac{r^2}{2R^2}\right] = -\frac{k_0 Z e^2}{R}\left[\frac{3}{2} - \frac{r^2}{2R^2}\right]$

$E = \frac{1}{2} m v^2 + U(r)$ / $\frac{v^2}{r} = \frac{F}{m} = \frac{k_0}{m} \frac{Qer}{R^3} \rightarrow v^2 = \frac{k_0 Q e r^2}{m\ R^3}$

$K(r) = \frac{1}{2}\ m v^2 = \frac{1}{2}\ \frac{k_0 Z e^2 r^2}{R^3}$

b) $E = \frac{k_0}{2R} Ze^2 \left(\frac{r^2}{R^2} - 3 + \frac{r^2}{R^2}\right) = \frac{k_0}{2R} Ze^2 \left(2\frac{r^2}{R^2} - 3\right) = -10{,}75\ \text{MeV}.$

$\frac{9\times10^9 \times 82 \times 1{,}6\times10^{-19}}{10{,}75\times 2\times 7{,}1\times10^{-15}\times10^6} = \left(-\frac{2r^2}{R^2}+3\right)^{-1} \longrightarrow 3 - \frac{2r^2}{R^2} = 1{,}293.$

$r^2 = \frac{(3-1{,}293)}{2} R^2$

$r = 6{,}56\ \text{fm}$

c) $E_2 = -\frac{Z^2 \times 207 \times 13{,}6}{4} = -4{,}73\ \text{MeV}$ $\qquad \Delta E = E_2 - E_1 = 6\ \text{MeV} = E_f.$

$E_f = h\nu \rightarrow \nu = 1{,}45\times10^{15}\,\text{Hz} \rightarrow \lambda = \frac{c}{\nu} = 206\ \text{nm}$

(3) a) $\psi(x) = A x e^{-\alpha x}$ $\qquad -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - \frac{q^2}{x}\psi = E\psi$ $\qquad \alpha = \frac{mq^2}{\hbar}$

$\frac{d\psi}{dx} = A\left(e^{-\alpha x} - \alpha x e^{-\alpha x}\right) = A(1-\alpha x)e^{-\alpha x}$

$\frac{d^2\psi}{dx^2} = A\left[-\alpha(1-\alpha x) - \alpha\right] e^{-\alpha x} = A\alpha(2-\alpha x)e^{-\alpha x}.$

$-\frac{\hbar^2}{2m}\alpha(2-\alpha x) - q^2 = E x \longrightarrow E = -\frac{\hbar^2}{2m}\alpha^2 \overset{(*)}{=} -\frac{\hbar^2 m q^4}{2\hbar^4} = -\frac{m q^4}{2\hbar^2}$

$\frac{\hbar^2}{m}\alpha = q^2 \rightarrow \alpha = \frac{mq^2}{\hbar^2}$ (*)

b) $\psi(x,t) = A x e^{-\alpha x} e^{j\omega t}$ / $\omega = \frac{E}{\hbar}$

$1 = \int_0^{+\infty} A^2 x^2 e^{-2\alpha x} dx = \frac{A^2}{\alpha^3}\frac{2}{8} = \frac{A^2}{4\alpha^3} \rightarrow A = 2\alpha^{3/2}$

$\langle x \rangle = \int_0^{+\infty} A^2 x^3 e^{-2\alpha x} dx = \frac{\cancel{4}}{\alpha^4}\frac{6\alpha^3}{\cancel{16}_4} = \frac{3}{2\alpha}$