EXAMEN 10/07/09 FÍSICA MODERNA IPA

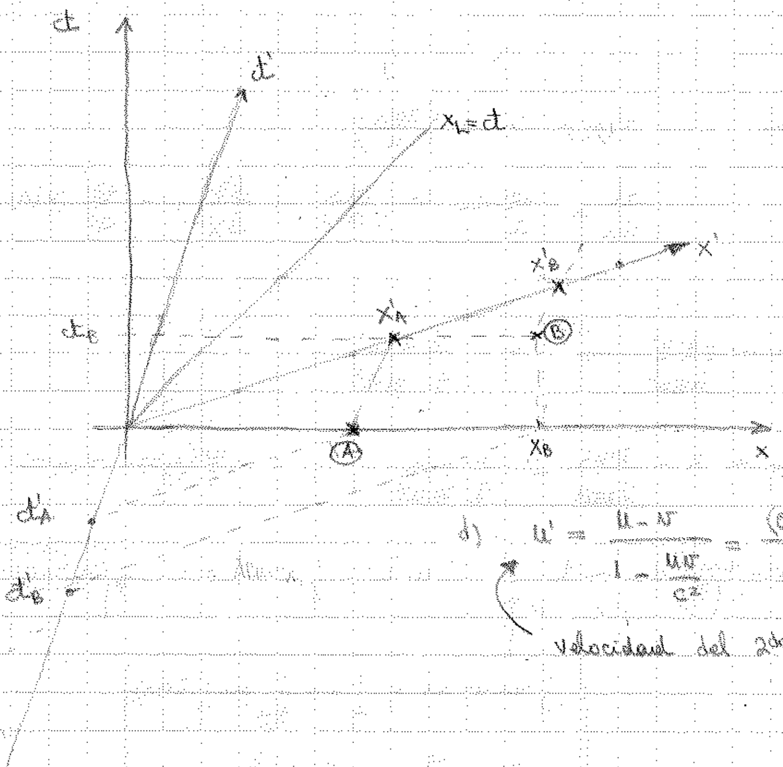
① $t_A = 0$ $t_B = 4,0\times10^{-6}\,s$ $ct_B = 12\times10^{2}\,m = 1,2\times10^{3}\,m$

$x_A = 3,0\times10^{3}\,m$ $x_B = 6,0\times10^{3}\,m$

$v = 0,6c$ $\gamma = 1,25$

a) $x'_A = \gamma(x_A - v t_A) = 3,75\times10^{3}\,m$ $x'_B = \gamma(x_B - v t_B) = 6,6\times10^{3}\,m$

$t'_A = \gamma\left(t_A - \frac{v}{c^2}x_A\right) = -7,5\times10^{-6}\,s$ $t'_B = \gamma\left(t_B - \frac{v}{c^2}x_B\right) = -0,1\times10^{-5}\,s$



c) $t''_A = t''_B \rightarrow u = ?$

$$t_A - \frac{u}{c^2}x_A = t_B - \frac{u}{c^2}x_B$$

$$u = \frac{(t_B - t_A)c^2}{(x_B - x_A)} = 0,4c$$

Velocidad del 2do inspector, resp. del ASTEROIDE

d) $$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{(0,4-0,6)c}{1 - 0,4\times0,6} = -0,263c$$

velocidad del 2do inspector, resp del 1er inspector

② a) $\nu_{rot} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{me^4}{4\pi\hbar^3}\frac{2}{n^3}$ Demostración $\boxed{L = n\hbar = mrv}$ (1)

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \quad / \quad F = m\omega^2 r \longrightarrow \omega^2_{rot} = \frac{F}{mr} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr^3} = \frac{v^2}{r^2} \longrightarrow$$

$$v^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr} \longrightarrow m^2v^2r^2 = \frac{m^2r^2}{4\pi\varepsilon_0}\frac{e^2}{\cancel{m}\cancel{r}} = \frac{me^2}{4\pi\varepsilon_0}r = \underset{(1)}{n^2\hbar^2}$$

$$r = (4\pi\varepsilon_0)\frac{n^2\hbar^2}{me^2} \qquad v = \frac{n\hbar}{mr} = \frac{e^2}{(4\pi\varepsilon_0)n\hbar}$$

$$\omega_{rot} = 2\pi\nu_{rot} = \frac{v}{r} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{n^3\hbar^3} \longrightarrow \nu_{rot} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{2\pi n^3\hbar^3}$$

b) $$E = \frac{1}{2}mv^2 - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = \frac{1}{2}m\frac{e^4}{(4\pi\varepsilon_0)^2 n^2\hbar^2} - \frac{1}{(4\pi\varepsilon_0)^2}\frac{me^4}{n^2\hbar^2} = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{me^4}{2n^2\hbar^2} \rightarrow -13,6\,eV$$

$$E(n_f) - E(n_i) = 13,6\,eV\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad \nu_{rot} = \frac{13,6\,eV}{\pi\hbar}\frac{1}{n^3} =$$

