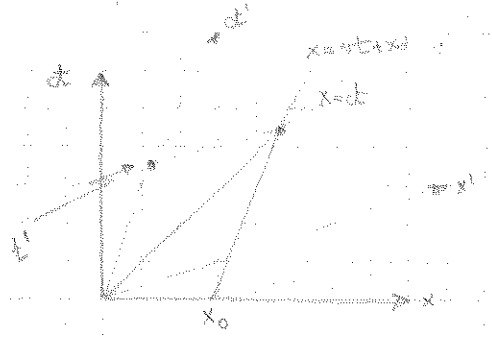
④ $v = 0{,}6c$ $\Longrightarrow$ $L_0 = 150\,m$



$$x_0 = \frac{L_0}{\gamma} = \frac{150}{5}\times 4 = 120\,m$$

$$\left.\begin{aligned} x &= vt + x_0 \\ x &= ct \end{aligned}\right\} \quad vt + x_0 = ct$$

$$t = \frac{x_0}{c-v} = \frac{120}{3\times10^8 \times 0{,}4}$$

$$t = \frac{300}{3\times10^8} = 1{,}0\times10^{-6}$$

$$t' = \frac{150}{c} = 0{,}5\times10^{-6}$$

$\gamma = \frac{5}{4}$

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② $E = m_0c^2 + K$ $\qquad E^2 - m_0^2c^4 = p^2c^2 = m(\gamma)^2 v^2 c^2 = \frac{25}{16} m_0^2 \times \frac{9}{25} c^4$

$\gamma = \frac{5}{4}$ $\qquad E^2 = \left(\frac{9}{16} + 1\right) m_0^2c^4 = \frac{25}{16} m_0^2c^2 \Rightarrow E = \frac{5}{4} m_0c^2$

$v = 0{,}6c = \frac{3}{5}$ $\qquad E_0 = \frac{9}{4} m_0c^2 = E_f = \sqrt{m_0^2c^4 + p_f^2c^2}$

$$\frac{81}{16} m_0^2c^4 = m_0^2c^4 + \gamma(v)^2 m_0^2 \frac{v_f^2}{c^2} c^2$$

$$\begin{cases} \frac{81}{16} m_0^2 = m_0^2\left(1 + \gamma(v)\frac{v_f^2}{c^2}\right) \\ \underbrace{\frac{5}{4}}_{\gamma(v_0)} \times m_0 \times \underbrace{\frac{3}{5}c}_{v_0} = \gamma(v_f)\, m_0\, v_f \end{cases}$$

$$\gamma^2(v_f)\, m_0^2 \frac{v_f^2}{c^2} = \frac{9}{16} m_0^2 \qquad \frac{81}{16} m_0^2 = m_0^2 + \frac{9}{16} m_0^2$$

$$m_0^2 = \left(\frac{81}{16} - \frac{9}{16}\right) m_0^2 \rightarrow m_0 = \frac{\sqrt{32}}{4} m_0 = \frac{2\sqrt{2}}{4} 3 = \frac{3}{\sqrt{2}} m_0$$

$$\frac{3}{5} m_0 = \frac{1}{(1-v^2/c^2)^{1/2}} \frac{3}{\sqrt{2}} m_0 \frac{v_f}{c} \rightarrow \frac{1}{2\sqrt{2}} = \frac{v/c}{(1-v^2/c^2)^{1/2}}$$

$$\frac{1}{8} = \frac{\beta^2}{(1-\beta^2)} \rightarrow 1-\beta^2 = 8\beta^2 \rightarrow 1 = 9\beta^2 \rightarrow \boxed{\beta = \frac{1}{3}}$$

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② OTRA FORMA:

$$E = \gamma(v_0)\, m_0c^2 = \frac{5}{4} m_0c^2 = m_0c^2 + K \rightarrow K = \frac{1}{4} m_0c^2$$

$$E_0 = \frac{9}{4} m_0c^2 = \gamma(v_f)\, m_0c^2 \rightarrow \gamma(v_f)\, m_0 = \frac{9}{4} m_0$$

$$\gamma(v_f)\, m_0\, v_f = \frac{5}{4} m_0 v_0$$

$\gamma(v_f) = (1 - 1/9)^{-1/2} = \frac{3}{2\sqrt{2}}$

$$v_f = \frac{5}{9} v_0 = \frac{5}{9} \times \frac{3}{5} c = \frac{1}{3} c$$

$$m_0 = \frac{9}{4} m_0 / \gamma(v_f) = \frac{9}{4} \frac{2\sqrt{2}}{3} m_0 = \frac{3}{\sqrt{2}} m_0$$

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① Quieren usar efecto Doppler $\qquad z = \left(\frac{1+v/c}{1-v/c}\right)^{1/2} - 1$

$$t = \frac{L_0}{\gamma(c-v)} = \frac{L_0}{c} \frac{(1-v^2/c^2)^{1/2}}{(1-v/c)} = t' \frac{(1+v/c)^{1/2}}{(1-v^2/c^2)^{1/2}}$$ ✓