$\rightarrow$

Sigue PROB. 2

$$\nu = \frac{13{,}6\,eV}{h}\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) = \frac{13{,}6\,eV}{h}\left(\frac{1}{(n_i-N)^2}-\frac{1}{n_i^2}\right) = \frac{13{,}6\,eV}{h}\;\frac{\cancel{n_i^2}-\cancel{n_i^2}+2n_iN-N^2}{(n_i-N)^2\,n_i^2}$$

$$\nu = \frac{13{,}6\,eV}{h}\;\frac{N\,(2n_i-N)}{(n_i-N)^2\,n_i^2} = \frac{13{,}6\,eV}{h}\;\frac{N\,n_i\,(2-N/n_i)}{n_i^2\,(1-N/n_i)^2\,n_i^2} = \frac{13{,}6\,eV}{h}\;\frac{N}{n_i^3}\;\frac{(2-N/n_i)}{(1-N/n_i)}$$

$$\nu_{mt} = \frac{13{,}6\,eV}{h}\times\frac{2}{n^3} \qquad \lim_{n_i\to\infty}\frac{N\,(2-N/n_i)}{(1-N/n_i)} = 2 \;\text{ si }\; N/n_i \ll 1 \qquad \boxed{N=1}$$

(3)

a) y b) $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} - V_0\,\psi(x) = E\,\psi(x)$ $\qquad \psi(x) = C_1\,\text{sen}\left(\dfrac{2\pi x}{a}\right)$

$$\frac{d\psi}{dx} = \frac{2\pi}{a}C_1\cos\left(\frac{2\pi x}{a}\right) \qquad \frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{a^2}\psi(x)$$

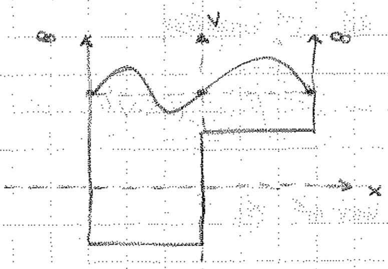
$$\frac{4\pi^2\hbar^2}{2ma^2} - V_0 = E$$

$$\frac{\pi^2\hbar^2}{2ma^2} + V_0 = E$$

$$2E = \frac{5\pi^2\hbar^2}{2ma^2} \longrightarrow E = \frac{5}{4}\frac{\pi^2\hbar^2}{ma^2}$$

$$V_0 = E - \frac{\pi^2\hbar^2}{2ma^2} = \frac{3}{4}\frac{\pi^2\hbar^2}{ma^2} \qquad \Psi(x,t) = \psi(x)\,e^{-\frac{i}{\hbar}Et}$$

c) $$\int_{-a}^{0} C_1^2\,\text{sen}^2(2\pi x/a)\,dx + \int_0^a C_2^2\,\text{sen}^2(\pi x/a)\,dx = 1 = \frac{1}{2}a\,C_1^2 + \frac{1}{2}a\,C_2^2 = 1 \rightarrow \boxed{C_1^2+C_2^2 = \frac{2}{a}}$$



$$\boxed{\begin{array}{l}\int \text{sen}^2\alpha x\,dx = \frac{1}{2}x - \frac{1}{4\alpha}\,\text{sen}(2\alpha x)\\ \alpha_1 = \frac{2\pi}{a}\;;\;\alpha_2 = \frac{\pi}{a}\end{array}}$$

$$\psi_1(x=0) = 0 = \psi_2(x=0) \qquad \frac{d\psi_1}{dx}(x=0) = \frac{2\pi}{a}C_1 = \frac{\pi}{a}C_2 = \frac{d\psi_2}{dx}(x=0)$$

$$2C_1 = C_2 \longrightarrow C_1^2 + 4C_1^2 = \frac{2}{a}$$

$$C_1^2 = \frac{2}{5a} \rightarrow \boxed{C_1 = \sqrt{\frac{2}{5a}}}$$

$$\boxed{C_2 = 2\sqrt{\frac{2}{5a}}}$$

d) $$P(x>0) = \int_0^a \psi_2^2(x)\,dx = \frac{1}{2}C_2^2\,a = \frac{4}{5}$$